A Level Further Mathematics A 
Additional Pure - Y545 
Student revision checklist

**Revision checklist**

The tables below can be used as a revision checklist: **It doesn’t contain all the detailed knowledge you need to know, just an overview.** For more detail see the syllabus and talk to your teacher.

[A Level Further Mathematics A – H245 specification.](https://www.ocr.org.uk/Images/308752-specification-accredited-a-level-gce-further-mathematics-a-h245.pdf)

The table headings are explained below:

| **OCR Reference.** | **Content Description**  (unshaded content is AS content) | **R** | **A** | **G** | **Notes** |
| --- | --- | --- | --- | --- | --- |
| Each item of content has a unique specification reference code.   * **Mathematics A – H240**  1. Mathematics A: Pure 2. Mathematics A: Statistics 3. Mathematics A: Mechanics  * **Further Mathematics A – H245**  1. Further Mathematics A: Pure Core 2. Further Mathematics A: Statistics Option 3. Further Mathematics A: Mechanics Option 4. Further Mathematics A: Discrete Option 5. Further Mathematics A: Additional Pure | | You can use the tick boxes to show when you have revised an item and how confident you feel about it.  R = **RED** means you are really unsure and lack confidence; you might want to focus your revision here and possibly talk to your teacher for help  A = **AMBER** means you are reasonably confident but need some extra practice  G = **GREEN** means you are very confident.  As your revision progresses, you can concentrate on the **RED** and **AMBER** items in order to turn them into **GREEN** items.  You might find it helpful to highlight each topic in red, orange or green to help you prioritise. | | | You can use the notes column to:   * add more information about the details for each point * add formulae or notes * include a reference to a useful resource. * Highlight areas of difficulty or things that you need to talk to your teacher about or look up in a textbook. |

You must be able to use all the formulae and identities given for the Pure Core mandatory strand of A Level Further Mathematics, without those formulae and identities being provided, either in these forms or in equivalent forms. Those formulae and identities may only be provided where they are the starting point for a proof or as a result to be proved.

Learners will be given a Formulae Booklet in each assessment which has both the A Level Mathematics and the A Level Further Mathematics formulae (the version used for AS has only the AS Maths and Further Maths formulae).

**Additional Pure option formulae**

**Vector product**

*,* where, in that order form, a right-handed triple.



**Surfaces**For 3-D surfaces given in the form , the Hessian Matrix is given by .



At a stationary point of the surface:

1. if  and , there is a (local) minimum;



2. if  and , there is a (local) maximum;



3. if  there is a saddle-point;

4. if  then the nature of the stationary point cannot be determined by this test.

The equation of a tangent plane to the curve at a given point is



.



**Calculus**Arc length







Surface area of revolution

## Content of Additional Pure Mathematics (Optional paper Y545)

| **OCR Reference.** | **Content Description**  (unshaded content is AS content) | **R** | **A** | **G** | **Notes** |
| --- | --- | --- | --- | --- | --- |
| **8.01a** | a) Be able to work with general sequences given either as recurrence relations or by position-to-term (closed form) formulae .  *The notation*  *for sequences, which may or may not include a zeroth term, should be recognised.* |  |  |  |  |
| **8.01b** | b) Use induction to prove results relating to both sequences and series. |  |  |  |  |
| **8.01c** | c) Understand and be able to describe various possibilities for the behaviour of sequences.  *Learners are expected to be able to use the terms “periodic”, “convergence”, “divergence”, “oscillating”, “monotonic”.*  *Note that a periodic sequence with period two may be referred to as “oscillating”, but that both convergent and divergent sequences can oscillate. “Divergence” can refer to sequences that are bounded or unbounded.* |  |  |  |  |
| **8.01d** | d) Identify and be able to use the limit of the *n*th term of a sequence as , including steady-states.  *Includes forming sequences from other sequences, for example, finding differences or ratios of successive terms of a sequence.*  [*Rates of convergence are excluded.*] |  |  |  |  |
| **8.01e** | e) Be able to work with the Fibonacci Numbers (and other Fibonacci-like sequences, such as the Lucas Numbers), and understand their properties.  *Includes recognising and using the properties of* *, both numerical and algebraic, and its role in the Fibonacci sequence.* |  |  |  |  |
| **8.01f** | f) Be able to solve a first-order linear recurrence relation with constant coefficients, using the associated auxiliary equation and complementary function.  *Includes finding both general and particular solutions.*  *Includes homogeneous and non-homogeneous recurrence relations of the form* *, where* *may be a polynomial function or of the form*  *Includes knowing the terms, “closed form” and “position-to-term”.*  *Includes understanding that a “recurrence system” consists of a “recurrence relation”, an “initial condition” and the range of the variable n*. |  |  |  |  |
| **8.01g** | g) Be able to solve a second order linear recurrence relation with constant coefficients, using the associated auxiliary equation and complementary function.  *Includes finding both general and particular solutions.*  *Includes the cases when the roots of the auxiliary equation are:*  *(i) distinct and real,*  *(ii) repeated,*  *(iii) complex.*  *Includes homogeneous and non-homogeneous recurrence relations of the form* *, where* *may be a polynomial function or of the form*  *Learners should be aware that this topic is the discrete analogue of the work on differential equations in H245 section 4.10.* |  |  |  |  |
| **8.01h** | h) Be able to apply their knowledge of recurrence relations to modelling.  *Includes birth- and/or death-rates and the use of the*  *function for discrete models. Learners may find it useful to have a calculator with this function, but large numbers of repeated applications will not be required in the assessment.* |  |  |  |  |
| **8.01i** | i) Be able to extend their knowledge of modelling to second order recurrence relations. |  |  |  |  |
| **8.02a** | a) Understand and be able to work with numbers written in base *n*, where *n* is a positive integer.  *The standard notation for number bases will be used.*  *i.e.*  *will denote the number*  *(with*  *in this example) and the letters A-F will be used to represent the integers 10-15 respectively when* *.* |  |  |  |  |
| **8.02b** | b) Be able to use (without proof) the standard tests for divisibility by 2, 3, 4, 5, 8, 9 and 11.  *Includes knowing that repeated tests can be used to establish divisibility by composite numbers.* |  |  |  |  |
| **8.02c** | c) Be able to establish suitable (algorithmic) tests for divisibility by other primes less than 50.  *For integers a and b, the notation*  will be used for *“a divides exactly into b” (“a is a factor of b”, “b is a multiple of a”, etc.).* |  |  |  |  |
| **8.02d** | d) Appreciate that, for any pair of positive integers *a*, *b* with , we can uniquely express *a* as where *q* (the quotient) and *r* (the residue, or remainder, when *a* is divided by *b*) are both positive integers and  . |  |  |  |  |
| **8.02e** | e) Understand and be able to use finite arithmetics (the arithmetic of integers modulo *n* for ). |  |  |  |  |
| **8.02g** | g) Be able to calculate quadratic residues and solve, or prove insoluble, equations involving them. |  |  |  |  |
| **8.02f** | f) Be able to solve single linear congruences of the form . |  |  |  |  |
| **8.02h** | h) Be able to solve simultaneous linear congruences of the form .  *No more than three simultaneous linear congruences will be used. Use of the Chinese remainder theorem will be allowed but not required.* |  |  |  |  |
| **8.02i** | i) Understand the concepts of prime numbers, composite numbers, highest common factors (hcf), and coprimality (relative primeness).  *Knowledge of the fundamental theorem of arithmetic will be expected, but proof of the result will not be required.* |  |  |  |  |
| **8.02j** | j) Know and be able to apply the result that  and  for any integers *x* and *y*.  *Includes using this result, for example to test for common factors or coprimality.* |  |  |  |  |
| **8.02k** | k) Know and be able to use Euclid’s lemma: if  and  then  . |  |  |  |  |
| **8.02l** | l) Know and be able to use Fermat’s little theorem in both forms:  1. If *p* is prime and hcf , then  ;  2. If *p* is prime .  *Includes recognising that if p is prime then Fermat’s little theorem holds, but that the converse is not true (that is, be aware of the existence of “pseudo-primes” to base p).*  [*The proof is excluded.*]  [*Carmichael numbers are excluded.*] |  |  |  |  |
| **8.02m** | m) Know and be able to use the fact that  is not necessarily the least positive integer *n* for which . |  |  |  |  |
| **8.02n** | n) Know that the *n* with this property, called “the order of *a* modulo *p*”, is a factor of  and be able to find it in specific cases. |  |  |  |  |
| **8.02o** | o) Be able to use the binomial theorem to show that  for prime *p*, and use this result. |  |  |  |  |
| **8.03a** | a) Be able to work with binary operations and their properties when defined on given sets.  *Includes knowing and understanding the terms “associativity” and “commutativity”.* |  |  |  |  |
| **8.03b** | b) Be able to construct Cayley tables for given finite sets under the action of a given binary operation.  *Multiplicative notation and/or terminology will generally be used, when appropriate.* |  |  |  |  |
| **8.03c** | c) Recall and be able to use the definition of a group, for example to show that a given structure is, or is not, a group.  *e.g. Questions may be set on groups of integers modulo n (for* *), functions, matrices, transformations, the symmetries of given geometrical shapes and complex numbers.*  *Groups may be referred to in either of the forms:*  *1. by the given set and associated binary operation*  *,*  *2. as “G”, where the operation is understood, or*  *3. as “the set* G *with the operation* *”.*  *To include knowing the meaning of the terms “identity” and “closed”, and that in an abelian group the operation is commutative.* |  |  |  |  |
| **8.03d** | d) Recognise and be able to use the Latin square property for group tables. |  |  |  |  |
| **8.03e** | e) Recall the meaning of the term “order”, as applied both to groups and to elements of a group, and be able to determine the orders of elements in a given group.  *Includes knowing and being able to use the fact that the order of an element is a factor of the order of the group.* |  |  |  |  |
| **8.03f** | f) Understand and be able to use the definition of a subgroup, find subgroups and show that given subsets are, or are not, proper subgroups. |  |  |  |  |
| **8.03g** | g) Recall the meaning of the term “cyclic” as applied to groups. |  |  |  |  |
| **8.03h** | h) Understand that a cyclic group is generated by “powers” of a single element (generator), that there may be more than one such element within a group, and that other (non-cyclic) groups may be generated by two or more elements along with their “powers” and “products”. |  |  |  |  |
| **8.03i** | i) Be familiar with the structure of finite groups up to, and including, order seven, and able to apply this knowledge in solving problems. |  |  |  |  |
| **8.03j** | j) Be able to work with groups of higher finite order, or of infinite order.  *No particular prior knowledge of such groups, or their structures, will be expected.* |  |  |  |  |
| **8.03k** | k) Recall and be able to apply Lagrange’s theoremconcerning the order of a subgroup of a finite group.  *The proof of the theorem is not required, but a clear statement of the result may be expected.* |  |  |  |  |
| 8.03l | l) Be able to determine whether two given groups are, or are not, isomorphic using informal methods.  *e.g. By noting disparities between the orders of elements.* |  |  |  |  |
| **8.03m** | m) Be able to work with groups defined by their algebraic properties.  *Includes using algebraic methods to establish properties in abstract groups.*  *e.g. To show that any group in which every element is self-inverse is abelian, to establish an identity or complete a Cayley table.* |  |  |  |  |
| **8.04a** | a) Understand and be able to use the definition, in geometrical terms, of the vector product and be able to form the vector product in magnitude and direction, and in component form.  *Includes use of the formula*  *, where*  *in that order (and the vectors*  *in that order) form a right-handed triple.* |  |  |  |  |
| **8.04b** | b) Understand the anti-commutative and distributive properties of the vector product. |  |  |  |  |
| **8.04c** | c) Be able to use the vector product to calculate areas of triangles and parallelograms. |  |  |  |  |
| **8.04d** | d) Understand the significance of .  *e.g. The equation of a line in the form* *.* |  |  |  |  |
| **8.04e** | e) Understand and be able to use the definition of the scalar product, and be able to use it to calculate volumes of tetrahedra and parallelepipeds.  *Includes the notation* *.*  *Includes understanding the significance of* *.* |  |
| **8.05a** | a) Be able to work with functions of two variables, given either explicitly in the form  or implicitly in the form , and understand and use the fact that this equation, and its partial derivatives, relate to a 3-D surface.  *An informal understanding only of how the partial derivatives relate to the surface is required.*  *Functions*  *will involve sums and products of powers of x and y only. Issues relating to domains and ranges will not be considered beyond the appreciation that, for example the surface*  *has no point at which**.* |  |  |  |  |
| **8.05b** | b) Extend their knowledge of surfaces to those defined by functions of more than two variables, and incorporating trigonometric functions, logarithms and exponentials. |  |  |  |  |
| **8.05c** | c) Be able to sketch sections and contours, and know how these are related to the surface.  *i.e. Sections of the form*  *or*  *and contours of the form* . |  |  |  |  |
| **8.05d** | d) Be able to find first and second derivatives, including mixed derivatives.  *Learners will be expected to recognise and use both notations for first- and second order partial derivatives, including mixed ones.*  *e g.* ,  and , .  *Includes the Mixed derivative theorem; namely, that*  *or*  *for suitably well-defined, continuous functions* f*.* |  |  |  |  |
| **8.05e** | e) Understand and be able to apply the concept that stationary points of *z* arise when  (or ) and that these can be maxima, minima or saddle-points.  *Learners should know and understand the basic properties of these stationary points.*  *Learners will only be required to find stationary points, but will not be required to determine their natures.* |  |  |  |  |
| **8.05f** | f) Be able to determine, for 3-D surfaces given in the form , the nature of maxima, minima and saddle-points by means of the sign of the determinant of the matrix of second partial derivatives (the Hessian Matrix),  ;  namely, that:  1. if  and , there is a (local) minimum;  2. if  and , there is a (local) maximum;  3. if  there is a saddle-point;  4. if  then the nature of the stationary point  cannot be determined by this test. |  |  |  |  |
| **8.05g** | g) Be able to determine, for 3-D surfaces given in the form , the equation of a tangent plane to the curve at a given point  using the formula  . |  |  |  |  |
| **8.06a** | a) Be able to establish and use given reduction formulae, and employ them to evaluate integrals using recursive techniques. |  |  |  |  |
| **8.06b** | b) Be able to find arc lengths and areas of surface of revolution for curves defined in cartesian or parametric form. |  |  |  |  |



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