GCE

Mathematics

Advanced GCE

Unit **4734:** Probability and Statistics 3

Mark Scheme for June 2011

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1 (i)	E(*S*)= 22 Var(*S*)=E(*S*)	B1 B1 **2**	
(ii)	E(*T*) = ½×5 - ¼×4=1.5 Var(*T*)=¼×5 + $^1/_{16}$×4 = 1.5 =E(*T*) AG	B1 M1 A1 **3**	Using Var(*aX*+*bY*) **CWO**
(iii)	*T* only does not have a Poisson distribution Some values of *T* are EITHER negative OR: fractional	B1 B1 **2** **(7)**	Unless wrong reason
2(i)	Use $(^6/_{80})(^{74}/_{80})/80$ $p_s \pm zs$ z= 1.96 (0.0173, 0.1327)	B1 M1 B1 A1 **4**	Or /79 *s* of the form $\sqrt{(p_s q_s/80)}$ (or 79) or no √ Accept (0.017,0.133)
(ii)	Use $z\sqrt{(p_s q_s/n)}$ ≤ 0.05 $n \ge 106.6$, least is 107	M1 A1 A1 **3**	or no √ and z=1.96 .Or = Allow 110
(iii)	e.g Variance is an estimate OR Distribution of p_s is only approx normal	 B1 **1** **(8)**	Not var unknown Must state distribution of what.
3(i)	$\int_0^1 ax\,dx + \int_1^2 a(x-2)^2\,dx = 1$ $\left[\frac{ax^2}{2}\right]_0^1 + \left[\frac{a(x-2)^3}{3}\right]_1^2$ $\frac{1}{2}a + \frac{1}{3}a = 1$ $a = {}^6/_5$	M1 B1 M1 A1 **4**	 With or without limits Correct method for equation with fractions/decimals
(ii)	EITHER: $\int_0^1 ax\,dx + \int_1^{1.5} a(x-2)^2\,dx$ OR $1 - \int_{1.5}^2 a(x-2)^2\,dx$ $= {}^{19}/_{20}$	M1 A1 **2**	Any *a* AEF
(iii)	$\int_0^1 ax^2\,dx + \int_1^2 ax(x-2)^2\,dx$ $= \left[\frac{ax^3}{3}\right]_0^1 + \left[a(\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2\right]_1^2$ =9/10 (Expected monthly demand = 900)	M1 B1 A1 **3** **(9)**	 AEF With or without limits AEF

4(i)	$2608p$ $p = e^{-3.87}3.87^6/6!$ (×2608=253.82) $(273-253.82)^2/253.82$ =1.449	M1 A1 M1 A1 **4**	p from Poisson From 253.8 or 254 seen Answer between 1.445 and 1.460
(ii)	Number of cells – 1 (estimated mean) – 1(same totals)	B1 **1**	Not 11-1
(iii)	H_0: A Poisson distribution fits the data H_1: A Poisson distribution does not fit the data CV = 15. 99 13.0 < CV and do not reject Ho accept that there is insufficient evidence that a Poisson distribution does not fit data	 B1 B1 M1 A1 **4** **(9)**	For both hypotheses Their CV Sufficient evidence that Poisson distribution fits data , OK
5(i)	Solve $\frac{4}{3}(1-\frac{1}{m^2}) = \frac{1}{2}$ Giving $m = \sqrt{\frac{8}{5}}$	M1 A1 **2**	 Or equivalent. 1.26, 1.265, 2√10/5
(ii)	$G(y) = P(Y \le y)$ or < $= P(X \ge 1/\sqrt{y})$ $= 1 - F(1/\sqrt{y})$ $= 1 - {}^4/_3(1-y)$ or $(4y-1)/3$ $1 \le 1/\sqrt{y} \le 2 \Rightarrow \frac{1}{4} \le y \le 1$ $g(y) = \begin{cases} 4/3 & 1/4 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$	M1 A1 M1 A1 B1 B1√ **6**	Or:$x=1/\sqrt{y}$, $\lvert dx/dy\rvert=1/(2y^{3/2})$ B1 $f(x)=8/(3x^3)$; $1\le x\le 2$ M1A1 $g(y)=f(x)\lvert dx/dy\rvert$ M1 =4/3 A1 $\frac{1}{4}\le y\le 1$ B1 Ft $G(y)$
(iii)	EITHER: $E(2-2Y)$ $= 2 - 2\times{}^5/_8$ $= {}^3/_4$ OR $2 - \int_1^2 16/(3x^5)dx$ OR $\int_1^2 (2-2/x^2))(8/3x^3)dx$ $= 2 + [4/(3x^4)]$ $=[-8/(3x^2) +4/(3x^4)]$ $= 3/4$ $=3/4$	M1 A1√ A1 M1 A1 A1 **3** **(11)**	 √ $g(y)$ CAO AEF From $2 - \int xF'(x)dx$ √ $f(x)$ CAO AEF
6(i)	$s^2 = (68636.41 - 2605^2/100)/99$ (=7.84) $\bar{x} = 26.05$ $26.05 \pm zs/10$ $z = 2.326$ or $\Phi^{-1}(0.99)$ ART (25.4, 26.7)	B1 B1 M1 B1 A1 **5**	AEF Allow t(99)=2.365
(ii)	Use N(26.05,7.84) $P(\ge 30)= 1-\Phi([30-26.05]/\sqrt{7.84})$ = 0.0792 = 7.92%	M1 M1 A1 **3**	s^2 from (i) M0 for 7.84/100 No "cc" allow either; ART 0.08 or 8%
(iii)	Use $B_1 - B_2 \sim N(0, 15.68)$ $P(< 5) = \Phi(5/\sigma)$ = 0.897	M1 A1 A1 A1 **4**	With $\mu = 0$ For variance σ^2 Their σ; $\Phi(\pm 5/\sigma) \Rightarrow$ M1
(iv)	(i) only since sample size of 100 is large enough (for CLT to hold)	B1 **1** **(13)**	Must be clear which part and with correct reason.

7(i)	For each student the scores are correlated	B1 **1**	Or equivalent**,** eg paired
(ii)	Increase in score has a normal distribution Sample is considered to be a random sample of all students attending the course	B1 B1	Allow pop of differences~ normal Or equivalent , allow independent
	H_0: μ_D = 0, H_1: μ_D > 0 where *D*= increase in scores D= 10 2 12 -3 18 10 11 6 14 9	B1 M1	Or H_0: μ_1=μ_2 H_1: $\mu_1 < \mu_2$: not μ =0 D may be implied
	$\bar{D}$ = 8.9 s^2 = 35.88	B1 B1	
	Test statistic = 8.9/(s/√10) = 4.699 v = 9, CV = 3.25 4.699 > CV Reject H_0 and accept that there is sufficient evidence a at the ½ % significance level of an increase in mean scores. SR 2-sample test: (i)B0(ii)B0B1B1M0 Max 2/11	M1 A1 B1 M1 **10**	Must involve 10 Allow ART 4.70 Or P(t>4.699)=0.00056<0.005 Not OA
(iii)	Test statistic = (8.9-5)/√3.588=2.059 This is significant of an increase at the 5% significance level (CV of 1.833) so director's claim is supported.	M1A1 M1 A1 **4** **(15)**	Or P(t>2.059)=0.035 Any reasonable significance level with corresponding conclusion Allow at ½ %
	SR 2-sample t-test. (8.9-5)/s M1 Max1/4 SR: Use of confidence intervals 99% C I 2-sided (2.74,15.1) : 99.5% 1-sided (2.74, ∞) 5 is in this interval so not significant at ½ % level A1 OR 90% CI 2-sided (5.43,12.37) ; 95% 1-sided (5.43, ∞) 5 not in this interval so significant at 5% SL	M1A1 M1 A1 M1A1 M1 A1	

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