RECOGNISING ACHIEVEMENT

GCE

Statistics (MEI)

Advanced Subsidiary GCE **AS H132**

Mark Scheme for the Units

June 2009

H132/MS/R/09

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

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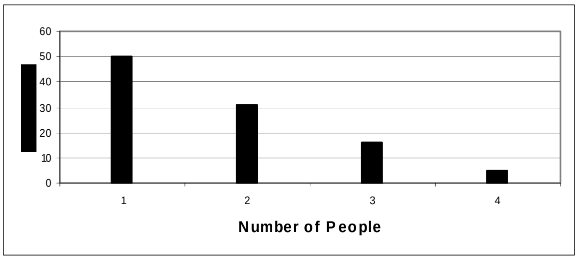
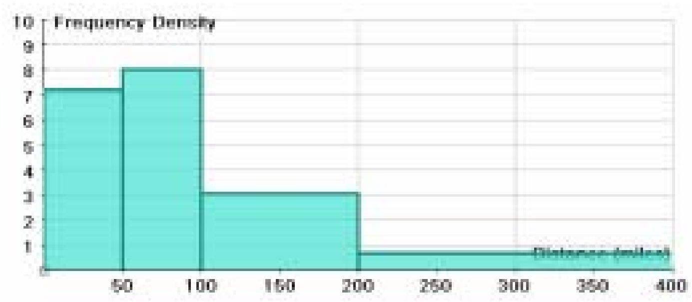
CONTENTS

Advanced Subsidiary GCE Statistics (H132)

MARK SCHEMES FOR THE UNITS

G241 Statistics 1

Q1 (i)	Median = 2 Mode = 1	B1 CAO B1 CAO	**2**
(ii)		S1 labelled linear scales on both axes H1 heights	**2**
(iii)	Positive	B1	**1**
		TOTAL	**5**
Q2 (i)	$\binom{25}{5}$ different teams = 53130	M1 for $\binom{25}{5}$ A1 CAO	**2**
(ii)	$\binom{14}{3} \times \binom{11}{2} = 364 \times 55 = 20020$	M1 for either combination M1 for product of both A1 CAO	**3**
		TOTAL	**5**
Q3 (i)	Mean $= \frac{126}{12} = 10.5$ Sxx $= 1582 - \frac{126^2}{12} = 259$ $s = \sqrt{\frac{259}{11}} = 4.85$	B1 for mean M1 for attempt at *Sxx* A1 CAO	**3**
(ii)	New mean = 500 + 100 × 10.5 = 1550 New s = 100 × 4.85 = 485	B1 ANSWER GIVEN M1A1FT	**3**
(iii)	On average Marlene sells more cars than Dwayne. Marlene has less variation in monthly sales than Dwayne.	E1 E1FT	**2**
		TOTAL	**8**

Q4 (i)	E(X) = 25 because the distribution is symmetrical. Allow correct calculation of Σrp	E1 <u>ANSWER GIVEN</u>	**1**
(ii)	$E(X^2) = 10^2 \times 0.2 + 20^2 \times 0.3 + 30^2 \times 0.3 + 40^2 \times 0.2 = 730$ $Var(X) = 730 - 25^2 = 105$	M1 for $\Sigma r^2 p$ (at least 3 terms correct) M1dep for $- 25^2$ A1 CAO	**3**
		TOTAL	**4**
Q5 (i)	Distance / freq / width / f dens 0- / 360 / 50 / 7.200 50- / 400 / 50 / 8.000 100- / 307 / 100 / 3.070 200-400 / 133 / 200 / 0.665  Distance (miles)	M1 for fds A1 CAO Accept any suitable unit for fd such as eg freq per 50 miles. L1 linear scales on both axes and label W1 width of bars H1 height of bars	**5**
(ii)	Median = 600th distance Estimate = $50 + {}^{240}/_{400} \times 50 = 50 + 30 = 80$	B1 for 600^{th} M1 for attempt to interpolate A1 CAO	**3**
		TOTAL	**8**
Q6 (i)	(*A*) P(at most one) $= \frac{83}{100} = 0.83$ (*B*) P(exactly two) $= \frac{10+2+1}{100} = \frac{13}{100} = 0.13$	B1 aef M1 for (10+2+1)/100 A1 aef	**1** **2**
(ii)	P(all at least one) $= \frac{53}{100} \times \frac{52}{99} \times \frac{51}{98} = \frac{140556}{970200} = 0.145$	M1 for $\frac{53}{100} \times$ M1*dep* for product of next 2 correct fractions A1 CAO	**3**
		TOTAL	**6**

Q7 (i)	$a = 0.8$, $b = 0.85$, $c = 0.9$.	B1 for any one B1 for the other two	**2**
(ii)	P(Not delayed) = $0.8 \times 0.85 \times 0.9 = 0.612$ P(Delayed) = $1 - 0.8 \times 0.85 \times 0.9 = 1 - 0.612 = 0.388$	M1 for product A1 CAO M1 for 1 – P(delayed) A1FT	**4**
(iii)	P(just one problem) $= 0.2\times0.85\times0.9 + 0.8\times0.15\times0.9 + 0.8\times0.85\times0.1$ $= 0.153 + 0.108 + 0.068 = 0.329$	B1 one product correct M1 three products M1 sum of 3 products A1 CAO	**4**
(iv)	P(Just one problem | delay) $= \dfrac{\text{P(Just one problem and delay)}}{\text{P(Delay)}} = \dfrac{0.329}{0.388} = 0.848$	M1 for numerator M1 for denominator A1FT	**3**
(v)	P(Delayed | No technical problems) *Either* $= 0.15 + 0.85 \times 0.1 = 0.235$ *Or* $= 1 - 0.9 \times 0.85 = 1 - 0.765 = 0.235$ *Or* $= 0.15 \times 0.1 + 0.15 \times 0.9 + 0.85 \times 0.1 = 0.235$ *Or (using conditional probability formula)* $\dfrac{\text{P(Delayed and no technical problems)}}{\text{P(No technical problems)}}$ $= \dfrac{0.8\times0.15\times0.1 + 0.8\times0.15\times0.9 + 0.8\times0.85\times0.1}{0.8}$ $= \dfrac{0.188}{0.8} = 0.235$	M1 for 0.15 + M1 for second term A1CAO M1 for product M1 for 1 – product A1CAO M1 for all 3 products M1 for sum of all 3 products A1CAO M1 for numerator M1 for denominator A1CAO	**3**
(vi)	Expected number = $110 \times 0.388 = 42.7$	M1 for product A1FT	**2**
		TOTAL	**18**

Q8 (i)	X ~ B(15, 0.2) (***A***) $P(\boldsymbol{X} = 3) = \binom{15}{3} \times 0.2^3 \times 0.8^{12} = 0.2501$ OR from tables $0.6482 - 0.3980 = 0.2502$	M1 $0.2^3 \times 0.8^{12}$ M1 $\binom{15}{3} \times p^3 q^{12}$ A1 CAO OR: M2 for 0.6482 – 0.3980 A1 CAO	**3**
	(***B***) $P(\boldsymbol{X} \geq 3) = 1 - 0.3980 = 0.6020$	M1 $P(X \leq 2)$ M1 $1 - P(X \leq 2)$ A1 CAO	**3**
	(***C***) $E(X) = np = 15 \times 0.2 = 3.0$	M1 for product A1 CAO	**2**
(ii)	(*A*) Let *p* = probability of a randomly selected child eating at least 5 a day H_0: $p = 0.2$ H_1: $p > 0.2$ (*B*) H_1 has this form as the proportion who eat at least 5 a day is expected to <u>increase</u>.	B1 for definition of *p* in context B1 for H_0 B1 for H_1 E1	**4**
(iii)	Let $X \sim B(15, 0.2)$ $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.8358 = 0.1642 > 10\%$ $P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.9389 = 0.0611 < 10\%$ So critical region is {6,7,8,9,10,11,12,13,14,15} 7 lies in the critical region, so we reject null hypothesis and we conclude that there is evidence to suggest that the proportion who eat at least five a day has increased.	B1 for 0.1642 B1 for 0.0611 M1 for at least one comparison with 10% A1 CAO for critical region *dep* on M1 and at least one B1 M1 *dep* for comparison A1 *dep* for decision and conclusion **in context**	**6**
		TOTAL	**18**

G242 Statistics 2

Q1			
(i)	H_0: there is no association between area and quality H_1: there is an association between area and quality	B1	**1**
(ii)	Expected frequencies | | Area A | Area B | Area C | | Excellent | 26.04 | 27.72 | 30.24 | | Good | 39.68 | **42.24** | **46.08** | | Satisfactory | 27.28 | **29.04** | **31.68** | Contributions to X^2 | | Area A | Area B | Area C | | Excellent | 3.8096 | 3.4083 | 0.0019 | | Good | 0.3413 | **1.4256** | **0.3613** | | Satisfactory | 1.4457 | **0.1323** | **0.5891** | $X^2 = 11.5150$ 4 degrees of freedom Critical value for 5% significance level is 9.488 As 11.5150 > 9.488 the result is significant There is evidence of an association between the area where grapes grow and their quality.	M1 A1 M1 M1 A1 CAO B1 B1 M1A1 A1	**10**
(iii)	Large contribution of 3.8096 shows there were more excellent quality grapes than expected in Area A. Large contribution of 3.4083 shows there were fewer excellent grapes than expected in Area B. Low contributions for Area C show that grape quality was more or less as expected.	E1 E1 E1	**3**

Q2			
(i)(*A*)	$P(X < 25) = P(Z < \frac{25 - 25.2}{0.1}) = P(Z < -2)$ $1 - \Phi(2) = 1 - 0.9772 = 0.0228$	M1 standardising M1 correct tail A1	**3**
(i)(*B*)	$1 - p^5$ where p = 0.9772 = 0.1089	M1 M1 A1	**3**
(ii)(*A*)	$\dfrac{33544 - \frac{1295^2}{50}}{49} = 0.07143$ (AG)	M1 A1	**2**
(ii)(*B*)	$25.9 \pm 1.96 \times \dfrac{\sqrt{0.07143}}{\sqrt{50}}$ (25.83, 25.97)	M1 centred on 25.9 M1 structure (S.E.) B1 (1.96) A1 CAO	**4**
(ii)(*C*)	This interval does not contain 25 kg. This suggests that the mean amount of coal delivered could be greater than 25 kg.	E1 E1 (mean) E1 (greater) allow sensible alternatives	**3**

Q3			
(i)	H_0: population median = 23 H_1: population median < 23 Actual differences -9 +10 -11 -12 -17 -7 +4 -5 +6 -15 -14 -3 Associated ranks 6 7 8 9 12 5 2 3 4 11 10 1 $T^- = 6 + 8 + 9 + 12 + 5 + 3 + 11 + 10 + 1 = 65$ $T^+ = 7 + 2 + 4 = 13$ $\therefore T = 13$ From tables – at the 5% level of significance in a one-tailed Wilcoxon signed rank test, the critical value of T is 17 $13 < 17$ $\therefore$ the result is significant The evidence suggests a decrease in the median waiting time with the new appointments system	B1 B1 B1 M1 A1 B1 B1 B1 B1 M1 A1 E1	**12**
(ii)	Sample small and population variance unknown. t test	B1 B1	**2**

Q4			
(i)*A*	Sample mean = 312÷120 (=2.6 AG)	M1 A1	**2**
(i)*B*	Variance = $1.880^2 = 3.5344$ Comparison of sample mean and variance with conclusion about suitability of Poisson model.	B1 E1 dep	**2**
(i)*C*	The observed frequencies would tail-off more in a Poisson model (or the observed frequency for $x = 5$ is too high) – hence a Poisson model may not be suitable.	E1 E1	**2**
(ii)	H_0: The Poisson model is suitable Missing expected frequencies are 8.832 ($x = 5$), and 5.88 ($x \geq 6$) Missing contributions are 1.4546 ($x = 4$) and 1.9670 ($x = 5$) $X^2 = 19.32(76)$ There are 7 – 1 – 1 = 5 degrees of freedom. At the 5% significance level the critical value is 11.07 The result is significant Evidence suggests that the Poisson model is inappropriate.	 M1 A1 A1 M1A1 A1 B1 B1 B1 B1	**10**

Q5			
(i)	Estimate of population mean = 273 Estimate of population variance = 18.222	B1 B1	**2**
(ii)	$H_0 : \mu = 268$ & $H_1 : \mu > 268$ Where μ represents the population mean ball striking distance using clubs made from the new alloy. $t = \dfrac{273 - 268}{SD/\sqrt{10}}$ = 3.704 (4s.f.) 9 degrees of freedom At 5% level, critical value of *t* is 1.833 3.704 > 1.833 so the result is significant. Evidence suggests the clubs made with the new alloy have a mean striking distance greater than 268 yards.	B1 (both) B1 M1 A1 B1 B1 M1A1 A1	**9**
(iii)	Any suitable comments – e.g. wind speed/length of grass/type of ball will affect the distance travelled.	E1 E1	**2**

G243 Statistics 3

Q1			
(i)	H_0: μ=0 H_1: μ>0 (or μ<0 if dominant - non dominant) where μ represents the population mean difference	B1 B1 B1	Condone absence of "population" if correct notation "μ" has been used, but do NOT accept $\overline{X}=0$ (or $\overline{X}=\overline{Y}$) or similar unless $\overline{X}$ and $\overline{Y}$ are clearly and explicitly stated to be <u>population</u> means. Accept hypothesis explained in words, provided "population" appears.
	non / dom / diff: 485 / 336 / 149 356 / 381 / -25 450 / 348 / 102 402 / 329 / 73 376 / 329 / 47 409 / 346 / 63 419 / 344 / 75 289 / 327 / -38 420 / 342 / 78 410 / 356 / 54 total 578 mean 57.8 sample SD 55.17	 M1 awrt A1 awrt A1	
	Test statistic is $t=\frac{\bar{d}-0}{s/\sqrt{n}}=\frac{57.8}{55.17/\sqrt{10}}=3.31$	M1 A1awrt	Follow-through incorrect value of test statistic
	Critical region t_9>1.833	M1 A1	fr t_9 (No follow-through from here if wrong) for 1.833 (No follow-through from here if wrong)
	Since 3.31>1.833 H_0 is rejected, there is sufficient evidence to suggest that reaction times for the dominant hand are faster, on average, than for the non-dominant hand.	A1 A1	
	Assume Normality; of (population of) differences	B1 B1	
(ii)	e.g. possible "consistent order" or learning effect which cannot be disentangled in the analysis and should therefore be randomised.	E2 (E1,E1)	
		16	

<table>
<tr><td>Q2</td><td></td><td></td><td></td></tr>
<tr><td>(i)</td><td>There may have been changes in products, or changes in the economy.
Any sensible reason.</td><td>B1</td><td></td></tr>
<tr><td>(ii)</td><td>
<table>
<tr><td></td><td></td><td></td><td>1951</td><td>1</td></tr>
<tr><td>2159</td><td>3</td><td></td><td>2077</td><td>2</td></tr>
<tr><td>2361</td><td>6</td><td></td><td>2193</td><td>4</td></tr>
<tr><td>2570</td><td>7</td><td></td><td>2286</td><td>5</td></tr>
<tr><td>2985</td><td>10</td><td></td><td>2780</td><td>8</td></tr>
<tr><td>3012</td><td>11</td><td></td><td>2983</td><td>9</td></tr>
<tr><td>5442</td><td>13</td><td></td><td>4912</td><td>12</td></tr>
<tr><td>5756</td><td>15</td><td></td><td>5629</td><td>14</td></tr>
<tr><td>5825</td><td>16</td><td></td><td></td><td></td></tr>
<tr><td>6023</td><td>17</td><td></td><td></td><td></td></tr>
<tr><td>6078</td><td>18</td><td></td><td></td><td></td></tr>
<tr><td></td><td>116</td><td></td><td></td><td>55</td></tr>
</table>
H_0: Medians for both groups are the same
H_1: Median for the NW group > median for the NE group

Use of joint ranking
All ranks correct
Use of rank sums
Smaller rank sum 55
Use of $W_{8,10}$
Critical point 56
$55 \le 56$, so reject H_0
Median for NW does seem to be greater,
so the advertising did seem to have an impact</td><td>B1

M1
A1
M1
A1
M1
A1
M1 A1
A1
E1</td><td></td></tr>
<tr><td>(iii)</td><td>Number the stores 01 to 30
Choose a random starting position in the table
Select 2 digit numbers (between 01 – 30) and the corresponding stores
Ignore repeats.</td><td>B1
B1
B1
B1</td><td></td></tr>
<tr><td></td><td>Alternative for (ii) using Mann-Whitney method:
<table>
<tr><td></td><td></td><td>0</td><td>1951</td></tr>
<tr><td>2159</td><td></td><td>0</td><td>2077</td></tr>
<tr><td>2361</td><td></td><td>1</td><td>2193</td></tr>
<tr><td>2570</td><td></td><td>1</td><td>2286</td></tr>
<tr><td>2985</td><td></td><td>3</td><td>2780</td></tr>
<tr><td>3012</td><td></td><td>3</td><td>2983</td></tr>
<tr><td>5442</td><td></td><td>5</td><td>4912</td></tr>
<tr><td>5756</td><td></td><td>6</td><td>5629</td></tr>
<tr><td>5825</td><td></td><td></td><td></td></tr>
<tr><td>6023</td><td></td><td></td><td></td></tr>
<tr><td>6078</td><td></td><td></td><td></td></tr>
<tr><td>statistic</td><td></td><td>19</td><td></td></tr>
<tr><td>MW CV(8,10) 5%</td><td>20</td><td></td><td></td></tr>
</table></td><td></td><td>Mark scheme as above.</td></tr>
<tr><td></td><td></td><td>16</td><td></td></tr>
</table>

Q3			
(i)	$\frac{\sum x}{n} = \frac{1252.9}{34}$ $\sqrt{\frac{1}{n-1}(\sum x^2 - \frac{(\sum x)^2}{n})}$ $= \sqrt{\frac{1}{33}(46172.85 - \frac{1252.9^2}{34})}$ $= \sqrt{0.105606} = 0.325 awrt$	B1 M1 A1	 S_{xx} or better
(ii)	Both samples are large or Central Limit Theorem.	B1	
(iii)	$\frac{\bar{x}_m - \bar{x}_f}{\sqrt{\frac{s_m^2}{n_m} + \frac{s_f^2}{n_f}}} = \frac{-0.05}{\sqrt{\frac{0.247^2}{36} + \frac{0.325^2}{34}}} = -0.7216$ $= -0.72$ awrt H_0: $\mu_m = \mu_f$ H_1: $\mu_m \neq \mu_f$ μ_m = population mean temperature for males μ_f = population mean temperature for females Critical value +/- 2.5758 ensuring that they compare like with like Since -0.7216 > -2.5758 there is no evidence to suggest rejecting H_0 We can accept that the two samples come from populations which have the same mean.	M1 M1 (m) M1 (f) M1 (all) A1CAO B1 B1 B1 B1 M1 E1	numerator denominator Structure Condone absence of "population" if correct notation "μ" has been used, but do NOT accept $\bar{X} = \bar{Y}$ or similar unless $\bar{X}$ and $\bar{Y}$ are clearly and explicitly stated to be population means. Accept hypothesis explained in words, provided "population" appears. No FT if critical value wrong
		15	

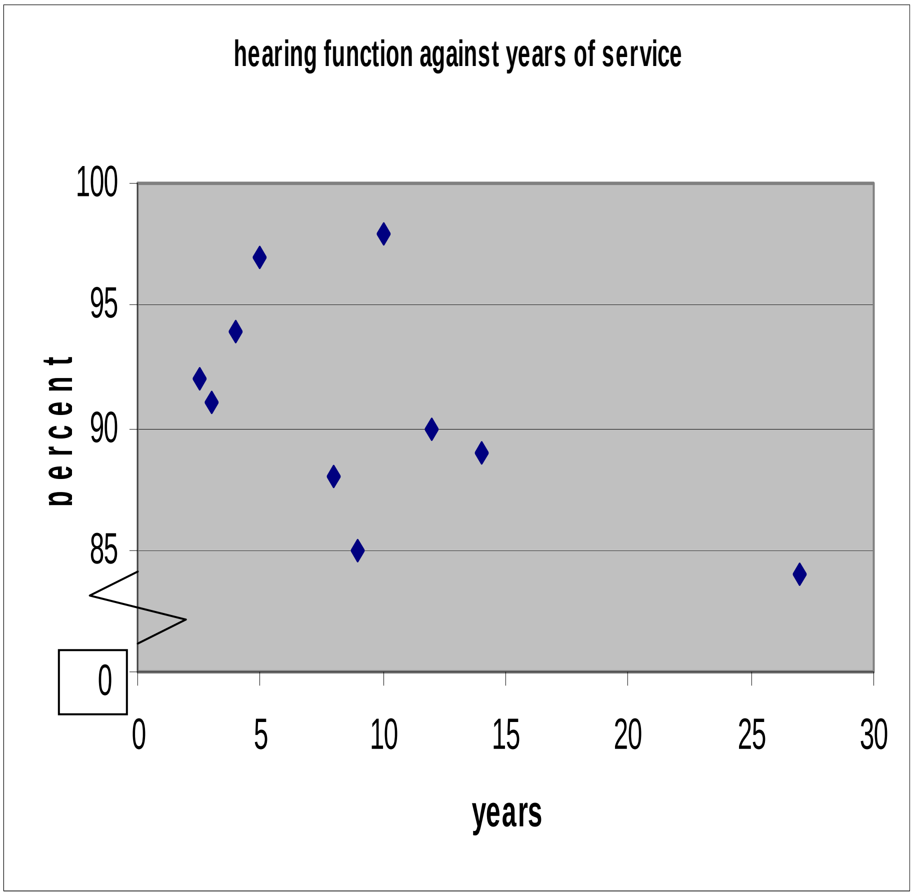
Q4			
(i)	See graph on page 9	G3	G1 for labelled axes G1 for "break" in vertical axis or full linear scale G1 for correct points
(ii)	H_0: ρ=0 H_1: ρ<0 where ρ is the population coefficient Critical Region < -0.5494 (one tail) Since -0.5711 < -0.5494 reject H_0 Hence this data would show that there is evidence of negative correlation between hearing function and years service.	 B1 B1 B1 M1 A1 E1	 No FT if critical value wrong
(iii)	The outlier (27 years) tends to suggest that these data are not bivariate Normal.	B1 B1	

(iv)

	A	B	C	D	E	F	G	H	I	J	
Years Service, x	12	3	27	5	2.5	4	8	9	10	14	
% Hearing function, y	90	91	84	97	92	94	88	85	98	89	
rank x	8	2	10	4	1	3	5	6	7	9	
rank y	5	6	1	9	7	8	3	2	10	4	
d^2	9	16	81	25	36	25	4	16	9	25	246

	An attempt at ranking Complete Σd^2 =246 R= 1 – (6 x 246) / (10 x 99) = 1 – 1.4909 = -0.4909 awrt	M1 A1 B1 A1	
(v)	H_0: there is no association between years of service and hearing function H_1: there is negative association Critical value -0.5636 < -0.4909 There is insufficient evidence to reject H_0 So there would appear to be no association between hearing function and years of service	 B1 B1 M1 E1	 No FT if critical value wrong No marks except initial B1 if $\lvert r_s \rvert > 1$
(vi)	Discussion of different outcomes, different hypotheses, different assumptions	E1, E1	
(vii)	Age, sex, discos, shooting, disease or other sensible (quantifiable) suggestion	E1, E1	
(viii)	Simple random sampling does not guarantee that all the sites are represented Stratified sampling	E1 E1	
		25	

Graph for question 4 (i)



Grade Thresholds

Advanced GCE Statistics MEI (H132)
June 2009 Examination Series

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
G241	Raw	72	60	53	46	40	34	0
	UMS	100	80	70	60	50	40	0
G242	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
G243	Raw	72	52	45	38	32	26	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
H132	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
H132	8.5	23.4	36.2	61.7	78.7	100	48

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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