

**ADVANCED GCE  
MATHEMATICS (MEI)**

Differential Equations

**4758/01**

Candidates answer on the answer booklet.

**OCR supplied materials:**

- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Wednesday 26 January 2011  
Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 (a) The displacement,  $x$  m, of a particle at time  $t$  seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 4e^t.$$

- (i) Find the general solution. [9]

The particle is initially at rest at the origin.

- (ii) Find the particular solution. [4]

- (b) The differential equation

$$\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

is to be solved.

- (i) Show that 1 is a root of the auxiliary equation and find the other two roots. Hence find the general solution. [5]

When  $x = 0$ ,  $y = 1$  and  $\frac{dy}{dx} = -4$ . As  $x \rightarrow \infty$ ,  $y \rightarrow 0$ .

- (ii) Find the particular solution subject to these conditions. [4]

- (iii) Find the value of  $x$  for which  $y = 0$ . [2]

- 2 (a) The differential equation

$$\frac{dy}{dx} + 2xy = e^{-x^2} \sin x$$

is to be solved subject to the condition  $x = 0$ ,  $y = 1$ .

- (i) Find the particular solution for  $y$  in terms of  $x$ . [9]

- (ii) Show that  $y > 0$  for all  $x$  and that  $y$  has a stationary point when  $x = 0$ . State the limiting value of  $y$  as  $|x| \rightarrow \infty$ . Hence draw a simple sketch graph of the solution, given that the stationary point at  $x = 0$  is a maximum. [6]

- (b) The differential equation

$$\frac{dy}{dx} + 2xy = 1$$

is to be solved numerically subject to the condition  $x = 0$ ,  $y = 1$ .

- (i) Use Euler's method with a step length of 0.1 to estimate  $y$  when  $x = 0.2$ . The algorithm is given by  $x_{r+1} = x_r + h$ ,  $y_{r+1} = y_r + hy'_r$ . [4]

- (ii) Use the integrating factor method and the approximation  $\int_0^{0.2} e^{x^2} dx \approx 0.2027$  to estimate  $y$  when  $x = 0.2$ . [5]

## 3 The differential equation

$$\frac{dy}{dx} + ky = \cos 3x,$$

where  $k$  is a constant, is to be solved.

(i) Find the complementary function. Hence find the general solution for  $y$  in terms of  $x$  and  $k$ . [8]

(ii) Find the particular solution subject to the condition that  $\frac{dy}{dx} = 1$  when  $x = 0$ . [4]

Now consider the differential equation

$$\frac{dy}{dx} + ky = 2e^{-kx}.$$

(iii) Find the general solution. [6]

Now consider the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 4e^{2x}.$$

(iv) Using your answer to part (iii), or otherwise, solve this differential equation subject to the conditions that  $y = 0$  and  $\frac{dy}{dx} = 1$  when  $x = 0$ . [6]

4 The populations of foxes,  $x$ , and rabbits,  $y$ , on an island at time  $t$  are modelled by the simultaneous differential equations

$$\frac{dx}{dt} = 0.1x + 0.1y,$$

$$\frac{dy}{dt} = -0.2x + 0.3y.$$

(i) Show that  $\frac{d^2x}{dt^2} - 0.4\frac{dx}{dt} + 0.05x = 0$ . [5]

(ii) Find the general solution for  $x$ . [4]

(iii) Find the corresponding general solution for  $y$ . [4]

Initially there are  $x_0$  foxes and  $y_0$  rabbits.

(iv) Find the particular solutions. [4]

(v) In the case  $y_0 = 10x_0$ , find the time at which the model predicts the rabbits will die out. Determine whether the model predicts the foxes die out before the rabbits. [7]

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.**



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