

**ADVANCED GCE**

**MATHEMATICS (MEI)**

Methods for Advanced Mathematics (C3)

**4753/01**

**QUESTION PAPER**

Candidates answer on the printed answer book.

**OCR supplied materials:**

- Printed answer book 4753/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Monday 20 June 2011**

**Morning**

**Duration:** 1 hour 30 minutes

**MODIFIED LANGUAGE**

**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER / INVIGILATOR**

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

## Section A (36 marks)

- 1 Solve the equation  $|2x - 1| = |x|$ . [4]
- 2 Given that  $f(x) = 2 \ln x$  and  $g(x) = e^x$ , find the composite function  $gf(x)$ . Express your answer as simply as possible. [3]
- 3 (i) Differentiate  $\frac{\ln x}{x^2}$ , simplifying your answer. [4]
- (ii) Using integration by parts, show that  $\int \frac{\ln x}{x^2} dx = -\frac{1}{x}(1 + \ln x) + c$ . [4]
- 4 The height  $h$  metres of a tree after  $t$  years is modelled by the equation
- $$h = a - be^{-kt},$$
- where  $a$ ,  $b$  and  $k$  are positive constants.
- (i) Given that the initial height of the tree is 0.5 metres and that the long-term height is 10.5 metres, find the values of  $a$  and  $b$ . [3]
- (ii) Given also that the tree grows to a height of 6 metres in 8 years, find the value of  $k$ . Give your answer correct to 2 decimal places. [3]
- 5 Given that  $y = x^2\sqrt{1+4x}$ , show that  $\frac{dy}{dx} = \frac{2x(5x+1)}{\sqrt{1+4x}}$ . [5]
- 6 A curve is defined by the equation  $\sin 2x + \cos y = \sqrt{3}$ .
- (i) Verify that the point  $P\left(\frac{1}{6}\pi, \frac{1}{6}\pi\right)$  lies on the curve. [1]
- (ii) Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .
- Hence find the gradient of the curve at the point  $P$ . [5]
- 7 (i) Multiply out  $(3^n + 1)(3^n - 1)$ . [1]
- (ii) Hence prove that  $3^{2n} - 1$  is divisible by 8 if  $n$  is a positive integer. [3]

## Section B (36 marks)

8

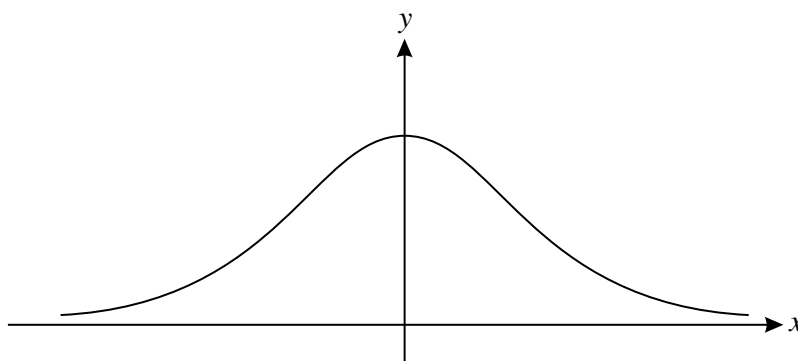


Fig. 8

Fig. 8 shows the curve  $y = f(x)$ , where  $f(x) = \frac{1}{e^x + e^{-x} + 2}$ .

- (i) Show algebraically that  $f(x)$  is an even function, and state how this property relates to the curve  $y = f(x)$ . [3]
- (ii) Find  $f'(x)$ . [3]
- (iii) Show that  $f(x) = \frac{e^x}{(e^x + 1)^2}$ . [2]
- (iv) Hence, using the substitution  $u = e^x + 1$ , or otherwise, find the exact area enclosed by the curve  $y = f(x)$ , the  $x$ -axis, and the lines  $x = 0$  and  $x = 1$ . [5]
- (v) Show that there is only one point of intersection of the curves  $y = f(x)$  and  $y = \frac{1}{4}e^x$ , and find its coordinates. [5]

[Question 9 is printed overleaf.]

- 9 Fig. 9 shows the curve  $y = f(x)$ . The endpoints of the curve are  $P(-\pi, 1)$  and  $Q(\pi, 3)$ , and  $f(x) = a + \sin bx$ , where  $a$  and  $b$  are constants.

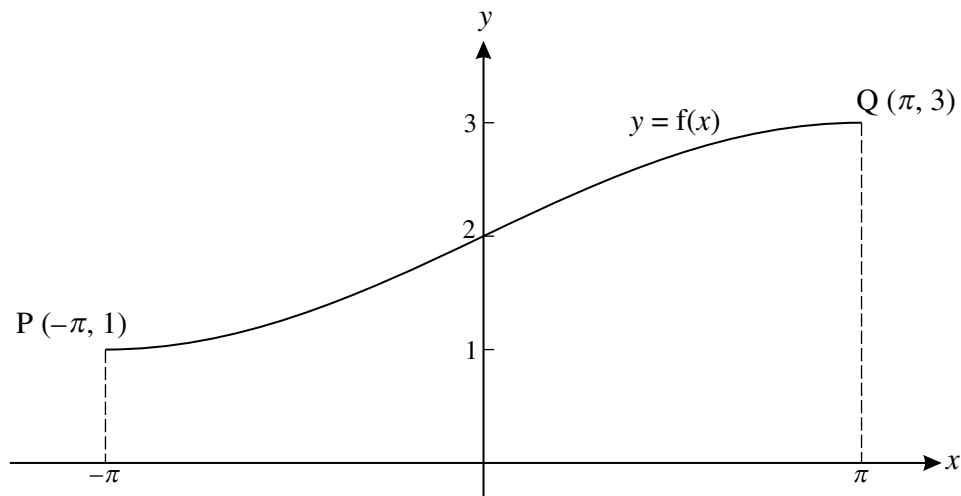


Fig. 9

- (i) Using Fig. 9, show that  $a = 2$  and  $b = \frac{1}{2}$ . [3]

- (ii) Find the gradient of the curve  $y = f(x)$  at the point  $(0, 2)$ .

Show that there is no point on the curve where the gradient is greater than this. [5]

- (iii) Find  $f^{-1}(x)$ , and state its domain and range.

Write down the gradient of  $y = f^{-1}(x)$  at the point  $(2, 0)$ . [6]

- (iv) Find the area enclosed by the curve  $y = f(x)$ , the  $x$ -axis, the  $y$ -axis and the line  $x = \pi$ . [4]

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