

**ADVANCED GCE  
MATHEMATICS (MEI)**

Decision Mathematics Computation

**THURSDAY 12 JUNE 2008**

**4773/01**

Morning  
Time: 2 hours 30 minutes

**Additional materials (enclosed):** None

**Additional materials (required):**  
Answer Booklet (8 pages)  
Graph paper  
MEI Examination Formulae and Tables (MF2)



**INSTRUCTIONS TO CANDIDATES**

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.

**COMPUTING RESOURCES**

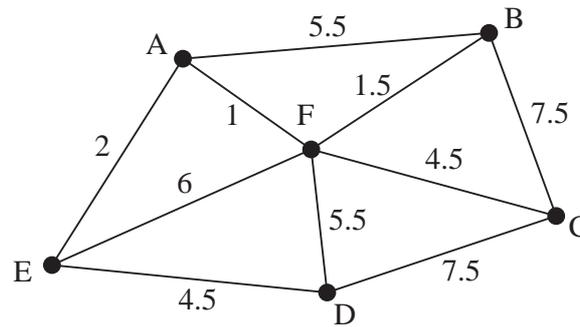
- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.

This document consists of **6** printed pages and **2** blank pages.

- 1 The vertices of the network represent six villages in a rural area in a developing country. The arc weights represent journey times (hours) along connecting tracks.



Health centres are to be built in some villages, so that all villages are within 5 hours journey time of a health centre.

The matrix shows the complete set of least journey times between the villages.

	A	B	C	D	E	F
A	0	2.5	5.5	6.5	2	1
B	2.5	0	6	7	4.5	1.5
C	5.5	6	0	7.5	7.5	4.5
D	6.5	7	7.5	0	4.5	5.5
E	2	4.5	7.5	4.5	0	3
F	1	1.5	4.5	5.5	3	0

- (i) Let  $X_A, X_B, \dots$  be indicator variables representing whether or not a health centre is built in village A, B, ...
- Write down an inequality using these variables which, when satisfied, will ensure that village A is within 5 hours journey of a health centre. Explain your inequality by reference to the matrix above. [5]
- (ii) Formulate an LP, the solution to which will give the minimum number of health centres required and their locations. [5]
- (iii) Run your LP and interpret the solution. [3]
- (iv) Modify your LP so as to find a second optimal solution to the problem. [2]
- (v) Find a third optimal solution. [1]
- (vi) Explain why it is not necessary to force your variables to be indicator variables, and why it is preferable not to do so. [2]

- 2 For a small population of organisms it is approximately the case that in any one-second interval either a birth occurs, or a death, or both, or neither. There is never more than one birth or one death in a one-second interval.

Let  $n$  be the number of organisms in existence at the beginning of a one-second interval.

The probability of a birth occurring in that interval is  $\alpha \times n$ , where  $\alpha$  is a small positive number.

Similarly, the probability of a death occurring in that interval is  $\beta \times n$ , where  $\beta$  is a small positive number.

- (i) Build a spreadsheet to simulate the development of a population of size 10 over a one-minute period.

Print out your spreadsheet and show the formulae which you use. [6]

- (ii) Using  $\alpha = 0.01$  and  $\beta = 0.04$ , run your simulation 10 times.

Tabulate your results and estimate the probability of extinction over the one-minute period. [3]

- (iii) Investigate how the probability of extinction varies for different  $\beta$ s, given that  $\alpha$  remains at 0.01.

Tabulate your results. [3]

You are now to investigate what happens if the population is increased by immigration. In each one-second period there may be either 0 or 1 immigrants arrive into the population. The probability of an arrival is  $\gamma$ .

- (iv) Modify your spreadsheet to incorporate immigration.

Print out your spreadsheet and show the formulae which you use. [4]

- (v) Investigate how immigration affects your results in part (iii), given that  $\gamma = 0.05$ . [2]

3 (a)

	A	B	C	D	E	F	G
A	–	8	–	–	3	–	–
B	8	–	5	7	–	–	–
C	–	5	–	1	–	–	8
D	–	7	1	–	2	–	–
E	3	–	–	2	–	4	–
F	–	–	–	–	4	–	6
G	–	–	8	–	–	6	–

**Table 3.1**

Table 3.1 represents the capacities of arcs in a flow network.

Formulate and solve an LP to find the maximum flow from A to G.

[9]

(b)

	S1	S2	S3	S4	S5	S6	S7
W1	–	8	–	–	3	–	–
W2	8	–	5	7	–	–	–
W3	–	5	–	1	–	–	8
W4	–	7	1	–	2	–	–
W5	3	–	–	2	–	4	–
W6	–	–	–	–	4	–	6
W7	–	–	8	–	–	6	–

**Table 3.2**

Table 3.2 has the same numerical entries as Table 3.1, but its rows are labelled W1 to W7, representing warehouses, and its columns are labelled S1 to S7, representing shops supplied from those warehouses. Each numerical entry represents the cost of moving a single unit from the corresponding warehouse to the corresponding shop.

Each warehouse has 10 units in stock. Each shop requires 10 units.

Formulate and solve an LP to find a minimum cost distribution pattern.

[9]

- 4 (a) Solve the recurrence relation  $u_{n+2} = \frac{3}{2}u_{n+1} - \frac{1}{2}u_n$ , given that  $u_0 = 5$  and  $u_1 = 3$ .

Give the values of  $u_2$ ,  $u_3$ ,  $u_{10}$  and  $u_{1000000}$ . [13]

- (b) Successive waves arriving on a beach are found to have heights which follow approximately the recurrence relation  $w_{n+2} = \frac{3}{2}w_{n+1} - w_n + 5$ , with  $w_0 = 5$  and  $w_1 = 3$ .

(i) Construct a spreadsheet to model wave height. [2]

(ii) Give the values of  $w_2$ ,  $w_3$ ,  $w_{10}$  and  $w_{20}$ . [2]

(iii) State the limitation of using a spreadsheet to model a recurrence relation. [1]

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