ADVANCED GCE

MATHEMATICS (MEI) 4756

Further Methods for Advanced Mathematics (FP2)

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

- Scientific or graphical calculator

Friday 11 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (54 marks)

Answer all the questions

1 **(a)** **(i)** Given that $\mathrm{f}(t) = \arcsin t$, write down an expression for $\mathrm{f}'(t)$ and show that

$$\mathrm{f}''(t) = \frac{t}{(1-t^2)^{\frac{3}{2}}}.$$ **[3]**

(ii) Show that the Maclaurin expansion of the function $\arcsin\left(x + \frac{1}{2}\right)$ begins

$$\frac{\pi}{6} + \frac{2}{\sqrt{3}}x,$$

and find the term in x^2. **[5]**

(b) Sketch the curve with polar equation $r = \dfrac{\pi a}{\pi + \theta}$, where $a > 0$, for $0 \leqslant \theta < 2\pi$.

Find, in terms of a, the area of the region bounded by the part of the curve for which $0 \leqslant \theta \leqslant \pi$ and the lines $\theta = 0$ and $\theta = \pi$. **[6]**

(c) Find the exact value of the integral

$$\int_0^{\frac{3}{2}} \frac{1}{9 + 4x^2}\,\mathrm{d}x.$$ **[5]**

2 **(a)** Given that $z = \cos\theta + \mathrm{j}\sin\theta$, express $z^n + \dfrac{1}{z^n}$ and $z^n - \dfrac{1}{z^n}$ in simplified trigonometric form.

Hence find the constants A, B, C in the identity

$$\sin^5\theta \equiv A\sin\theta + B\sin 3\theta + C\sin 5\theta.$$ **[5]**

(b) **(i)** Find the 4th roots of $-9\mathrm{j}$ in the form $r\mathrm{e}^{\mathrm{j}\theta}$, where $r > 0$ and $0 < \theta < 2\pi$. Illustrate the roots on an Argand diagram. **[6]**

(ii) Let the points representing these roots, taken in order of increasing θ, be P, Q, R, S. The mid-points of the sides of PQRS represent the 4th roots of a complex number w. Find the modulus and argument of w. Mark the point representing w on your Argand diagram. **[5]**

3 **(a)** **(i)** A 3×3 matrix $\mathbf{M}$ has characteristic equation

$$2\lambda^3 + \lambda^2 - 13\lambda + 6 = 0.$$

Show that $\lambda = 2$ is an eigenvalue of $\mathbf{M}$. Find the other eigenvalues. **[4]**

(ii) An eigenvector corresponding to $\lambda = 2$ is $\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$.

Evaluate $\mathbf{M}\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$ and $\mathbf{M}^2\begin{pmatrix} 1 \\ -1 \\ \frac{1}{3} \end{pmatrix}$.

Solve the equation $\mathbf{M}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$. **[5]**

(iii) Find constants A, B, C such that

$$\mathbf{M}^4 = A\mathbf{M}^2 + B\mathbf{M} + C\mathbf{I}.$$ **[4]**

(b) A 2×2 matrix $\mathbf{N}$ has eigenvalues -1 and 2, with eigenvectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ respectively. Find $\mathbf{N}$. **[6]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Prove, using exponential functions, that

$$\sinh 2x = 2\sinh x \cosh x.$$

Differentiate this result to obtain a formula for $\cosh 2x$. **[4]**

(ii) Sketch the curve with equation $y = \cosh x - 1$.

The region bounded by this curve, the x-axis, and the line $x = 2$ is rotated through 2π radians about the x-axis. Find, correct to 3 decimal places, the volume generated. (You must show your working; numerical integration by calculator will receive no credit.) **[7]**

(iii) Show that the curve with equation

$$y = \cosh 2x + \sinh x$$

has exactly one stationary point.

Determine, in exact logarithmic form, the x-coordinate of the stationary point. **[7]**

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 In parts **(i)**, **(ii)**, **(iii)** of this question you are required to investigate curves with the equation

$$x^k + y^k = 1$$

for various positive values of k.

(i) Firstly consider cases in which k is a positive even integer.

(A) State the shape of the curve when $k = 2$.

(B) Sketch, on the same axes, the curves for $k = 2$ and $k = 4$.

(C) Describe the shape that the curve tends to as k becomes very large.

(D) State the range of possible values of x and y. **[6]**

(ii) Now consider cases in which k is a positive odd integer.

(A) Explain why x and y may take any value.

(B) State the shape of the curve when $k = 1$.

(C) Sketch the curve for $k = 3$. State the equation of the asymptote of this curve.

(D) Sketch the shape that the curve tends to as k becomes very large. **[6]**

(iii) Now let $k = \frac{1}{2}$.

Sketch the curve, indicating the range of possible values of x and y. **[2]**

(iv) Now consider the modified equation $|x|^k + |y|^k = 1$.

(A) Sketch the curve for $k = \frac{1}{2}$.

(B) Investigate the shape of the curve for $k = \dfrac{1}{n}$ as the positive integer n becomes very large.

[4]