



# **AS/A LEVEL GCE**

**Examiners' report** 

# MATHEMATICS

3890-3892, 7890-7892

# 4724/01 Summer 2018 series

Version 1

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# Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

# Paper 4274/01 series overview

4724 is the fourth compulsory component of GCE Mathematics. The examination is of duration 1 hour and 30 minutes, and carries a total of 72 marks. It contributes 16<sup>2</sup>/<sub>3</sub>% of GCE assessment. Knowledge of the specification content of components C1 (4721), C2 (4722) and C3 (4723) is assumed and candidates may be required to demonstrate such knowledge in answering questions in this assessment.

# Question 1(i)

1 (i) Express  $\frac{3}{2x+1} - \frac{2}{x+1}$  as a single algebraic fraction in its simplest form. [2]

# Question 1(ii)

(ii) Hence express  $\left(\frac{3}{2x+1} - \frac{2}{x+1}\right)\left(\frac{6x+3}{x^2+x-2}\right)$  as a single algebraic fraction in its lowest terms. [4]

Candidates who did well in this question combined the fractions in part (i) successfully and then factorised the given fraction in part (ii) before cancelling out.

Those who did less well sometimes made an incorrect simplification of their fraction in part (i). Sometimes they multiplied the two fractions together in part (ii) without factorising the numerator and/or the denominator.



This candidate correctly combined the fractions in part (i), but then lost the A mark because of an incorrect simplification.

In part (ii) the numerator was factorised and the fractions were multiplied, earning B1M1 out of the three possible remaining marks.

# Question 2(i)

2 The equations of two lines are

$$\mathbf{r} = \begin{pmatrix} 2\\ -3\\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1\\ 4\\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} -4\\ 6\\ -2 \end{pmatrix} + \mu \begin{pmatrix} 4\\ -11\\ 3 \end{pmatrix}.$$

(i) Explain why these lines are not parallel.

#### Question 2(ii)

(ii) Determine whether these lines are skew or whether they intersect.

Candidates who did well in part (i) wrote down the direction vectors explicitly and explained that one was not a scalar multiple of the other. Candidates who did less well identified the reason the lines are not parallel, but did not specify which vectors represented the directions of the lines.

Candidates who did well in part (ii) demonstrated the inconsistency of the three equations using the values of  $\lambda$  and  $\mu$  they had found, and explained that this demonstrated that the lines are skew.

Candidates who did less well made a slip in writing down the equations or a slip when solving them. Some lost the final mark having a last line that reads (for example) '21 = -27 so skew lines'. Candidates should write  $21 \neq -27$  to show that the lines are skew.

[1]

[4]

2(i) Jolloom Genous

(2(ii)-S - 4+4×4 Ξ 2+ 46 ..... L M1 38 M1 84 ... lines 84200 are Elen.

This candidate had the right idea in part (i), but the response was too vague to be credited as the direction vectors were not specifically identified.

In part (ii) a sign error in the third equation resulted in the loss of an A mark for finding a correct value of one of the parameters. The second M mark was earned for substituting in the unused equation, but the A mark was no longer available. It is worth noting that  $\frac{84}{23} = \frac{-72}{23}$  would have been insufficient for the final A mark; the symbol  $\neq$  would be required (or an equivalent acknowledgement of inconsistency) accompanying the conclusion of skewed lines.

# Question 3(i)

3 (i) Find the quotient and the remainder when  $2x^3 - 3x^2 + 3x + 2$  is divided by  $x^2 - 2x + 1$ . [3]

Candidates who did well in this question equated coefficients or used long division to find the quotient and the remainder.

Candidates who did less well often did know what to do, but made algebraic slips or spoiled their answer through wrong labelling.

#### Exemplar 3



This candidate found both the quotient and the remainder correctly, but went on to spoil the answer by identifying the remainder as a fraction.

### Question 3(ii)

(ii) Hence show that, if x is small,

$$\frac{2x^3 - 3x^2 + 3x + 2}{1 - 2x + x^2} \approx a + bx + cx^2$$

where a, b and c are constants to be determined.

Candidates who did well in this question generally used their answer to part (i) and then used the binomial expansion on the denominator.

Candidates who did less well truncated the expansion after two terms, or made a sign error when considering the connection between  $(1 - x)^{-2}$  and  $(x - 1)^{-2}$  instead of spotting the equivalence.

Exemplar 4



This candidate adopted the expected approach, but made a sign error and so lost the final mark.

[4]

#### Question 4(i)

4 The parametric equations of a curve are

 $x = 2 \tan 2t$  and  $y = 1 + \tan t$ , where  $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$ .

(i) Show that a cartesian equation of the curve is  $4y + xy^2 - 2xy = 4$ .

Candidates who did well in this question used the double angle formula for tan 2*t* and substituted for tan *t* in terms of *y*.

Candidates who did less well often tried to verify the result by substitution for x and y in the equation and were rarely successful. Some made sign errors in working when adopting the expected approach, which were then often ignored by the candidates in order to arrive at the provided answer.

#### Exemplar 5



This candidate adopted the expected approach, but made a sign error and so lost the final mark.

[4]

# Question 4(ii)

(ii) Find 
$$\frac{dy}{dx}$$
 in terms of x and y.

Candidates who did well in this question differentiated the given equation implicitly and rearranged the result to obtain an expression in terms of *x* and *y*.

Candidates who did less well made sign or coefficient errors with differentiation, or differentiated in parametric form and went astray when trying to eliminate *t*.

#### Exemplar 6

4(ii) 
$$x = \frac{1}{2} + \frac{1}$$

This candidate probably used the formula booklet to differentiate  $\tan^{-1} x$ . Their  $\frac{dt}{dx}$  contained an error; nevertheless M1 was earned for attempting  $\frac{dy}{dx}$  correctly. Note however that the mark scheme allowed B1 only when  $\frac{dt}{dy}$  was inverted.

### Question 4(iii)

(iii) Find the gradient of the curve at the point where the curve intersects the y-axis.

Candidates who did well in this question identified the *y*-intercept correctly and substituted in their expression for  $\frac{dy}{dx}$ .

Candidates who did less well made errors in substituting the value of *y* or *t* if the parametric form was used.

### Question 5(i)

5 (i) Given that  $u = \ln(\cos x)$ , find  $\frac{du}{dx}$ .

#### Question 5(ii)

(ii) Hence find  $\int 2\cos 2x \ln(\cos x) dx$ .

Candidates who did well in this question used the chain rule successfully in part (i) and went on to use integration by parts in part (ii). They recognised that use of double angle formulae was necessary to complete the solution.

Candidates who did less well either made an error in part (i) (usually the omission of multiplying by sin*x*), or were unable to make progress after the first application of integration by parts.

[2]

[5]

[2]

| .5(ii) | f2.cos2x.cn(osx)dx   |
|--------|--|
|        | $\frac{U = Un(cosx)}{dv} = \frac{dv}{dx} = \frac{2cos2x}{dx}$  |
|        | dv = -tanx dx  |
|        | dx $y = sin 2x$  |
|        |  |
|        | sintrin(cosx) - (-tonxsintx dx   |
|        |  |
|        |  |
|        | -150000 3400000000000000000000000000000000   |
|        | <u>(1963)</u>  |
|        | U = -tand V = sindx  |
|        | $u' = -\sec^2 x  V = -\cos^2 x$  |
|        |  |
|        | $\frac{\tan(\cos 2x)}{2} = \int \frac{1}{(\cos 2x)} \frac{1}{2} \frac{1}{2} \frac{1}{1} \frac{1}{1} \frac{1}{2} \frac{1}{1} \frac{1}{2} \frac{1}{1} \frac{1}{2} \frac{1}{1} \frac{1}{$ |
|        | 2 () COSD 2  |
|        |  |
|        | $= \frac{\cos 2x}{\cos^2 x - 1}$   |
|        | $2\cos^2x$   |
|        | $-\frac{2\cos^{2}x-1}{2\cos^{2}x} = \frac{1-\frac{1}{2}}{2\cos^{2}x}$  |
|        | $2\cos^2 x$ $2\cos^2 x$ $x - \frac{1}{2} \tan x$   |
|        | $\mathcal{L} = 12 \text{ Log ML}$  |
|        | Sinlahass) = tonacosz = x + 1/2 tona   |
|        | 2  |

This candidate chose the slightly harder route of using integration by parts a second time before using the double angle formula for cos 2*x*. Unfortunately a sign error resulted in the loss of the final mark.

# Question 6

6 Use the substitution 
$$u = 1 + 2\sqrt{x}$$
 to find  $\int \frac{\sqrt{x}}{1 + 2\sqrt{x}} dx$ . [7]

Candidates who did well in this question set out their working clearly and simplified the integrand in terms of u before integrating. They remembered to substitute back in terms of x at the end and to include the constant of integration.

Candidates who did less well usually made errors with manipulating the algebra before integration.

#### Exemplar 8

| 6 | $U = 142\sqrt{2}$ $2x^{1/2}$ $du$ $du$  |
|---|---|
|   |   |
|   | $dv = 2c^{2} - 5 \cdot dx = 5c^{2}$<br>dx = bc du   |
|   | 1 1+2JZ dz U= 1+2JZ   |
|   |   |
| : | U-1=2JZ   |
|   | $\int \frac{\sqrt{2x}}{1+2\sqrt{2x}} \frac{\sqrt{2x}}{\sqrt{2x}} \frac{\sqrt{2}}{\sqrt{2x}} \frac{\sqrt{2}}{2x$ |
|   |   |
|   | $\int \frac{dc}{1+2Jx} dc \qquad \frac{1}{(1-1)}$   |
|   |   |
|   | p or do y   |
|   |   |
|   | $\int \frac{\left( \left( U - I \right)^2}{4} dv = \frac{4}{2} dv$  |
|   |   |
|   | ta on   |
|   | $\int \underbrace{U(U-1)^2}_{4} dv \times$  |
|   |   |
|   | $\int \frac{1}{\sqrt{(v^2 - 2v + 1)}} dv$   |
|   |   |
| · | $\frac{1}{4}$ $U^{3} - 2U^{2} + U dw$   |
|   |   |
|   | $\frac{1}{4} \left[ \frac{1}{4} \left[ \frac{1}{4} \left[ \frac{1}{2} + \frac{1}{3} \left[ \frac{1}{2} + \frac{1}{2} \right]^2 \right] \rightarrow \frac{1}{4} \left[ \frac{1}{4} \left[ \frac{1}{4} \left[ \frac{1}{2} + \frac{1}{3} \left[ \frac{1}{2} + \frac{1}{2} \right]^2 \right] \right] \rightarrow \frac{1}{4} \left[ \frac{1}{4} \left[ \frac{1}{4} \left[ \frac{1}{4} + \frac{1}{3} \left[ \frac{1}{4} + \frac{1}{3} \right]^2 \right] \right]$   |
|   | t t4(1+2JZ)"-3(1+2JZ)+2(1+2JZ)"]  |
|   |   |

This candidate understood the principle of integration by substitution, but was unable to complete the algebraic manipulation of dividing through by *u* successfully.

# Question 7(i)

7 (i) Express 
$$\frac{12-6x}{(1+x)(1-2x)^2}$$
 in partial fractions.

Candidates who did well in this question used substitution to find two of the constants and then equated coefficients to find the third.

Candidates who did less well made slips in arithmetic when calculating the constants or equated coefficients to find all three constants and made algebraic slips.

### Question 7(ii)

(ii) Find 
$$\int \frac{12-6x}{(1+x)(1-2x)^2} dx$$
. Hence evaluate  $\int_1^2 \frac{12-6x}{(1+x)(1-2x)^2} dx$ , giving your answer in the form  $A - \ln B$ , where A and B are integers to be determined. [5]

Candidates who did well in this question used their fractions from part (i) and recognised the two forms of integral. They clearly showed their substitution to evaluate the integral.

Candidates who did less well often thought all three fractions integrated to logarithmic form, or differentiated the quadratic term. Some candidates neglected to use modulus signs to evaluate the integral and were unable to deal with, for example,  $\ln |-3|$ .

[5]

7(ii) . ohech 6 (1-2x)2 dx 1-2x 2 4 6(1-20) -2at1 )4X 21  $\chi + 1$ 2-2 -1X-2 ጌ × 2 2 2 21-2-24(-1) 2ん(3) 先ま +3(-1) 263 +3 -212 3 ЬΙ 2 +21 -212 12 2+ んち 9

This candidate integrated successfully, but couldn't deal with the logarithms of negative numbers.

# Question 8(i)

- 8 *A* is the point (1, 3, 1), *B* is the point (3, 2, 4) and *P* is the point (15, 4, 6). The point *Q* is on the line through *A* and *B* such that angle  $AQP = 90^{\circ}$ .
  - (i) Write down a vector equation of the line through A and B.

Candidates who did well in this question wrote down a correct equation of the line, but candidates who did less well often wrote down an expression rather than equation, or made a slip in finding the direction vector.

#### Question 8(ii)

(ii) Find the coordinates of Q.

[5]

[2]

Candidates who did well in this question used their answer to part (i) to identify vector *OQ*. They found the scalar product of vectors *PQ* and *AQ* and solved for the parameter, before going on to find *Q*. Other approaches were successful only very rarely.

#### Exemplar 10

| 8(ii) | (1+2)   |
|-------|---|
|       | Q: (3-M)  |
|       | (1+3m) V  |
|       | - Cali -14+2m   |
|       | QP (-1)-M<br>(-5+3M)  |
|       |   |
|       | $2(-14+2\mu) - 1(-1-\mu) + 3(-5+3\mu)$                            |
|       | _=0   |
|       |   |
|       | -28+2m +1+m -15+9m=0  |
|       |   |
|       | $12\mu = 42$  |
|       |   |
|       | $M = \frac{1}{2}$   |
| -     | · · · · · · · · · · · · · · · · · · ·                             |
|       | 8   |
|       | $Q_{12} \left( \frac{1}{12} \right) \left( \frac{-1}{12} \right)$ |
|       |   |
|       | Z/  |
| :     | $Q: (8, -\frac{1}{2}, \frac{2}{2})$                               |
|       | Q- (0, 2, 2)  |

This candidate adopted the correct approach, but an arithmetic slip cost the last two A marks.

(iii) Find the area of triangle AQP.

Candidates who did well in this question used  $\frac{1}{2} \times AQ \times PQ$  to find the requested area. In order to score well a correct answer to part (ii) was needed, so those who did less well worked with incorrect coordinates for *Q*.

#### Question 9(i)

9 When a container is partially filled with liquid to a depth of x centimetres, the volume  $V \text{ cm}^3$  of liquid in the container is given by the formula

$$V = (x+1)^3 - 1.$$

Initially the container is empty. Liquid is poured into the container so that the rate at which V increases is directly proportional to  $e^{-t}$ , where t is the time in seconds since the addition of liquid began. When t = 2, the rate at which V is increasing is  $10 \text{ cm}^3 \text{ s}^{-1}$ .

(i) Show that  $V = 10e^2(1 - e^{-t})$ . Hence find how long it takes for the depth of liquid in the container to reach 3 cm. [8]

Candidates who did well in this question made a correct initial set up and showed all their reasoning clearly for the first five marks. In the second part of the question they found V correctly and solved the equation for *t*.

Those who did less well generally gave an incomplete argument for 'show that' or decimalised *k* in the first part. In the second part, errors were often through a slip in rearranging the equation or rounding their final answer incorrectly.

Others started with the incorrect premise that  $V = ke^{-t}$  and in the second part worked with V = 3 instead of 63.



This candidate started correctly, but made the (quite common) error of integrating  $\int_{k}^{1} dV$  to ln*k*. In the second part of this question, the marks were only available to those who worked with *V* and not *x*, so this candidate did not score after the initial award of B1M1.

# Question 9(ii)

(ii) Find the value that the depth of liquid approaches as t increases, giving your answer correct to 3 significant figures. [3]

Candidates who did well in this question considered the behaviour of  $e^{-t}$  for large *t* and went on to solve the equation for *x*. Those who did less well made slips in rearranging the equation or gave the final answer to a different level of precision to that requested.

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