

Mathematics (MEI)

Advanced GCE 4754A

Applications of Advanced Mathematics (C4) Paper A

Mark Scheme for June 2010

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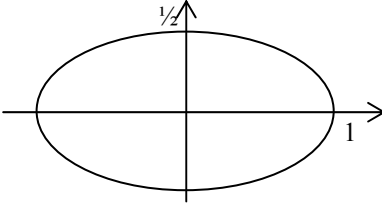
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Section A

<p>1</p> $\frac{x}{x^2-1} + \frac{2}{x+1} = \frac{x}{(x-1)(x+1)} + \frac{2}{x+1}$ $= \frac{x+2(x-1)}{(x-1)(x+1)}$ $= \frac{(3x-2)}{(x-1)(x+1)}$ <p>or</p> $\frac{x}{x^2-1} + \frac{2}{x+1} = \frac{x(x+1)+2(x^2-1)}{(x^2-1)(x+1)}$ $= \frac{3x^2+x-2}{(x^2-1)(x+1)}$ $= \frac{(3x-2)(x+1)}{(x^2-1)(x+1)}$ $= \frac{(3x-2)}{(x^2-1)}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>[3]</p>	<p>$x^2 - 1 = (x + 1)(x - 1)$</p> <p>correct method for addition of fractions</p> <p>or $\frac{(3x-2)}{x^2-1}$ do not isw for incorrect subsequent cancelling</p> <p>correct method for addition of fractions</p> <p>$(3x-2)(x+1)$</p> <p>accept denominator as x^2-1 or $(x-1)(x+1)$ do not isw for incorrect subsequent cancelling</p>
<p>2(i) When $x = 0.5, y = 1.1180$ $\Rightarrow A \approx 0.25/2\{1+1.4142+2(1.0308+1.1180+1.25)\}$ $= 0.25 \times 4.6059 = 1.151475$ $= 1.151$ (3 d.p.)*</p>	<p>B1</p> <p>M1</p> <p>E1</p> <p>[3]</p>	<p>4dp</p> <p>(0.125×9.2118) need evidence</p>
<p>(ii) Explain that the area is an over-estimate. or The curve is below the trapezia, so the area is an over- estimate.</p> <p>This becomes less with more strips. or Greater number of strips improves accuracy so becomes less</p>	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>or use a diagram to show why</p>
<p>(iii) $V = \int_0^1 \pi y^2 dx$</p> $= \int_0^1 \pi(1+x^2) dx$ $= \pi \left[(x + x^3/3) \right]_0^1$ $= 1\frac{1}{3}\pi$	<p>M1</p> <p>B1</p> <p>A1</p> <p>[3]</p>	<p>allow limits later</p> <p>$x + x^3/3$</p> <p>exact</p>

<p>3 $y = \sin \theta \cos \theta = \frac{1}{2} \sin 2\theta$ $x = \cos 2\theta$ $\sin^2 2\theta + \cos^2 2\theta = 1$ $\Rightarrow x^2 + (2y)^2 = 1$ $\Rightarrow x^2 + 4y^2 = 1$ *</p> <p>or $x^2 + 4y^2 = (\cos 2\theta)^2 + 4(\sin \theta \cos \theta)^2$ $= \cos^2 2\theta + \sin^2 2\theta$ $= 1$ *</p> <p>or $\cos 2\theta = 2\cos^2 \theta - 1$ $\cos^2 \theta = \frac{x+1}{2}$ $\cos 2\theta = 1 - 2\sin^2 \theta$ $\sin^2 \theta = \frac{1-x}{2}$ $y^2 = \sin^2 \theta \cos^2 \theta = \left(\frac{1-x}{2}\right)\left(\frac{x+1}{2}\right)$ $y^2 = \frac{1-x^2}{4}$ $x^2 + 4y^2 = 1$ *</p> <p>or $x = \cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $x^2 = \cos^4 \theta - 2\sin^2 \theta \cos^2 \theta + \sin^4 \theta$ $y^2 = \sin^2 \theta \cos^2 \theta$ $x^2 + 4y^2 = \cos^4 \theta - 2\sin^2 \theta \cos^2 \theta + \sin^4 \theta + 4\sin^2 \theta \cos^2 \theta$ $= (\cos^2 \theta + \sin^2 \theta)^2$ $= 1$ *</p> <div style="text-align: center;">  </div>	<p>M1</p> <p>M1</p> <p>E1</p> <p>M1</p> <p>M1</p> <p>E1</p> <p>M1</p> <p>M1</p> <p>E1</p> <p>M1</p> <p>M1</p> <p>E1</p> <p>M1</p> <p>E1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>use of $\sin 2\theta$</p> <p>substitution use of $\sin 2\theta$</p> <p>for both</p> <p>correct use of double angle formulae</p> <p>correct squaring and use of $\sin^2 \theta + \cos^2 \theta = 1$</p> <p>ellipse correct intercepts</p>
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<p>4</p> $\sqrt{4+x} = 2\left(1 + \frac{x}{4}\right)^{\frac{1}{2}}$ $= 2\left(1 + \frac{1}{2} \cdot \frac{x}{4} + \frac{1}{2} \cdot \frac{-1}{2} \cdot \frac{x}{4} \cdot \frac{x}{4} + \dots\right)$ $= 2\left(1 + \frac{1}{8}x - \frac{1}{128}x^2 + \dots\right)$ $= 2 + \frac{1}{4}x - \frac{1}{64}x^2 + \dots$ <p>Valid for $-1 < x/4 < 1$ $\Rightarrow -4 < x < 4$</p>	<p>M1</p> <p>M1 A1</p> <p>A1</p> <p>B1 [5]</p>	<p>dealing with $\sqrt{4}$ (or terms in $4^{\frac{1}{2}}, 4^{-\frac{1}{2}}, \dots$ etc)</p> <p>correct binomial coefficients correct unsimplified expression for $(1+x/4)^{\frac{1}{2}}$ or $(4+x)^{\frac{1}{2}}$</p> <p>cao</p>
<p>5(i)</p> $\frac{3}{(y-2)(y+1)} = \frac{A}{y-2} + \frac{B}{y+1}$ $= \frac{A(y+1) + B(y-2)}{(y-2)(y+1)}$ <p>$\Rightarrow 3 = A(y+1) + B(y-2)$ $y=2 \Rightarrow 3 = 3A \Rightarrow A=1$ $y=-1 \Rightarrow 3 = -3B \Rightarrow B=-1$</p>	<p>M1 A1 A1 [3]</p>	<p>substituting, equating coeffs or cover up</p>
<p>(ii)</p> $\frac{dy}{dx} = x^2(y-2)(y+1)$ <p>$\Rightarrow \int \frac{3dy}{(y-2)(y+1)} = \int 3x^2 dx$</p> <p>$\Rightarrow \int \left(\frac{1}{y-2} - \frac{1}{y+1}\right) dy = \int 3x^2 dx$</p> <p>$\Rightarrow \ln(y-2) - \ln(y+1) = x^3 + c$</p> <p>$\Rightarrow \ln\left(\frac{y-2}{y+1}\right) = x^3 + c$</p> <p>$\Rightarrow \frac{y-2}{y+1} = e^{x^3+c} = e^{x^3} \cdot e^c = Ae^{x^3} *$</p>	<p>M1</p> <p>B1 ft B1</p> <p>M1 E1 [5]</p>	<p>separating variables</p> <p>$\ln(y-2) - \ln(y+1)$ ft their A, B $x^3 + c$</p> <p>anti-logging including c www</p>
<p>6</p> $\tan(\theta+45) = \frac{\tan\theta + \tan 45}{1 - \tan\theta \tan 45}$ $= \frac{\tan\theta + 1}{1 - \tan\theta}$ <p>$\Rightarrow \frac{\tan\theta + 1}{1 - \tan\theta} = 1 - 2\tan\theta$</p> <p>$\Rightarrow 1 + \tan\theta = (1 - 2\tan\theta)(1 - \tan\theta)$ $= 1 - 3\tan\theta + 2\tan^2\theta$</p> <p>$\Rightarrow 0 = 2\tan^2\theta - 4\tan\theta = 2\tan\theta(\tan\theta - 2)$</p> <p>$\Rightarrow \tan\theta = 0$ or 2</p> <p>$\Rightarrow \theta = 0$ or 63.43</p>	<p>M1</p> <p>A1</p> <p>M1 A1 M1</p> <p>A1A1 [7]</p>	<p>oe using sin/cos</p> <p>multiplying up and expanding any correct one line equation solving quadratic for $\tan\theta$ oe</p> <p>www -1 extra solutions in the range</p>

Section B

<p>7(i) $\overline{AB} = \begin{pmatrix} 100 - (-200) \\ 200 - 100 \\ 100 - 0 \end{pmatrix} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}^*$</p> <p>$AB = \sqrt{(300^2 + 100^2 + 100^2)} = 332 \text{ m}$</p>	<p>E1</p> <p>M1 A1 [3]</p>	<p>accept surds</p>
<p>(ii) $\mathbf{r} = \begin{pmatrix} -200 \\ 100 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$</p> <p>Angle is between $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$</p> <p>$\Rightarrow \cos \theta = \frac{3 \times 0 + 1 \times 0 + 1 \times 1}{\sqrt{11}\sqrt{1}} = \frac{1}{\sqrt{11}}$</p> <p>$\Rightarrow \theta = 72.45^\circ$</p>	<p>B1B1</p> <p>M1</p> <p>M1 A1</p> <p>A1 [6]</p>	<p>oe</p> <p>...and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$</p> <p>complete scalar product method (including cosine) for correct vectors</p> <p>72.5° or better, accept 1.26 radians</p>
<p>(iii) Meets plane of layer when</p> <p>$(-200 + 300\lambda) + 2(100 + 100\lambda) + 3 \times 100\lambda = 320$</p> <p>$\Rightarrow 800\lambda = 320$</p> <p>$\Rightarrow \lambda = 2/5$</p> <p>$\mathbf{r} = \begin{pmatrix} -200 \\ 100 \\ 0 \end{pmatrix} + \frac{2}{5} \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix} = \begin{pmatrix} -80 \\ 140 \\ 40 \end{pmatrix}$</p> <p>so meets layer at $(-80, 140, 40)$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1 [4]</p>	
<p>(iv) Normal to plane is $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$</p> <p>Angle is between $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$</p> <p>$\Rightarrow \cos \theta = \frac{3 \times 1 + 1 \times 2 + 1 \times 3}{\sqrt{11}\sqrt{14}} = \frac{8}{\sqrt{11}\sqrt{14}} = 0.6446..$</p> <p>$\Rightarrow \theta = 49.86^\circ$</p> <p>$\Rightarrow$ angle with layer = 40.1°</p>	<p>B1</p> <p>M1A1</p> <p>A1 A1 [5]</p>	<p>complete method</p> <p>ft 90-theirθ accept radians</p>

<p>8(i) At A, $y = 0 \Rightarrow 4\cos \theta = 0$, $\theta = \pi/2$ At B, $\cos \theta = -1$, $\Rightarrow \theta = \pi$ x-coord of A = $2 \times \pi/2 - \sin \pi/2 = \pi - 1$ x-coord of B = $2 \times \pi - \sin \pi = 2\pi$ $\Rightarrow OA = \pi - 1$, $AC = 2\pi - \pi + 1 = \pi + 1$ \Rightarrow ratio is $(\pi - 1):(\pi + 1)$ *</p>	<p>B1 B1 M1 A1 E1 [5]</p>	<p>for either A or B/C for both A and B/C</p>
<p>(ii) $\frac{dy}{d\theta} = -4\sin \theta$ $\frac{dx}{d\theta} = 2 - \cos \theta$ $\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= -\frac{4\sin \theta}{2 - \cos \theta}$ At A, gradient = $-\frac{4\sin(\pi/2)}{2 - \cos(\pi/2)} = -2$</p>	<p>B1 M1 A1 A1 [4]</p>	<p>either $dx/d\theta$ or $dy/d\theta$ www</p>
<p>(iii) $\frac{dy}{dx} = 1 \Rightarrow -\frac{4\sin \theta}{2 - \cos \theta} = 1$ $\Rightarrow -4\sin \theta = 2 - \cos \theta$ $\Rightarrow \cos \theta - 4\sin \theta = 2$ *</p>	<p>M1 E1 [2]</p>	<p>their $dy/dx = 1$</p>
<p>(iv) $\cos \theta - 4\sin \theta = R\cos(\theta + \alpha)$ $= R(\cos \theta \cos \alpha - \sin \theta \sin \alpha)$ $\Rightarrow R\cos \alpha = 1$, $R\sin \alpha = 4$ $\Rightarrow R^2 = 1^2 + 4^2 = 17$, $R = \sqrt{17}$ $\tan \alpha = 4$, $\alpha = 1.326$ $\Rightarrow \sqrt{17} \cos(\theta + 1.326) = 2$ $\Rightarrow \cos(\theta + 1.326) = 2/\sqrt{17}$ $\Rightarrow \theta + 1.326 = 1.064, 5.219, 7.348$ $\Rightarrow \theta = (-0.262), 3.89, 6.02$</p>	<p>M1 B1 M1 A1 M1 A1 A1 [7]</p>	<p>corr pairs accept 76.0°, 1.33 radians inv cos $(2/\sqrt{17})$ ft their R for method -1 extra solutions in the range</p>

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