9

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Report on the Units

June 2009

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CONTENTS

Advanced GCE Further Mathematics (MEI) (7896)
Advanced GCE Further Mathematics (Additional) (MEI) (7897)
Advanced GCE Mathematics (MEI) (7895)
Advanced GCE Pure Mathematics (MEI) (7898)

Advanced Subsidiary GCE Further Mathematics (MEI) (3896)
Advanced Subsidiary GCE Further Mathematics (Additional) (MEI) (3897)
Advanced Subsidiary GCE Mathematics (MEI) (3895)
Advanced Subsidiary GCE Pure Mathematics (MEI) (3898)

REPORT ON THE UNITS

4751 Introduction to Advanced Mathematics (C1)

General Comments

As usual, the full range of achievement has been seen on this paper. There was plenty of good, positive achievement seen and there were no parts of questions which proved inaccessible.

A few centres continue to enter large numbers of very weak candidates.

Time was not an issue in general, with very few candidates not having the opportunity to attempt all parts of questions, and some clearly having a second go at a few parts where they were dissatisfied with their answers. However, some candidates used long methods in some parts, and will not have helped themselves timewise in doing so.

Lack of facility with fractions continues to disadvantage some candidates. On this paper, question 11 parts (i) to (iii) were the places examiners experienced this most often. In questions 12(i)(*C*) and 13(ii) a surprising number confused the x and y axes.

Centres should note that this paper will be marked on-line as from January 2010. Candidates will be issued with a lined Printed Answer Book that is specific to the paper. Samples based on previous papers will be available to centres on the OCR website, before next January's examination, so that candidates have an opportunity to gain familiarity with the style of booklet and the space provided for answering each question.

Comments on Individual Questions

Section A

1) Many candidates gained 4 marks, but some stopped after finding the equation of the line and did not go on to find the intersections with the axes. Some made errors in the equation of the line because of negative signs and brackets; some did not notice that they were asked to find two intersections.

2) Most candidates gained at least one mark in the rearrangement. Some made errors in coping with the ½ , whilst a surprising number attempted a square root – perhaps they had practised the same formula making t the subject?

3) The majority understood they should substitute 3 and use f(3) = 1. The commonest error after that was $3^3 = 9$. Those who attempted long division rarely succeeded completely. Some achieved success from equating coefficients or using a mixture of that and long division, working backwards and finding k had to be 10 to make the working correct. Some candidates omitted this question or used f(3) = 0 or used f(−3) instead of f(3).

4) Many candidates made life more difficult for themselves here by multiplying out the brackets and working with $x^2 > 6x$. In so doing, they often reached $x > 6$ but usually lost the $x < 0$ part of the solution. Others correctly identified 0 and 6 as the endpoints, but had wrong inequalities. Gaining both marks here was relatively rare. Few candidates drew sketches of $y = x(x - 6)$ to help themselves, but those who did usually had the correct answer.

5) Many gained full marks on this question but relatively few saw the link between the parts, with most starting again with Pascal's triangle in part (ii). As expected, some candidates often failed to use factorials correctly in the first part or to cube the 2 in the second part.

6) Many candidates tried only examples of odd and even numbers and did not attempt a general argument. Of those that did, the majority used the approach 'odd^3 = odd, odd − odd = even (and similarly with even numbers). Some factorised the expression as $n(n^2 - 1)$ and argued successfully from there. Very few factorised it further. Some of the better candidates used $n = 2m$ and $n = 2m + 1$, but some of these attempts were spoiled by errors in their algebra, particularly in expanding and simplifying $(2m + 1)^3 - (2m + 1)$.

7) This was usually correct, but errors in the first part included $5^{2-2} = 5$ and $5^{\frac{1}{2}} = \sqrt{5}$. In the second part, those who cubed first often had problems and some candidates who did obtain $\sqrt{100} = 10$ occasionally gave the answer to cubing it as 30.

8) A surprising number struggled with the first part. Some tried to multiply top and bottom by $2\sqrt{27}$, which tended to lead to problems with arithmetic. Others got part of the way, but failed to simplify, including quite a few who reached $\frac{2\sqrt{3}}{3\sqrt{3}}$ or even $\frac{4}{6}$ and then failed to complete the simplification. Nevertheless many candidates did get the method mark. One common error was go from $2 \times 3\sqrt{3}$ to $5\sqrt{3}$ in the denominator.

Most managed the first three terms in the expansion, but many could not deal with $(3\sqrt{2})^2$. Some got 18 but thought it was negative, and a few got 18 but could not add 25 and 18 correctly. $(3\sqrt{2})^2 = 9\sqrt{2}$ or 36 were other common errors.

9) With no fractions to cope with, and the coefficient of x^2 being 1, this completing the square question was done well. Two common errors were: $(x + 3)^2 + 14$ and $(x - 3)^2 - 4$.

Many candidates were able to use their answer to the first part to write down the coordinates of the minimum, though a few failed to see the connection and started again using calculus. Others attempted to solve the quadratic equation and could have arrived at a correct conclusion using symmetry but failed to do so. The most common error was one of sign.

10) Few gained full marks for this question. Some tried to square root the whole expression on the LHS, to get equations such as $x^2 - 5x - 6 = 0$. Others tried taking out a factor of x^2 and worked from $x^2(x^2 - 5) = 36$. It was evident that many candidates did not understand the concept of a quadratic equation in x^2. Of those who did, some solved the quadratic equation correctly but then left the answer as $x = 9$ or -4, others gave answers of ±2 as well as ±3, or just 2 and 3, and a few gave 16 and 9. The word "real" in the question led a number of candidates to consider the value of the discriminant without taking further steps towards a solution.

Section B

11) (i) Most knew what to do and many found the equation of the perpendicular line correctly. However, $2 \div -6 = -3$ was a common error after a correct expression for the gradient of AB.

(ii) Many candidates successfully equated the correct equations. However, there were frequent errors in coping with the fact that the gradient of AB was −1/3. Those who multiplied up frequently failed to multiply all terms. Those who kept the fraction and reached $3x = -\frac{1}{3}x + 3$ or $3\frac{1}{3}x = 3$ frequently made errors in finding *x*. Follow-through marks limited the damage for such candidates.

(iii) Some did not realise that the coordinates they had just found were needed here, but those who used them correctly were able to gain a mark even if their answers were wrong. From the correct coordinates, following through to achieve our given answer required some squaring of fractions and coping with surds where errors were frequent.

(iv) This part was independent of the previous ones and was done quite well, with the surd in the required form of answer giving a helpful hint. In spite of this, some who had the correct method left their answers as $\sqrt{40}$, whilst the error of thinking this was $4\sqrt{10}$ was also quite common.

(v) Many made this more difficult than they needed to, with some very complicated routes to find the area of OAB, frequently involving two triangles to be added. There was frequent involvement of length OB and Pythagoras being invoked to find length OA.

12) (i)(*A*) Having the answer given to the expansion helped to concentrate candidates' minds on finding any errors in their work, and most gained 2 marks here. Expanding one pair of brackets first was the usual method.

(i)(*B*) The sketch of the cubic graph was often done well – the mark lost most often was for failure to find and show the intersection with the *y* axis. A few stopped their graph at the intersections with the *x* axis at −1 and 4.

(i)(*C*) Some simply put f(x − 3) and did not produce an explicit equation for *y* in terms of *x*. Using f(x + 3) or f(x) + 3 were the common errors. Several candidates omitted this part. Of those who gave an equation, few realised they could substitute $x = 0$ into their unsimplified form to obtain the intersection with the *y* axis – mistakes in the algebra then sometimes led to a wrong answer from those who had a correct equation for *y*. Some gave the intersections with the *x* axis rather than the required *y* axis.

(ii) Most used f(3) to obtain the required result although some candidates went straight into long division. Some candidates gained the first mark but did not have much idea about the division. Some used inspection to find the other factor. There were some errors in the formula, but many candidates gained full marks on this part.

13) Many candidates answered this question well.

(i) Most candidates picked up both marks in this first part. A few candidates got the signs of the coordinates wrong and a few gave the radius as 20 rather than $\sqrt{20}$.

(ii) Some who appeared to understand the question lost marks by not being sufficiently clear (e.g. "the radius is not big enough" without comparing 5 and $\sqrt{20}$). Others compared the radius with the distance from the x-axis and said that it did cross. One or two found the distance from the origin to the centre and tried to base an argument on that. Some put $x = 0$ into the equation of the circle in order to find the intersection and gained full credit by showing that the discriminant of the resulting quadratic equation was negative, thus showing that there were no real roots and so no intersection. Some showed this from the equation $(y - 2)^2 = -5$.

(iii) This part was done very well, with just a few using the perpendicular gradient.

(iv) A good number of candidates produced efficient solutions to find the point (1, 4), although many failed to gain the mark for explaining that the repeated root meant that the line was a tangent. Many had the right idea but made algebraic errors, especially in squaring $2x$. Those who multiplied out before substituting $y = 2x + 2$ were more error-prone. A substantial minority made no attempt at this part.

4752 Concepts for Advanced Mathematics (C2)

General Comments

The paper was generally well received, with most candidates able to make some headway with questions in both sections. However, very few obtained full marks or even close to full marks. A significant minority of candidates lost easy marks because they were unable to use terminology and definitions expected of Higher Level GCSE candidates correctly. Some candidates presented wildly inaccurate answers, seemingly without the sense that something must be wrong, and failure to show adequate working cost easy marks for others. It seemed that many candidates understood the concepts in this course, but failed to do themselves full justice in the examination because of poor algebra (e.g. factorising) and poor arithmetic (e.g. inability to deal with fractional indices).

Comments on Individual Questions

Section A

1) There were some good clear answers to this question, but many candidates failed to score both marks. Elementary mistakes with the right angled triangle, such as $1^2 + 1^2 = 2^2$, were common. For the second mark, $\cos\theta = \frac{\text{opp}}{\text{hyp}}$ was seen quite often.

2) This question was generally well done, with many candidates scoring full marks. Occasional errors were $6x^6$ for the first term, and $\frac{5x^2}{2}$ for the second term. A few made slips with the arithmetic, or evaluated 138 + 7 as a final step.

3) Part (i) was tackled successfully by most. A few candidates added two extra terms or began the sum at 1. Some evaluated $3^2 - 1$ six times and found the sum. Other mistakes were to simply compute each term, or slip up with the arithmetic – a final term of 65 in the sum was quite frequent.

Part (ii) was seldom answered correctly. Approximately half the candidates gave "convergent" as the answer. Most of those who correctly stated "divergent" were unable to give the correct reason – comments such as "$r > 1$", "the terms are increasing exponentially" and "the terms are getting further from 0" were quite common.

4) Part (i) was well done. Some candidates spoiled their answer by adding a gratuitous π, and a few wrote "$2.4° = \frac{\pi}{75}$ radians".

Most candidates successfully multiplied 2.4 by $\frac{180}{\pi}$, but a good number then failed to round their answer to the nearest degree, thus losing an easy mark.

5) Part (i) was generally well done. Most successfully drew the correct cosine waves, but a good number lost the second mark because they only sketched the curve for half the range. Some candidates sketched $y = 2\cos x$ or $y = \cos x + 2$. A few candidates sketched $y = \sin x$ and $y = \sin 2x$. Clear labelling on both axes was required. Many candidates unnecessarily used graph paper and attempted an accurate plot.

Many candidates failed to score in part (ii) because they did not know the correct terminology. Even those who did identify "stretch" as the appropriate word often lacked precision in their description, referring to the scale factor as an "increase" or even a vector.

6) The first three marks were obtained by the vast majority of candidates, but the final two were not usually earned. Most candidates examined the second derivative, identified the corresponding *y*-values and then ran out of steam. Some solved $\frac{d^2y}{dx^2} = 0$ and then used their value for an inequality. Those who did obtain the correct inequality often wrote it down clumsily: $-1 > x > 5$ was common.

7) Most candidates knew that the substitution $\cos^2\theta = 1 - \sin^2\theta$ was expected, and were able to show at least one correct step in obtaining the required result. A few incurred a penalty by failing to make clear what they were doing. Some weak candidates simply manipulated the original expression, "went round the houses" and ended up back at the start point. Most candidates went on to obtain 14.47° and 165.53°, but omitted either 0° or 180° - or in some instances both.

8) There were many excellent responses to this question, with full marks awarded on many occasions. Some slipped up by writing $3\sqrt{x}$ as $x^{\frac{1}{3}}$ before integrating, and some evaluated $2\times4^{1.5}$ as $(2\times4^{\frac{1}{2}})^3$ when finding "c". However, a significant minority of candidates scored no marks at all because they went straight to "$y = mx + c$" and inserted $m = 3\sqrt{x} - 5$.

9) Part (i) was very well done, although some candidates wrote $3\log_a a = \log_a a^3 = 1^3$ and $10 - 1 = 9$.

Part (ii) was sometimes done well by weaker candidates, but there were often mistakes on better scripts. Most scored M1 for a correct use of one of the log laws, but a surprisingly high number obtained the answer $4\frac{1}{2}\log a$ in various different ways.

Section B

10) (i) This was done very well indeed, with many candidates scoring full marks. Two decimal place accuracy was expected for $\log_{10}7$ and $\log_{10}15$. A few slipped up with plotting the points (usually (1,12) was plotted as (1,10)) and some drew a free hand line or failed to cover the domain given in the table.

(ii) There were many excellent answers; most knew how the gradient and the intercept related to *a* and *b*. Some lost accuracy by taking values from the table rather than the line of best fit to solve a pair of simultaneous equations. In catastrophic cases, such as "$b = +12.5$", it was seldom appreciated that the value had to be wrong. The most fruitful approach was to use the gradient and vertical intercept of the line of best fit for *a* and *b*. However, a few candidates used the intercept on the horizontal axis for *b*.

(iii) This was generally well done, although many failed to score because their values for *a* and *b* were outside tolerance. A few candidates attempted to find *t* at h =100.

(iv) There were many good answers to this part of the question, although a few slipped up in manipulating the equation. Some candidates obtained "$\log_{10}t$ = 1.727.." and then ran out of steam.

Those who attempted extrapolation of the graph were generally so far away from the allowed range of values that they failed to score.

(v) A good number of candidates scored both marks here, but many candidates failed to comment on the model. Instead a paragraph on the growth of trees was presented. However plausible this may have been, it earned no marks.

11) (i) Part (A) was very well done.

In part (B), most candidates were able to use $S_n = \frac{n}{2}(2\times10 + (n-1)\times10)$, but few obtained a satisfactory simplified version. The correct quadratic was generally obtained, but many resorted to trial and improvement to find the solution. Some made slips with algebra, obtaining $n^2 + 2n - 2070 = 0$, or $n^2 - n - 2070 = 0$. A few candidates went straight to an arithmetic approach, which scored a maximum of two marks. A minority of candidates started out with $u_n = 10 + (n-1)\times10$

(ii) Parts (A) and (B) were very well done, although a small number of candidates used the formula for the *n*th term instead of using the formula for the sum of the first *n* terms or summing directly.

In part (C) there were many excellent solutions. Some spoiled their work by incorrect simplification of the formula, or by incorrect manipulation of the equation. A few candidates failed to score because they used the A.P. formula, or because they went straight to trial and error. Failure to "state the formula" was heavily penalised.

12) (i) The correct answer of 6.1 was often obtained. Occasionally this was arrived at by evaluating $\frac{dy}{dx}$ at x =3 and 3.1 and finding the mean. Some slipped up by rounding $3.1^2 - 7$ to 2.6 when finding "m" using the usual formula.

(ii) A minority of candidates adopted the correct approach, but often slipped up on the algebra. Of those who correctly obtained "6 + h", a significant minority went on to write "so h = - 6", showing that they did not really know what they were doing. Many candidates did not understand the notation at all – for example, f(3 + h) = 3f + 3h was surprisingly common, as was $(x^2 - 7)(3 + h)$ etc.

(iii) Hardly anyone realised that consideration of $h \to 0$ was expected. Most candidates re-started with differentiation and didn't score.

(iv) Many candidates scored full marks, having obtained m = 6 from differentiation. A significant minority scored zero, however, because they used $m = -\frac{1}{6}$.

(v) Those candidates who attempted this question generally did very well. A few found the y-intercepts instead, and a surprising number failed to leave the answer to the specified degree of accuracy, thus losing an easy mark.

4753 Methods for Advanced Mathematics (C3) (Written Examination)

General Comments

This paper proved to be accessible to all suitably prepared candidates, and there were plenty of marks available to even the weakest candidates. However, there were some questions, such as 3, 6 and 7(ii) which tested the abler candidates. Fewer full marks than usual were scored – the final part of question 7 proved the main stumbling block – but, equally, virtually all candidates scored above 20. There was no evidence of lack of time to complete the paper.

With reference to recent examiner's reports, it was pleasing to note that fewer candidates were using graph paper for their sketch in question 3. In general, some candidates seem to be insufficiently aware of the significance of words such as 'verify' (see question 8(ii)), and 'hence': most candidates missed the significance of this in questions 6 and 7(ii). Candidates also need to be clear what is meant by exact answers, or else they will lose marks in this paper.

In general, the calculus topics continue to be well answered, albeit with some sloppy notation used in integration, with modulus, proof and inverse trigonometric functions being less securely understood. The standard of presentation varied from chaotic to exemplary.

Comments on Individual Questions

Section A

1) This should have been a routine test of trigonometric integration. However, many candidates confuse differentiation and integration results, using a multiplier of 3 rather than 1/3, and making sign errors. There were also significant numbers of evaluation errors, such as '0' for the lower limit.

2) This question was very well done – exponential growth and decay questions are well understood by the large majority of candidates. The main sources of error lay in the use of '99' instead of '1' in part (ii), and inaccuracy in the final answer through premature rounding of the value of k.

3) Sketching this arccos graph proved to be quite testing – perhaps more so than arcsin - and few candidates scored all three marks. The first M1 was given for a reasonable attempt to reflect a cosine graph in $y = x$. Quite a few candidates scored the 'B1' for showing any graph through (1, 0), (0, π) and (-1, 2π), even if it was a straight line! For the final A1, we wanted to see the correct domain and range, and reasonably correct gradients at $x = -1$, 0 and 1. The use of degrees instead of radians was allowed, as no calculus was involved.

4) Although good candidates did this effortlessly, there was a degree of confusion amongst some candidates over handling the modulus. Many think taking a modulus means you have to multiply by -1, so only did this. The given diagram made most candidates aware that the signs of *a* and *b* were both positive, but there were many incorrect or dubious statements such as "$y = -2$, but y is positive, so $b = 2$" (–2 coming from use of $y = 2x - 2$ when $x = 0$). Another common mistake was to obtain $a = ½$ from the incorrect $2x - 1 = 0$.

Squaring y is another rather dubious technique when applied to a given modulus graph, even more so if the '2' is left un-squared, giving statements like $2(x - 1)^2 = 0$ (true, but from wrong working).

5) (i) Most candidates made a reasonable attempt to differentiate implicitly, but some common errors were (a) starting "d*y*/d*x* = ..", (b) omitting the '2' when differentiating e^{2y}, (c) RHS = 1+ cos x , (d) LHS= $2e^{2y\,dy/dx}$.

(ii) The most frequent mistake was in the mishandling of the inversion, with ln 1 + ln sin x appearing frequently. Even when the correct expression for y was found, a surprising number needlessly used the product or quotient rules with $u = ½$ and $v = \ln(1 + \sin x)$, and/or omitted the derivative of sin x in differentiating the latter. Some lost the last mark by not showing clearly that their results in (i) and (ii) were equivalent.

6) This question scored poorly. Although some confused composition with squaring, most candidates managed a correct expression for ff(x) by substituting $(x+1)/(x-1)$ for x in f(x); however, many of these then failed to deal with the subsidiary denominators, and to correctly simplify the expression to x.

Virtually all candidates then tried to invert $y = (x+1)/(x-1)$ to find $f^{-1}(x)$, rather than simply writing down that $f^{-1}(x) = f(x)$. Also, $f^{-1}(x) = (x-1)/(x+1)$ was quite a common error.

There was some confusion in the final B1 between the symmetry of f(x) in $y = x$, and the fact that f(x) and $f^{-1}(x)$ are symmetrical in this line. Some candidates thought this last question referred to odd and even functions.

7) (i) This algebra proved to be an easy 4 marks for all candidates, give or take a few slips due to carelessness.

(ii) On the other hand, the logic of this part eluded all but the very best candidates. Many substituted values, or tried other letters, or $y = x + 1$, etc. Some recognised that $x^2 + xy + y^2$ had to be proved to be positive, but failed to see the connection between this and (i)(*B*).

Section B

8) Plenty of candidates scored well on this question, seeing the links between the various parts.

(i) Most candidates got the correct coordinates for A and B, although many approximated for e^{-1} – we tried as much as possible to condone this by ignoring subsequent working. It is important that candidates knew why 'verify' was used in finding C – quite a few tried to solve $1 + \ln x = e^{x-1}$.

(ii) Inverting functions is usually well understood, and these two marks were obtained by all but a few candidates.

(iii) Quite a few candidates substituted $u = x - 1$ here, and others left the answer as $e^0 - e^{-1}$ (which though arguably 'in terms of e', was not what was intended), or evaluated e^0 as 0. Some candidates made errors in integrating e^{x-1}, e.g. $(e^{x-1})/(x - 1)$.

(iv) The classic blunder here is to take $u = \ln$ (whatever this means) and $v' = x$! Fortunately, this occurred rarely, and most candidates succeeded in using the correct parts. Quoting this result was not allowed, however, as they were asked to use parts to derive it.

We generously followed through their answer to this to integrate g(x). A rather disconcerting number of candidates – even good ones – failed to simplify $[x + x\ln x - x]$ as $x\ln x$, which made the substitution and derivation of 1/e a bit more complicated than necessary.

(v) This part was more demanding, but a few recovered to achieve the correct answer as the area of the square minus twice the given answer in part (iv).

9) This question was a routine test of calculus which offered plenty of accessible marks.

(i) This was an easy mark for all.

(ii) As this was a routine application of the quotient rule, we withheld the final 'E' mark if the bracket round $(3x - 1)$ was omitted. This was not common, however, and most candidates scored 3 easy marks.

(iii) The coordinates of the turning point were usually obtained correctly from the given derivative, although a few 'burned their boats' by using $3x - 1 = 0$ or numerator = denominator.

The gradients at 0.6 and 0.8 were also well done, and most then explained how this related to P being a minimum point. However, some candidates wasted time and effort in finding the second derivative, usually incorrectly. We allowed this, if fully correct.

(iv) This part proved to be a bit more demanding, and few candidates scored all 7 marks. Errors such as $x = (u - 1)/3$ or $x = (u/3 + 1)$ were found, though most achieved the M1 for substituting du/3 for dx. We condoned missing du's and dx's in this instance, though this is not always the case. Leibnitz would not recognise his notation in some solutions!

The integral was quite often incorrect – usually missing the ln u − and quite a few got the limits for u incorrect, or used the x- limits of 2/3 and 1 instead.

4754 Applications of Advanced Mathematics (C4)

General Comments

This paper was more comparable with that of June 2007 than the rather more straightforward paper of June 2008.

Section A provided questions which were accessible to all. In Section B question 8, in particular, gave the opportunity for very good candidates to show their skills and understanding, thus achieving a greater differentiation than last year's paper. The Comprehension proved more difficult than other such recent papers. In particular, the answers involving worded responses were not sufficiently clear.

Candidates should be advised to
- answer questions as required in radians or degrees
- beware of prematurely approximating their working
- include constants of integration where appropriate
- use the rules of logarithms correctly
- give complete explanations when answers are given
- think carefully before writing in the Comprehension paper answer spaces.

Centres are reminded that candidate's scripts for Paper A and Paper B are to be attached to one another before being sent to examiners.

Comments on Individual Questions

Paper A

Section A

Section A contained questions accessible to all candidates.

1) It was rare to see a fully correct solution to this question.
The method for the first part was almost always well understood and although there were some errors, 3 or 4 marks were usually obtained. The angle was often given incorrectly in degrees. In the second part, one common error was to use $\sqrt{17}\cos(\theta - 0.245)$ i.e. using the incorrect sign, and another was to incorrectly obtain the final answer using 2π- 0.511. Here candidates were penalised if they had inaccurate answers from premature approximation or if they incorrectly gave their answers in degrees.

2) The correct method of partial fractions was almost always used. Some candidates, however, felt that they could do partial fractions for $1/(x+1)(2x+1)$ and then include the x from the numerator at the integration stage.
The most common errors lay in the integration. Although $\ln(x+1)$ was usually found, the ½ was usually missing in $-1/2\ \ln(2x+1)$. $-1/2\ln(x+1)$ was another common error. The constant of integration was also often omitted. On this occasion incorrect further logarithmic work was not penalised once the correct answer was obtained.
The majority of candidates obtained the first 5 marks.

3) Most candidates successfully separated the variables and integrated. The main error was to fail to include and then find the constant of integration. For those who did include the constant, it was often followed by poor work when using the rules of logarithms or exponentials.

4) The majority of candidates attempted to rotate the curve around the correct axis.
The most common error was in the use of limits. These were often omitted or *x* limits were commonly used (±2) in the function of *y*. Many candidates seemed to fail to understand that when integrating a function of *y* that d*y* and not d*x* was needed in $\int x^2 dy$ or $\int (4-y)dy$.

5) There were many good solutions. The general method was understood but a few did try to eliminate *t*. Common errors included the omission of the constant, *a*, e.g. $x=at^3$, $dx/dt=3t^2$ was common. Similarly for d*y*/d*t* , failure to include *a* and -2*t* was common. Those that used the quotient rule for d*y*/d*t* often incorrectly obtained $[(1+t^2).1- 2at] / (1+t^2)^2$. The final part was usually successful even when the differentiation had not been completed correctly.

6) Candidates sometimes find trigonometric questions difficult but this time there were many good solutions. The main errors were giving -45° instead of 135° as a solution and some rather long complicated unsuccessful proofs to establish the given equation. Most candidates used $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$ neatly and efficiently. For those that could not establish the result, it was disappointing to see so many candidates did not proceed to solving the equation. Another occasional error was $\cot\theta = 2$, $\tan^{-1}\theta = 2$, $\theta = \tan 2$.

Section B

7) This proved to be a high scoring vector question. Examiners needed to take great care here as in many cases the use of the wrong vectors could lead fortuitously to apparently correct answers that were incorrectly obtained.

(i) The vector and vector equation were usually correct although occasionally only one was found.

(ii) This was usually fully correct. Some candidates failed to clearly show which vectors they were using or to show their method sufficiently. The general method was well known. Some candidates having correctly found the normal vector did not use it in the angle calculation. Many incorrect choices of vectors could apparently lead to the correct answer e.g. using **n**= $\begin{pmatrix}1\\0\\-1\end{pmatrix}$ or $\begin{pmatrix}1\\0\\1\end{pmatrix}.\begin{pmatrix}1\\-2\\0\end{pmatrix}$.
Others failed to find the acute angle as required.

(iii) Once again the incorrect vectors were often used and could lead to 45° by chance. The main error here was failing to establish how the given angle was obtained when their value from the scalar product was $-1/\sqrt{2}$. The answer was given so they needed to find $\phi=135°$ and then take it from 180°. For some, $+1/\sqrt{2}$ was found directly and the problem was avoided.

(iv) This was usually successful provided the correct angle had been found correctly in (ii). The commonest error was dividing 71.57 by 45 rather than using the sines.

(v) The method here was well known but it was prone to simple numerical errors when finding μ. Those that found the point of intersection correctly did not always proceed to the final part to find the distance. Those who did, often used the co-ordinates of the point of intersection, instead of the direction vector between (0,0,2) and (-2,-2,1), which gave the same answer from incorrect working.

8) This question provided marks accessible to all candidates but also gave the opportunity for good candidates to show their understanding and skills.

(i)A and (ii)A Both these parts needed GCSE work to establish the size of the angle and then some basic trigonometry. The answers were given and so they needed to explain clearly how the angle was obtained, that AB=2AC and show some trigonometry. Many answers lacked sufficient detail to obtain the E mark. Part (ii)A was more successful than (i)A.

(i)B The double angle formula was often incorrectly quoted. Some poor algebra followed the substitution of √3/2 to the given result.
$2\sin^2 15 = 1-\sqrt{3}/2$ leading to $2\sin 15 = \sqrt{(1-\sqrt{3}/2)}$ was common. Some candidates did not use a double angle formula or used a form that required substitution other than that of cos 30°.

(i)C It was not sufficient to say π =3.14159…. ,$6\sqrt{(2-\sqrt{3})}$=3.105828….so $\pi > 6\sqrt{(2-\sqrt{3})}$. Some said ‘half a circle=π’ or Area=πr^2. An appreciation of the comparison of the circumference with the perimeter of the polygon was needed. Another common error was to give the perimeter as $6\sqrt{(2-\sqrt{3})}$ from 12xAC instead of 12xAB.

(ii)B The double angle formula was not well known. For those that substituted correctly for tan 30 there were some efficient ways of showing the required result.

(ii)C The equation was usually solved correctly although a large number misquoted the quadratic equation formula. As in (i) C many failed to compare the perimeter with the circumference or made them equal rather than using inequality signs with reasons.

(iii) Weaker candidates should be advised not to overlook the possibility of some relatively easy marks at the ends of questions. Common errors here included incorrect rounding or giving exact answers.

Paper B

The Comprehension

1) This was often successful although some candidates failed to show how the total of 1 was obtained. ¼=0.25 is not sufficient to derive the figure. Some candidates failed to convince that that they were finding the average score for each player rather than just adding up the numbers in Table 2 and dividing by 8. Others tried to argue from probability;- choosing C or D is ½ so 1/2 x 1/2 =1/4, or stating that there were four equally likely outcomes.

2) This question was poorly answered.
Candidates, generally, failed to interpret the inequalities in context. Where references to the First World War were made, they were rarely detailed enough to illustrate the given inequalities fully although part (ii) was usually better than part (i).

3) This was often very successful with some good algebraic solutions of (1x2+(-2)(n-2)/n= -1.999 or equivalent. Some candidates used other similar equations and some used n and n+2 leading to 5998 and then added the extra 2. Some candidates were equally successful using a trial and error approach.

4) Those who correctly substituted values in b+w<2c were usually successful.

5) This was well answered by many. Some failed to realise they needed alternating C's and D's and so were not able to score further marks.

6) (i) Few candidates realised that the scores were increased by two points per round –although some referred to that in (ii). 'Scores will be positive 'or 'scores can never be negative' were insufficient answers.

(ii) Although many did realise that there would be no difference as the rank orders remained the same, many others did not realise the effect the change would have. They felt that it would be a draw or that there would be a different winner or no winner, or that the person with the highest score or the one that defected the most would win.

7) The expected answer that the companies would benefit by mutually agreeing to spend less, or not at all, on advertising was missed by many.
Credit was given for other reasonable answers which involved cooperation. Selling to 50% of the island each was the most common of these.
Some candidates failed to give an answer in (ii) which was consistent with their agreement in (i).

4755 Further Concepts for Advanced Mathematics (FP1)

General comments

Overall the candidates performed well, showing a good grasp of the material.

The standard of written argument varied; many candidates demonstrated excellent use of mathematical notation but some seemed to lack familiarity with the standard conventions for setting out clear mathematical arguments.

As has been the case with previous papers, some candidates dropped marks through careless algebraic manipulation, and a smaller number by failing to label diagrams and graphs clearly.

Comments on Individual Questions

1) **Matrices**

Usually this was well answered and gave candidates a good start. Some forgot to use the determinant, some miscalculated the determinant, and some used the determinant correctly but did not adjust the elements of **M** when finding the inverse.

Some candidates ignored the instruction to use their inverse matrix to solve the equations and so lost 3 marks.

2) **Complex roots of a cubic**

The large majority of candidates were able to show that $z = 3$ was a root of the equation. Usually they demonstrated this by showing that substituting $z = 3$ did yield zero. Some showed that the expression factorised with $(z - 3)$ as one factor.

Solving the quadratic equation resulting from division by $z - 3$, either using the quadratic formula or by factorisation, was usually successful. Some candidates erred in the quadratic expression and those that then found real roots should have asked themselves why the examiner had posed the question in terms of z. A surprising number of candidates thought $\frac{\sqrt{4}}{2} = 2$.

3) **Graph sketch and inequality**

This question revealed some short-comings in dealing with inequalities. Few candidates scored highly on part (ii).

(i) This graph should have been simple to sketch, based on AS core work. However, a large minority of candidates failed to score any marks. Many candidates attempted to plot the graph, rather than to sketch it. This method was rarely successful.

(ii) Candidates who realised that drawing $y = x + 3$ on their graph would demonstrate where the solutions lay were the most successful by far. Solving a simple quadratic equation found the intersections and the regions could be seen from the graph, although dealing with the asymptote at $x = -4$ and the producing correct inequalities still needed some thought.

Most candidates multiplied the inequality given by (x + 4) without considering whether this was negative or positive and then incorrectly produced two inequalities for x with -5 and -2. This earned no marks, although some managed to recover the situation by realising that $x = -5$ and $x = -2$ were 'critical values' and then going on to solve the inequality correctly.

A few candidates correctly solved the inequality by considering the two instances of $x > -4$ and $x < -4$, or by multiplying both sides by $(x+4)^2$, or by subtracting one side of the inequality from the other, although these methods must have taken up considerable time compared with the method of using intersecting graphs.

Generally only the very best candidates scored full marks for this question, even though it was quite simple if a graphical approach was used.

4) **Roots of a cubic**

Usually this was successfully answered, although those who worked throughout with w to find p and q often forgot to state the values of the roots and lost one mark.

Frequent errors were: forgetting that $a = 2$ in the root relationships; expanding $(x - 2w)(x + 6w)(x - 3w)$ and forgetting that the coefficient of x^3 had to be adjusted to be 2; forgetting to use the sum of the roots, so that w was either not found, or was assumed to be one.

There were also surprising numbers of candidates who solved $-6=\frac{p}{2}$ to give $p = -3$ and/or $-\frac{9}{2}=\frac{q}{2}$ to give $q = 2.25$.

5) **Method of differences**

Many candidates earned full marks for this question.

(i) This was usually correctly shown, although many candidates lost a mark by omitting brackets in the numerator of the algebraic fraction when forming a common denominator, resulting in incorrect workings.

(ii) This was usually started in the right way. Candidates who stated the correct relationship between the summation required and the expression in part (i) did not fall into the trap of forgetting the factor of $\frac{1}{5}$ needed to reach the solution. Some candidates obviously had to perform a rescue at the end. Some knew that 5 was a problem but put it in the wrong place, losing the accuracy mark. Many candidates wasted time and effort by multiplying out the denominator of the algebraic fraction, rather than preserving factors in their expressions.

A small number of candidates failed to begin this part correctly and tried either to start an inductive proof, or to use a series involving $\frac{1}{3\times 8}+\frac{1}{8\times 13}$ + ..., apparently believing that this could lead to cancellation of terms.

6) **Proof by induction**

Many candidates showed a good understanding of proof by induction. There were many excellent answers with full explanation and grammatical details included. Those who took shortcuts with the wording of the argument were generally less successful, often losing one or two marks at the end by failing to present a complete argument.

The case $n = 1$ should consider both the first term of the series and the expression for the sum of one term, *and* point out that these are the same.

'Assume true for $n = k$' is not easily abbreviated; 'Assume $n = k$' does not convey the same meaning. If at this stage in the proof the expression for the sum is shown with $n = k + 1$, it should be made clear that this is the target for the following analysis.

There were a number of candidates who do not appreciate the difference between $7k - 4$ and $\sum 7r - 4$ and some candidates were clearly confused by the distinction between r, k and n.

Various errors arose when adding the $(k + 1)$th term to the sum of the first k terms. Some candidates added the kth term, rather than the $(k + 1)$th term. Others added the $(k + 1)$th term to the expression for the sum of the first $k + 1$ terms.

Some candidates failed to manipulate the factor of ½ out of the expression, and some algebraic manipulation was incorrect or unconvincing and so did not receive full credit.

The final steps in the proof are more easily explained if the structure of the sum of the first $k + 1$ terms is shown unsimplified, with $k + 1$ replacing n. Students should make it clear that they understand that they have shown that '*if* the result is true for $n = k$ it is true for $n = k + 1$', which is not quite the same as 'the result is true for $n = k$ and $n = k + 1$'. Finally, it is essential to use the fact that the result is true for $n = 1$ to complete the chain of reasoning.

Some candidates seemed to believe that this proof is solely about algebraic manipulation and failed to appreciate that the logic of the argument is essential.

7) **Curve sketching**

This question was successfully answered by many candidates, but some answers would have been improved by using half a page to sketch the graph, rather than squashing it into a few lines. The use of graph paper is unnecessary. Some sketches were carelessly rough; intercepts with axes, asymptotes and approaches to asymptotes must be clearly shown.

(i) Coordinates were asked for explicitly but some candidates did not express their answers as coordinates.

(ii) This was usually fully correct. Weaker candidates sometimes assumed that $y = 0$ was the horizontal asymptote.

(iii) Many candidates did not show sufficient workings to earn the method mark, even if they did give the correct approaches to the horizontal asymptote.

(iv) Many candidates who could have earned full marks failed to do so because of sloppy sketches that did not show clearly the approaches to the asymptotes, the asymptotes themselves, or the points where the graph crosses the x and y axes.

8) **Loci on the Argand diagram**

This question caused the most problems for candidates.

Many candidates were unfamiliar with the notation needed to describe loci in the complex plane.

(i) Modulus signs were missing in many cases and $z + 4 + 2j$ or $z - 4 + 2j$ were often seen, rather than $z - (4 + 2j)$ or $z - 4 - 2j$.

(ii) This part was often omitted altogether. Weaker candidates were clearly uncertain how to deal with loci involving the argument of a complex number. Many of the strongest candidates also made errors; for example brackets are important here so $\arg z - 4 - 2j = 0$ is not a correct expression.

(iii) This was frequently omitted but when tackled was usually well done. Again it is important to use brackets; $4 - \sqrt{2} + 2 + \sqrt{2}j$ is incorrect. Some candidates tried this part using the coordinate geometry of an intersection of a line with the circle, usually successfully, but sometimes leading to intractable algebra.

(iv) This was generally answered correctly by the strongest candidates. Some candidates earned a mark for the inequality giving the interior of the circle, but could not cope with the inequalities involving the argument of a complex number. The weaker candidates generally scored no marks for this part of the question.

9) **Matrix transformations**

This question was generally answered well. A small number of candidates failed to complete this question, presumably due to time pressure.

(i) Few candidates understood that matrix multiplication is associative.

Most could multiply the matrices correctly, but a surprising number calculated **MNQ** rather than **QMN**. Many made simple arithmetic mistakes.

(ii) N was usually correctly described but Q was often thought to represent a reflection and M was often called an enlargement instead of a two-way stretch.

(iii) Some candidates thought that they had to calculate and use the matrix product **NMQ**, rather than proceeding with **QMN**, which they had already obtained in part (i). Many played safe and worked through each matrix in turn. Those that tried to apply the transformations directly to the triangle without using matrices usually made errors.

Candidates' diagrams sometimes lacked labels and/or scales, despite the explicit instruction to label the image of each point clearly.

4756 Further Methods for Advanced Mathematics (FP2)

General Comments

This paper included some challenging tests for the most able, while weaker candidates were offered plenty of part-questions which should have been familiar and reasonably straightforward, and indeed there were very few very low marks: the vast majority of candidates were able to show that they knew and understood topics from across the specification. Unfortunately weaker (and many stronger) candidates struggled to sustain accuracy through the paper; their "basic" algebra was often suspect, and many believed that
$\sqrt{x^2+y^2}+y=a \Rightarrow x^2+y^2+y^2=a^2$ or even $\sqrt{x^2+y^2}+y=a \Rightarrow x+y+y=a$ (Q1(b)(ii))
or $\sqrt{t^2+2t+5}=\sqrt{(t+1)^2+4}=(t+1)+2=t+3$ (Q4(iv)), which was rather worrying.

The presentation of scripts was usually good, although once again there were some candidates who split up questions and even part-questions, which does not help them or the examiner. Some candidates used three 16-page answer books. It is expected that candidates sketch graphs on the lined paper: there is no need to use separate pieces of graph paper. There was very little evidence of any time trouble.

In question 3, candidates were asked to "find" $\frac{dy}{dx}$ for y = arcsin x. Many candidates just quoted the result from the formula book without deriving it. In future no credit will be given in such cases: if candidates are asked to "find" a result which appears in the formula book, it is clear that a derivation is expected. When candidates did attempt to derive other given results, their answers often lacked the necessary detail: each step should be shown. For example, when the result of a definite integral is given, we would like to see the explicit substitution of limits.

In Section B, the overwhelming majority of candidates chose the hyperbolic functions question: although the Investigations of Curves question did produce some creditable responses this time, the majority of attempts at it were fragmentary.

Comments on Individual Questions

1) (Maclaurin series, polar curves)
The mean mark on this question was about 10 (out of 16).

(a) Most candidates approached part (i) in the way intended, quoting the series for ln(1 + x) from the formula book, replacing x by –x, and subtracting. Some gave only two terms. Quite a few wasted time by deriving the series for ln(1 – x) by differentiation, and several even derived the series for ln(1 + x); fortunately only a very few tried to differentiate $\ln\left(\frac{1+x}{1-x}\right)$. The range of validity was badly done, with many quoting non-strict inequalities or forgetting to answer at all.

The equation in (ii) was very often, but not always, solved correctly, with some believing that $4x = 2 \Rightarrow x = 2$. Most then went on to substitute their x into their series from (i), although a few just used their calculators to find ln 3. The instruction to give the answer to three decimal places was usually followed.

(b) Most candidates were able to produce a good sketch; where marks were lost it was usually because they failed to show that $r(0) = a$ and $r(\pi/2) = a/2$ (these sometimes appeared without the a) or drew a graph with a "dimple" in the top. Then most were able to show that $r + y = a$ by using $y = r \sin \theta$, although some just checked the given statement for one or two points. A significant number of candidates found it difficult to obtain a Cartesian equation for the curve: the formula $r^2 = x^2 + y^2$ was often quoted, but not always used. Of candidates who could produce a correct equation, a substantial number spoiled their answer by poor algebra.

2) (Matrices)
This was the best-answered question: the mean mark was about 14 (out of 19).

(i) Most candidates were able to produce a correct characteristic polynomial, although an equation appeared more rarely. Most expanded by the top row, although expansion by the middle row would have produced an answer more easily. The determinant of **M** was not so well done: those who worked it out from scratch were generally more successful than those who used the characteristic equation, because most reversed all their signs to obtain a leading coefficient of 1: thus $\lambda = 0$ gave $-\det(\mathbf{M})$. 1 was another common incorrect answer.

(ii) This part of the question had three "sub-parts" and some candidates did not answer all of them. Most candidates successfully showed that -1 was an eigenvalue and that the other two eigenvalues were not real. Then a substantial number missed out the part requiring the eigenvector to be found; of those who tried this, most knew the procedure, but there were many algebraic and arithmetical slips. Most candidates correctly solved the simultaneous equations, with the most common method of solution being elimination: a few found and used the inverse matrix here. Not many followed the instruction to "write down".

(iii) Only a minority of candidates stated the Cayley-Hamilton theorem correctly in words: if the intention to replace λ in the characteristic equation by **M** was clear, the mark was awarded. Candidates sometimes just wrote down the result that they were required to show, but most knew how to use this result to introduce $\mathbf{M}^{-1}$, although there were many sign errors.

(iv) Most candidates failed to produce a completely correct inverse. There was a roughly equal split between those who tried to use their expression in (iii) and those who found cofactors, transposed and divided by their determinant. There were a great many arithmetical errors with both methods.

3) (Calculus with inverse trigonometrical functions, complex numbers)
The mean mark for this question was about 11 (out of 19). Part(a) was done much better than part (b).

(a) The sketch in part (i) was mostly correct although there were a few candidates who reflected the whole sine graph in $y = x$, thus not producing the graph of a function. As mentioned above, $\frac{dy}{dx}$ was often quoted without proof. Candidates often omitted the explanation of the sign of their answer. The integral in part (ii) was very well done although some introduced an extra factor of $\frac{1}{\sqrt{2}}$.

(b) Most candidates were able to recognise $C + jS$ as an infinite geometric series and sum it correctly to gain the first 4 marks. It was then necessary to "realise the denominator" by multiplying numerator and denominator by an appropriate expression, and only the better candidates could do this. Those who worked with exponential forms for as long as possible generally made fewer mistakes and more progress than those who attempted to introduce trigonometry at an early stage.

4) (Hyperbolic functions)
Although this was by far the more popular question in Section B, and each part of the question allowed candidates who had not succeeded in the other parts to attempt it, the mean mark was less than 10 (out of 18).

(i) This part was usually answered well, although weaker candidates did not always "prove from definitions involving exponentials".

(ii) This part was also done well, although some candidates failed to get beyond an exponential expression for y or confused themselves by poor choice of variables. Only the best candidates could comment correctly on why the minus sign should be rejected.

(iii) Many candidates could make the hyperbolic substitution and perform the integration correctly, although the weakest only substituted for $\sqrt{x^2+4}$ and not for "dx", or mixed up $\frac{dx}{du}$ and $\frac{du}{dx}$. The last two marks, for obtaining the printed answer, were rarely awarded: again, "proof by blatant assertion" was commonly employed.

(iv) Completing the square was by no means universally remembered and such things as $t^2+2t+5=\left(t^2+\sqrt{2}t\right)^2$ were seen. Most successful solutions used $x = t + 1$ and the result in (iii), although some candidates went back to the beginning and substituted $t = 2\sinh x - 1$. Often (iii) was not used at all, with a "result" such as $\operatorname{arsinh}\frac{t+1}{2}$ appearing. Lack of necessary detail, such as explicit substitution of limits, in deriving the given answer prevented very many from scoring full marks here.

5) (Investigations of Curves)
Few attempts at this question were seen. Most were fragmentary, but a small number of candidates gained a substantial number of marks.

(i),(ii) These parts could be approached via algebra or trigonometry. The trigonometric approach was far easier, yet there were creditable attempts using both methods.

(iii),(iv) The sketch, where attempted, was done correctly. Those who could produce the sketch went on to gain most of the rest of the marks.

4757 Further Applications of Advanced Mathematics (FP3)

General Comments

This paper was found to be much more accessible than those of previous years. Very many scripts contained substantially correct solutions to all three chosen questions, and about 40% of the candidates scored 60 marks or more (out of 72). The choice of topics was similar to last year, with questions 1 and 2 being considerably more popular than the others.

Comments on Individual Questions

1) *(Vectors)*
   There were very many good answers to this question, and the average mark was about 18 (out of 24). Although some candidates used lengthy (yet valid) approaches, in general efficient methods were well known and applied competently to obtain the required results. The exception was part (iii), finding the shortest distance between two parallel lines, where a substantial number of candidates used the scalar product $\left|\mathbf{AB}\,.\,\hat{\mathbf{d}}\right|$ instead of $\left|\mathbf{AB}\times\hat{\mathbf{d}}\right|$.

2) *(Multi-variable calculus)*
   This was the most popular question, attempted by about 85% of the candidates. It was also generally well answered, and the average mark was about 18. The partial derivatives were usually found correctly in part (i), with some candidates preferring to multiply out the expression for *z* first. Almost all knew how to find the stationary points in part (ii), and a good proportion obtained all three points correctly. In part (iii), errors were frequently made in the normal vector (usually with the sign of the *z*-component). Some candidates thought that the normal vector was the required answer to this part, and some gave the equation of the tangent plane instead of the normal line. The work on small changes in part (iv) was generally done well, although quite a number of candidates substituted the given coordinates into the equation of the surface and multiplied out, rarely making any worthwhile progress. Part (v) was sometimes omitted, but very many were able to find the required point and to show that there are no others. A fairly common error was to start with $\partial z/\partial x = -27$ instead of $\partial z/\partial x = 27$.

3) *(Differential geometry)*
   This was the least popular question (attempted by about one third of the candidates), and it was also the worst answered question this year, with an average mark of about 14. A fair number of candidates seemed to be unfamiliar with the half-angle formulae, and so found several parts of the question to be inaccessible. In part (iii), most candidates could use their intrinsic equation to find the radius of curvature; those who used the parametric formula obtained a correct expression easily enough, but simplifying this to the given result proved to be challenging. Finding the centre of curvature in part (iv) was well understood and often done accurately; when marks were lost it was usually due to arithmetic slips or sign errors. Many candidates omitted part (v) (finding the curved surface area), and several others knew that they had to integrate $(1-\cos\theta)\cos\frac{1}{2}\theta$, but could not find a way of doing so.

4) *(Groups)*
The average mark on this question was about 16. Parts (ii), (v), (vi) and (vii) were answered very well, with candidates demonstrating a good understanding of groups. In part (i), many candidates lost marks for not showing enough working. Just asserting that the group *G* is generated by the element 3 is not sufficient; this should be established by listing all the powers of 3. Similarly in part (iv), the given result $\mathrm{ad}(x) = \mathrm{c}(x)$ was not always clearly shown to be true for all values of *x*. In part (iii), an explicit one-to-one correspondence between the elements of *G* and *H* was expected. Many candidates did not appear to understand the instruction 'specify an isomorphism' (which was quoted from the specification), and gave reasons why the two groups should be isomorphic.

5) *(Markov chains)*
This was the best answered question; the average mark was about 19, and about a third of the attempts scored full marks. Parts (i), (ii), (iv), (vi) and (vii) were all answered very well, with candidates demonstrating sound understanding of the techniques and confidence in using their calculators. In part (iii), the proportion who correctly used the diagonal elements in the transition matrix (to calculate the probability that the system remains in the same state) was higher than in previous years, but very many candidates found probabilities for level 14 and level 15 separately and assumed independence. Finding the run length in part (v) was sometimes omitted, but the correct formula $p/(1-p)$ was quoted by a good proportion of the candidates. Some thought that they needed to add or subtract one from this value. In part (vi), several candidates preferred to form simultaneous equations and solve them to find the equilibrium probabilities, despite the suggestion in the question paper that they could simply consider a high power of the new transition matrix.

4758 Differential Equations (Written paper)

General Comments

The standard of work was generally very good, with many candidates demonstrating a clear understanding of the techniques required. Almost all candidates answered Questions 1 and 4, with Question 2 being the least popular choice. Candidates often produced accurate work in solving second order differential equations, but they seemed reluctant to adapt their standard method to take account of the information which was given to them in the question and which was intended to help them in Q.1(iv) and Q.4(ii).

With regard to graph sketching, it should be noted that the expectation in this unit is that any known information should be indicated on the sketch, i.e. given initial conditions and relevant results found earlier in the question. In addition, any particular features (e.g. oscillating, approaching an asymptote, bounds) should be clearly shown. Calculations beyond those already requested in the question are not required.

Comments on Individual Questions

1) Second order differential equation

(i) The method here was well-known. The main problem was in the choice of an appropriate trial function for the particular integral, with many opting for the incorrect y= Acos5t + Bsin5t. Candidates making this error were still able to gain method marks.

(ii) This was answered well by those candidates who had obtained a general solution in (i). Unfortunately those who persisted with the error noted in part (i) applied the initial conditions to what was in effect a complementary function.

(iii) Attempts at this sketch were reasonable, with candidates often gaining three out of the four marks. The amplitude was usually shown as constant, rather than increasing.

(iv) Most candidates gained full marks, but relatively few chose to take the hint, preferring to find the particular integral for themselves, rather than verify the given one.

(v) This was rarely answered correctly, with appropriate comment on the oscillatory nature and the boundedness of the two solutions being compared

2) First order differential equation

(i) Many candidates completed this correctly. Any loss of marks was due to either sign errors in the integration by parts or omission of an arbitrary constant.

(ii) Candidates were able to use the given approximations but then seemed to struggle with giving a clear explanation as to why the arbitrary constant was zero.

(iii) This was answered well by those who had been successful in part (ii).

(iv) Candidates were able to find the correct general solution to this differential equation, but as in part (ii) struggled to deal with the condition that y remains finite as x tends to zero

3) First order differential equations

(a)(i) This was well-answered by almost all who attempted it. The few who chose to ignore the request to find complementary function and particular integral and instead attempted an integrating factor method were always unsuccessful.

(ii) There were no problems with this.

(iii) A surprising number of candidates were unable to find the amplitude and fewer still produced a sketch which showed an oscillation of constant (and labelled) amplitude.

(b)(i) This question was different to most previously asked and it was very pleasing that the vast majority of candidates who attempted it were able to produce very good solutions.

(ii) This was well-answered, with candidates able to use their results to part (i) to produce good sketches.

4) Simultaneous differential equations

(i) The method here was applied with pleasing algebraic and numerical accuracy.

(ii) The majority of candidates proceeded to find the particular integral from scratch, rather than heeding the (helpful) advice in the question. No marks were lost for this, but time was wasted.

(iii) The method was known, but accuracy errors were abundant.

(iv) Candidates seemed not to realise what was required here. Some addressed the condition for large values of t, but did not see that they had then merely to equate their expressions for x and y.

(v) As in Question 3, there were problems in calculating amplitudes.

4761 Mechanics 1

General Comments

Most of the candidates were able to make some progress with every question and many gave at least good answers to most of the questions. It was very pleasing to see many perfect solutions to Q3 which required calculus and Q4 on projectile motion. The two questions requiring the use of vector notation were not answered well by many candidates, partly because of errors caused by poor notation and partly because of poor understanding.

The presentation of the scripts was generally reasonably good but some candidates made mistakes because they were unable to follow their own working

Although many candidates showed a good understanding of all of the topics examined, others seemed to have no knowledge of some of them; for instance, quite few candidates did not know how to resolve or deal with a problem involving equilibrium on a slope.

Comments on Individual Questions

Section A

1) **Use of a velocity-time graph**

Most of the candidates who attempted to solve the problem using the areas under the graph were quite successful and there were many perfect answers. Few of those who tried to apply the constant acceleration results made much progress with parts (i) or (ii).

(i) This was done well by many candidates.

(ii) There was some confusion with the signs but the most common error was to find the time taken to go from B to C instead of from A to C

(iii) Many candidates answered this part correctly after failing to make progress with parts (i) and (ii). Quite a few candidates did not recognise that the 10 m displacement from B to C was negative.

2) **Use of vectors to represent forces and static equilibrium**

There were many sign errors seen in the attempts at this question and many examples of poor notation that hindered accurate solutions.

(i) This part was generally done very well, the only common mistake being to find the angle with the horizontal instead of the vertical.

(ii) Many candidates wrote $w\mathbf{j}$ instead of $-w\mathbf{j}$. Quite a few candidates gave answers such as – j and w – j .

(iii) Many candidates used $\mathbf{T}_1 + \mathbf{T}_2 = \mathbf{W}$. There were many sign errors including $k+10=10 \Rightarrow k=10$, which was quite commonly seen.

3) **Interpretation and the application of calculus in kinematics**

It was very pleasing to see the efficient way in which many of the candidates used calculus to answer this question

(i) Most candidates made a statement to the effect that constant acceleration would have been represented by a straight line.

(ii) This was generally done very well with relatively few attempts not involving calculus. The interpretation was usually good but a fairly common error was to suggest that a = 0 meant that the velocity was now constant or 0, the latter being a general comment not relevant to this problem.

(iii) The integration was typically done well and it was pleasing to see so many adopting this correct method. Errors included reversing the limits or substituting t = 3.

4) **The greatest height reached by a projectile**

There were many very confident, neat and efficient solutions to this question.

(i) Most candidates did this correctly.

(ii) The most common error here was to not establish the given answer sufficiently clearly.

(iii) It was pleasing to see so many candidates using the efficient method of using $v^2 = u^2 + 2as$. The common error was to use u = 32 instead of $u \sin \alpha$. Those who went via the time to the highest point and then used the equation for the height at time t were more likely to make mistakes. Apart from the wrong value for 'u', common mistakes were to use the half the time or even the whole time taken to travel the 44.8 m.

5) **A 2 dimensional kinematics problem in vector form and forming the cartesian equation of a path**

The answers of many candidates suffered from their poor use of vector notation and/or understanding of how to manipulate vectors and interpret their results.

(i) Those who kept the vector form usually did quite well but a substantial number worked out **r**(4) instead of **v**(4). Many candidates rapidly lost any trace of a vector and some of those that retained the vector form gave the speed instead of the velocity.

(ii) This part was done quite well by those who retained a vector form except that many ignored the definition of the direction of the **j** direction, assumed it was horizontal and so said there was zero horizontal force.

(iii) This was done very well by some candidates; others could only write x and y in terms of t or did not know what was required. Many showed no attempt.

6) **The static equilibrium of a box on an inclined plane**

Quite a few candidates recognised the standard situations being investigated and wrote down perfect solutions. Almost all of the candidates who knew how to resolve made some progress through the question but quite few did not seem to know this technique. Candidates who resolved horizontally or vertically instead of parallel and perpendicular to the plane rarely included all the forces correctly resolved. As in recent series, many candidates occasionally or consistently used sine instead of cosine and vice versa and quite a few occasionally or consistently used mass when they needed weight; these errors were found in all of the parts and have not been repeated in the notes below.

(i) This was done quite well by many candidates. A quite common error was to resolve the T instead of the weight.

(ii) This part presented problems to quite a few candidates who did not see how similar it was to part (i). Candidates who did not write out their expressions properly often found themselves working with $4g$ and m instead of $4g$ and mg. Quite a few candidates found the new mass instead of the extra mass.

(iii) The most common mistakes were to miss out the normal reaction and/or forget an arrow or a label. A good answer here helped with the latter parts of the question (as intended).

(iv) This part was not done well by many candidates who had coped well with parts (i) and (ii). Many candidates failed to carry out systematic resolution parallel to the plane. Most commonly, forces were omitted or not resolved.

(v) This part was not done well even by some candidates who had made a good or even correct attempt at part (iv). The errors were again most commonly the omission of forces or failure to resolve them. Many candidates seem to believe that the normal reaction is the resolved part of the weight perpendicular to the plane.

7) **The dynamics of a system of connected particles**

It is very much regretted that this question was poorly worded and there was an ambiguity about the direction of the frictional force in parts (i), (iii) and (iv). The question does not say that the system starts from rest and so the velocity need not be in the direction of the acceleration. This ambiguity did not cause problems in part (i) but in part (iv) it meant that a candidate could reason properly and fail to establish the given answer. Fortunately, in part (iv) almost all of the candidates did take the velocity and hence the friction to be in the directions intended. The very few candidates who obtained a different acceleration to the given answer mostly seemed to go straight on to use the given value in the rest of the part and their attempts were credited bearing in mind that they had made no mistake. Any scripts where it was thought that there could be evidence of a candidate being disadvantaged were reviewed by the Principal Examiner.

(i) This was done well by most candidates. The most common error was to omit the frictional force.

(ii) Many candidates wrote done several or even all of the modelling assumptions stated in the question and were not given the mark. The only exception made was to allow the quotation that it was a 'light inextensible string' rather than just 'an inextensible string'.

(iii) The diagrams were not always clearly labelled and many candidates falsely gave the tensions as 6*g* and/or 93.8.

(iv) The 'round the corner' method for finding the acceleration was accepted but the final mark for establishing the given answer was awarded only if it was made clear how Newton's second law was being applied to the situation. Candidates who established separate equations of motion for the sphere and the box and solved them simultaneously were generally more successful overall in this part; the common mistakes made in this method were not to have the same sign convention in the two equations or to omit a force acting on the box. Many candidates who used the 'round the corner' method failed to find the tension in the string as they did not produce an equation of motion for either the box or the sphere; many using this method took the tension to be the force in the string that would hold the sphere in equilibrium. Quite a few candidates did not match their forces to the appropriate mass.

(v)
(A) Many candidates stated that the acceleration was still 2.5 m s^{-2}, even though they used *g* in part (v)(B).

(B) This was done quite well but many candidates did not derive the given expression properly; expressions with wrong signs in the penultimate line 'became correct' in the final statement. Quite few candidates solved the quadratic equation instead of deriving it.

(C) Many candidates did not realize that they had to add the time to the string breaking with an acceleration of 2.5 m s^{-2} upwards to the positive root of the equation in (v)(B). A common error was to add the two quadratic roots, either signed or in modulus. However, many strong candidates answered this part efficiently and accurately.

4762 Mechanics 2

General Comments

Many good responses to this paper were seen and the majority of candidates could attempt at least some part of every question and gain credit for their efforts. The standard of presentation was variable and some candidates did not appreciate that poor notation and failure to state the principles or processes being employed could lead to avoidable errors and to loss of marks. As in previous sessions, those parts of the questions that required a candidate to explain or show a given answer were the least well done. Many candidates did not give enough detail in either case. As has also happened in previous sessions, those candidates that attempted to work back from a given answer usually obtained less credit than those who had attempted to employ the principles required to solve the problem.

There was evidence to suggest that some candidates found the paper long and this was taken into account at the Award. Some candidates ran out of time because they used inefficient methods of solution, particularly in Q1 and Q4.

Comments on Individual Questions

1) This was the highest scoring question on the paper with the vast majority of candidates able to produce work worthy of significant credit.

(a)(i) The majority of candidates obtained the mark for this part but some omitted labels or failed to indicate direction of the velocities.

(ii) Almost all of the candidates showed understanding of the principle of conservation of momentum and obtained full marks for this part. However, a small number failed to show sufficient evidence of how they obtained the given answer.

(iii) Many obtained full marks for this part but a significant minority failed to explain the reasoning behind their answer. Others interpreted the fact that the direction of motion of P had to be reversed by replacing v by –v instead of taking $v < 0$

(iv) It was encouraging to see that the majority of candidates employed Newton's experimental law correctly and obtained the right answer; failure to do this was usually due to a sign error.

(b)(i) Many fully correct solutions were seen to this part.

(ii) This part caused problems for a large number of candidates. Many failed to realise that the velocity parallel to the barrier would be unchanged and applied Newton's experimental law in both directions. Others took the barrier to be at right angles to the direction specified.

2) Candidates seemed to understand the principles to be employed in this question but did not always appreciate that all of the masses needed to be considered.

(a)(i) A Only a minority of candidates could give a clear reason as to why energy would be conserved in this situation. Many stated, incorrectly, that no external force was acting. Others seemed to have little idea regarding cause and effect and offered the answer that energy was conserved because potential energy was equal to kinetic energy.

B There were few correct answers seen to this part. Some candidates seemed to understand that the reaction force was perpendicular to the table but could not explain the relevance of this to work done. Others said that it was perpendicular to it, without any indication as to what ‘it’ was. It was common to see the arguments advanced in part A repeated.

(ii) Most candidates obeyed the instruction in the question to employ an energy method and gained some marks. A majority, though, used m = 25 (instead of 50) when calculating the kinetic energy terms i.e. ignored the masses of the spheres. Despite the instructions in the question, some candidates attempted to use the constant acceleration formulae and Newton’s second law without any consideration of energy at all.

(iii) As in the previous part, the majority of candidates considered m = 25 and many went on to omit the gravitational potential energy terms as well.

(b) A large number of candidates obtained all 5 marks for this part of the question. Those that did not, usually failed because they had omitted ‘g’ from the weight term or had the cosine component rather than the sine component.

3) Many excellent answers were seen to this question but a significant minority struggled with the trigonometry and the manipulation of equations.

(i) Almost all of the candidates obtained full marks for this part.

(ii) Most of the candidates understood that they needed to take moments for this part but could not manipulate their moments equation to show the given answer. Many thought that $\tan\alpha = \sin\alpha \times \cos\alpha$ and some ‘fudging’ of the algebra was seen.

(iii) The quality of the diagrams offered was, in many cases, poor. Forces at A and/or B were missing or shown at an acute angle to the beam. Additional spurious forces such as a reaction perpendicular to the beam at the point where the weight acts were included by some candidates. Those who drew a clearly labelled and correct diagram were usually able to make more progress in the following working than those who did not. Candidates gained some credit for appreciating the need to take moments and later to resolve but fully correct equations were not always obtained. Confusion with the use of sine and cosine was common as were equations where moment and force terms were mixed. The majority of candidates recovered by using the given answer to calculate the value of the coefficient of friction.

4) Some candidates were obviously pressed for time with this question and solutions appeared rushed. However, most made some progress worthy of credit, appearing to understand the method to be employed.

(i) The majority of the candidates gained the marks for this part. The main errors made by those who did not were lack of knowledge of the formula for the surface area of a cylinder (a GCSE topic) and, even though the formula for the surface area of the shell was given, many chose not to use it. False cancelling abounded in attempts to obtain the given answer.

(ii) Those candidates that were successful in part (i) were usually as successful in this part but some did not use what they had already derived and started from scratch, causing themselves time problems.

(iii) Many fully correct solutions were seen. Even those candidates who did not do well on the previous parts of this question obtained some credit for this part. Diagrams in many cases were good. Those that were not were usually too small to be helpful and did not clearly show the centre of mass above the edge of the base.

(iv) This part of the question was found difficult by the majority of the candidates. Few set out a complete statement of how they intended to test if sliding would occur and then offered random calculations without any statement as to their relevance. Those who clearly stated the criteria they were going to use to test almost always obtained full credit.

4763 Mechanics 3

General Comments

This paper was found to be slightly more difficult than last year's, but most candidates responded well by showing what they could do, and presenting their work clearly. There appeared to be sufficient time to complete the paper, and there was a good spread of marks: about 20% of the candidates scored 60 marks or more (out of 72), and about a quarter scored fewer than half marks. Questions 1 and 2 were found harder than questions 3 and 4.

Comments on Individual Questions

1) *(Circular motion)*
The average mark on this question was about 11 (out of 19).

(i) This was usually answered correctly, although some candidates had incorrect signs in their energy equation.

(ii) Most candidates knew that they should form a radial equation of motion and substitute the given expression for v^2. However, there were many sign errors; in particular, the normal reaction was often taken to be acting towards the centre instead of away from it.

(iii) Almost all candidates considered when the normal reaction became zero, and a good proportion obtained the speed correctly. Very many obtained the value of θ (unnecessarily, as only $\cos\theta$ was needed); and some stopped here and forgot to find the speed.

(iv) This problem involving motion in a horizontal circle was found very difficult, and there were few fully correct solutions. It was common for the normal reaction to be omitted when resolving vertically. The horizontal equation of motion was quite often given correctly, although sign errors occurred frequently and a component of the weight was sometimes included. Some candidates did try to consider motion in the radial and transverse directions, but this approach was even more prone to errors. In many cases, a clear diagram would have been helpful, for both the candidate and the examiner.

2) *(Elastic energy)*
This question had an average mark of about 10 (out of 17).

(i) Most candidates applied the conservation of energy successfully here.

(ii) This was also quite well answered, although very many candidates made it more complicated than necessary. The simplest method was to verify that the loss of gravitational potential energy is equal to the gain in elastic energy, but a more popular method was to form an equation for the compression. Some candidates split the motion into two parts: constant acceleration until the car hits the buffer, followed by consideration of kinetic, gravitational and elastic energy while the spring compresses. The most common error in this part was to forget the contribution to gravitational potential energy made while the car is in contact with the buffer.

(iii) Most candidates knew basically what needed to be done, but there were many opportunities to go wrong. Common errors included omitting the work done against the resistive force, forgetting the contribution made by the compression *x* to the work done or the gravitational potential energy, and incorrect signs when forming the work-energy equation.

3) *(Dimensional analysis and simple harmonic motion)*
The average mark on this question was about 12 (out of 18). Part (a) on dimensions was answered extremely well, and part (b) on simple harmonic motion rather poorly.

(a)(i) Almost all candidates gave the dimensions correctly.

(a)(ii) Dimensional analysis was very well understood, and most candidates obtained the powers correctly.

(b)(i) Most candidates could use the period to find the constant (ω) for the simple harmonic motion, although a surprising number made no progress beyond this. Those who were familiar with formulae such as $\dot{\theta}^2 = \omega^2(A^2 - \theta^2)$ usually obtained the correct value of $\dot{\theta}$. Some preferred to use $\theta = A\sin\omega t$ and $\dot{\theta} = A\omega\cos\omega t$, which was quite efficient in this case, as the value of *t* could then be used in the next part.

(b)(ii) Most candidates obtained a relevant value of *t*, but a very large number did not have a correct strategy for finding the required time interval. Some misinterpreted the question and thought that the amplitude had now changed to 0.05 radians.

4) *(Centres of mass)*
This was the best answered question, with an average mark of about 13 (out of 18).

(a) The method for finding the centre of mass of a lamina was well understood, and very often carried out accurately. The only common errors were slips in evaluating the definite integrals, and omitting the factor ½ from the *y*-coordinate. A few candidates appeared to be unfamiliar with integration by parts; knowledge of topics in Core 1 to 4 is of course assumed in this unit.

(b)(i) Finding the centre of mass of a solid of revolution was also well understood, and most candidates were able to derive the given result.

(b)(ii) Those who started with $a > 2 \Rightarrow a^3 - 4a < a^3 - 8$ were able to establish $\bar{x} < 3$ quite easily. A popular method was to state that $\bar{x}$ tends to 3 as *a* tends to infinity, but full marks were rarely obtained this way; this statement was often not proved properly (for example, there was a lot of work involving $\infty^3 - 4\infty$ and so on), and hardly any candidates mentioned that $\bar{x}$ is an increasing function of *a*.

4764 Mechanics 4

General Comments

The standard was very high in general. Candidates showed a good understanding of the syllabus, though many found the work on rotation difficult.

Comments on individual questions

1) (i) Generally well answered, though many candidates produced much more work than was required, either when rearranging their expression or by unnecessarily deriving the expansion of $\frac{d}{dt}(mv)$ from first principles.

(ii) Most candidates produced good work for this question.

(iii) Some candidates gave part of the reasoning, but very few gave a full account. This was often left unanswered.

2) (i) The concepts were well understood here, but details were often incorrect in the algebraic manipulation and calculus.

(ii) Again the concepts were well understood, but details were often missing in the second half of the question. In most cases this led only to the last accuracy mark being withheld. Many candidates gave extra values for θ which were not physically possible for the given system; these were ignored.

3) (i) The expression for mass of an elementary ring and the resulting integral were usually found accurately. Those candidates that did so then tended to get the correct moment of inertia. Again, many of the candidates gave more working than necessary by deriving the expression for mass of the disc by integration.

(ii) Almost universally correct.

(iii) Most candidates knew the form which the equation of motion should take, though the detail was often incorrect. In many cases the incorrect detail was introduced while deriving the equation of motion from the energy equation rather than writing it down as the question asked. It was very common to see equations of the form $\ddot{\theta} = k\theta$, with k positive, being given as SHM. Finding the period from their SHM equation was usually done well.

(iv) This was generally well answered.

(v) The kinetic energy was usually found correctly. Many candidates chose to continue by using the constant acceleration formulae rather than the energy considerations asked for in the question. Those that used the work-energy equation generally gave the correct magnitude for C, though many gave much more working than is necessary when dealing with a constant couple.

4) (i) This was usually correct.

(ii) Most candidates were able to set up the differential equation, separate variables and recognise the form of the integrand. The subsequent manipulation was usually well done; candidates that simplified their integrand before proceeding were more often successful.

(iii) This was sometimes done very well. Many candidates lost a term by disregarding the lower limit or gave the velocity as 60 when calculating the kinetic energy. It was very common to see candidates using rounded values to show equality rather than retaining the exact forms. Some candidates found the work done against mg-kv2, which caused difficulties if they included GPE in their calculation of mechanical energy.

(iv) Almost universally correct.

(v) Not many accurate solutions given. Most candidates omitted the modulus signs required in the integral (v being greater than g in the motion under consideration). Of these, some simplified their expression for t in such a way as to produce the correct answer, but many candidates' either ignored the negative or stopped when they found that they needed to evaluate the logarithm of a negative number.

4766 Statistics 1 (G241 Z1)

General Comments

The level of difficulty of the paper appeared to be entirely appropriate for the candidates with a good range of high marks obtained and fewer low marks than in previous sessions. The more able candidates scored heavily on all questions while the weaker candidates often picked up some marks on all questions with question 7 on probability contributing significantly to their total.

Most candidates supported their numerical answers with appropriate explanations and working although some rounding errors were noted. The possible exception was in question 8 where the procedure for distinguishing between hypotheses did not always include specific comparisons with 10% and where the construction of the critical region was often sketchy. There was a surprising inability to use the given numerical data in question 3 to find the standard deviation.

Weaker candidates often scored a significant proportion of their marks from question 1, the first three parts of the probability question (question 7) and from the initial parts of question 8. Amongst lower scoring candidates, there was evidence of the use of point probabilities in question 8. Also in question 8, many candidates are still not meeting the requirement to define p in words.

There seemed to be no trouble in completing the paper within the time allowed and no obvious misinterpretations of the rubric although a very small number of candidates ignored the instruction to use graph paper for the histogram. It would also be very helpful if candidates could write down the question numbers on the front of the question paper.

Comments on Individual Questions

1) (i) The mode was usually correct, and most candidates also found the median correctly. However some candidates quoted locations rather than actual values and others thought that the median was 1 or 1.5. There were occasional errors such as thinking that there was a total of 180 (using ∑fx) rather than 102 cars in the survey. Some weaker candidates found the mean instead of the median.

(ii) Most line diagrams were correct although a small number joined the lines in one manner or another. Some others forgot to label at least one of their axes.

(iii) The majority identified the positive skewness of the distribution, but a significant number of candidates thought that the skewness was negative.

2) (i) Many totally correct answers were seen although candidates occasionally tried to use permutations.

(ii) This part was rather less well answered, although a good number of fully correct answers were seen. The most common error was the use of addition instead of multiplication giving ${}^{14}C_3 + {}^{11}C_2$ and this occurred very frequently.

3) (i) Almost all candidates found the mean, but a large number of candidates did not know the formula for finding the standard deviation. Those who knew how to find S_{xx} usually went on to complete part (i) successfully. However there were many incorrect attempts at S_{xx} with common variations including $1582 – 10.5^2$ or $1582 – 12×10.5$. Others gave the standard deviation as 1582/11 or √(1582/11) and some had no idea what to do with the numbers they were given. Rather fewer candidates than in recent sessions divided by 12 rather than 11 and found the

RMSD.

(ii) Almost all showed that Dwayne's monthly earnings were £1550. However the majority of candidates did not realize that all they needed to do was to multiply their standard deviation by 100, but instead tried to start again in finding the new standard deviation, almost always without success. Of those that did multiply by 100, a few then could not also resist the addition of 500.

(iii) The explanation regarding the means was usually correct but that for the standard deviations was either ignored or candidates failed to explain in context. A lack of context in explaining the means was condoned, but not in the case of the standard deviations.

4) (i) Almost all candidates correctly explained why $E(X) = 25$, although hardly any commented on the symmetry of the distribution, but instead calculated $\Sigma xp(x)$.

(ii) Very many correct answers were seen. Some candidates just found $E(X^2)$, failing to subtract 25^2 to find the variance, and occasionally candidates found the correct answer but then went on to do further calculations. Several candidates tried to work out $10 \times 0.2^2 + 20 \times 0.3^2 + \ldots$ i.e. squaring the probabilities rather than the x values.

5) (i) Although a number of fully correct histograms were seen, there were also many errors. Candidates should always draw a table to show the frequency densities even if such a table is not specifically asked for in the question. Common errors included a simple frequency diagram, frequency ÷ midpoint, frequency × classwidth, vertical axis not labelled correctly, 3.07 plotted as 3.7 and more rarely 0.665 plotted as 0.0665. The label on the vertical axis of the histogram was not always in agreement with the bars drawn; for example bars drawn at 360, 400, 153.5 and 33.25 were described as frequency density rather than frequency per 50 miles or sometimes as both. A horizontal scale consisting of inequalities was another common error.

(ii) In estimating the median, many candidates identified that the median was the 600^{th} value or $600\frac{1}{2}^{th}$ value and then identified the correct interval from 50 to 100 but usually gave an answer of 75 rather than attempting any interpolation. Some got as far as 30 but then forgot to add on the 50. In many cases no candidate from a particular Centre attempted interpolation, suggesting that this is a topic which Centres should pay attention to. A small number decided to estimate a mean distance instead.

6) (i)(A) Marks scored on this question were surprisingly low. Errors of 0.36 (plants with one problem only) or 0.53 were very frequent in this part.

(B) The correct answer of 0.13 was frequently seen. There was also a wide variety of incorrect answers, perhaps 0.17 being the most common of these.

(ii) Many candidates (including a significant number of very high scoring candidates) treated part ii) as if it were "with replacement" giving an answer of 0.53^3. Another less but fairly frequent wrong answer was $1 - 0.47^3$. A small number interpreted it as being a Binomial distribution of 100 trials.

7) (i) Almost all candidates answered this correctly.

(ii) Most candidates answered this correctly but some candidates chose to find P(delayed) first, meaning that lengthy calculations were needed.

(iii) Once again this was usually answered correctly.

(iv) Many candidates struggled with the conditional probability, making a variety of errors, including (0.329 × 0.388)/0.388, 0.388/0.329 or just 0.329.

(v) Many candidates attempted to use a conditional probability approach to this part, but then the majority of these gave answers such as 0.388/0.8, 0.176/0.8 or 0.235/0.8, rather than the correct 0.188/0.8 = 0.235. A good proportion of candidates calculated just the numerator (0.188) or miscalculated it as 0.176 (missing the triplet 0.8 × 0.15 × 0.1). Very few realized the direct methods available such as 1 – 0.9 × 0.85 = 0.235.

(vi) This was very well answered although candidates usually rounded their answer to 43. On this occasion this error was not penalized. A few candidates miscalculated 110 × 0.388 as 38.8 rather than 42.68.

8) (i)(A) This was usually answered correctly either by calculation or tables, with direct calculation being the more popular method.

(B) Again this was usually answered correctly, but some candidates made things difficult for themselves by calculating point probabilities and then either forgetting P(0) or including P(3) and with varying degrees of accuracy. Some used tables incorrectly finding $1 - P(X \leq 3)$, rather than $1 - P(X \leq 2)$.

(C) Once again this was usually correct but occasionally the mode was found rather than the expected number.

(ii) Many candidates correctly stated their hypotheses in symbolic form. However, much use of incorrect notation was also seen. The required notation is clearly given in the mark scheme and candidates should be trained to use this, leading to a straightforward two marks. Many candidates still do not realise the need to define the parameter ‘p’ and thus they lose a third mark, even if they have stated their hypotheses correctly. The reason for the form of the alternative hypothesis was not always well explained in context.

(iii) Some Centres do not seem to have taught how to find a critical region and candidates from such Centres often ignored the request for the critical region and went straight to the hypothesis test. Of those who did try to find the critical region, many made errors, including omission of probabilities, failure to compare the probabilities with 10%, confusion between $P(X \geq n)$ and $P(X > n)$, and even in a surprisingly large number of cases an attempt to do a two–tailed test despite having stated the correct alternative hypothesis. There are still a considerable number of candidates who attempt to use point probabilities for a hypothesis test. Although it is given in the mark scheme, it is worth repeating here the recommended method for comparing the probabilities with the significance level. Candidates should find the two upper tail (in this case) cumulative probabilities which straddle the significance level.

$$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.8358 \text{ or } 0.1642 > 10\%$$
$$P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.9389 \text{ or } 0.0611 < 10\%$$

Irrespective of whether their critical region was correct, many candidates declined to use that information, but instead started again with $P(X \geq 7) = 0.0181 < 10\%$ and tackled the hypothesis test by that method. Those who did use their critical region sometimes did not make it clear that ‘7 lies in the critical region’. Candidates should also be advised that it is necessary not only to make a decision but give a conclusion in context.

4767 Statistics 2

General Comments

Once again, the general performance for most candidates taking this paper was high. It is pleasing to see continued improvement in the handling of hypothesis tests. One aspect that many candidates seem not to fully grasp is the difference between 'sample' and 'population'; this often leads to loss of marks when stating hypotheses.

Comments on Individual Questions

Section A

1) (i) Well answered. A small minority of candidates lost marks through minor slips or by mixing methods e.g. using S_{xy} in the numerator with rmsd(x) and rmsd(y) in the denominator.

(ii) Well answered. In tests involving the product moment correlation coefficient, candidates should be encouraged to write hypotheses in terms of ρ and define ρ as the population correlation coefficient. Most candidates obtained the correct critical value, made a sensible comparison and provided a conclusion in context. Many candidates scored 5 of the available 6 marks; in most cases the lost mark was due to failure to accurately define ρ.

(iii) What should have been an easy mark for stating that the ‘population’ is required to have a bivariate Normal distribution was missed by many candidates. Most candidates picked up the remaining marks for commenting on the elliptical shape required and making a relevant comment regarding the given case.

(iv) Few candidates gained the mark for pointing out that the alternative hypothesis should be decided before referring to the sample data. Most picked up the other available mark. A large number of candidates commented that ‘correlation does not imply causation’, gaining no credit on this occasion.

2) (i) Well answered. Most candidates obtained the mark for explaining that some element of randomness or independence was needed. Candidates should learn to use the phrase ‘uniform mean rate’ in such questions, as other attempts to word this phrase rarely describe what is needed. Fortunately, only a few candidates quoted ‘n is large and p is small’.

(ii) (*A*) Well answered.

(ii) (*B*) Well answered.

(iii) Well answered.

(iv) Most realised that a Normal approximation was appropriate and used the correct parameters. Many candidates failed to apply the correct continuity correction. Otherwise, the handling of the Normal probability calculation was good.

(v) Poorly answered. Many unsuccessful attempts to use an inappropriate Normal approximation were seen. Of the few that managed to proceed as required, many missed the final mark through failing to properly justify their final answers.

3) (i) Mostly well answered. With 4 marks available, candidates were expected to demonstrate how to obtain the given answer; many managed this, but a lot of unconvincing attempts were seen. In some cases, candidates simply wrote 'my calculator tells me this is the answer', or words to that effect; this was not taken as 'sufficient detail'.

(ii) Well answered.

(iii) A similar success rate to question 2 (iv). Again, continuity corrections were frequently omitted or incorrectly applied. Otherwise, the probability calculations were handled well. Common errors include dividing by variance when standardising, and obtaining the wrong 'tail'.

(iv) Well answered, apart from the definition of μ as the population mean.

(v) Well answered. A variety of approaches seen, with many leading to full marks. In most cases, marks were lost for either using the wrong distribution (treating the observed value as a single value rather than the mean of a sample of 20) or by sloppy handling of the comparison of the test statistic with the critical value. Common mistakes involved comparing the test statistic with (commonly) 5%, and comparing a negative test statistic with a positive critical value.

4) (i) Well answered. A small number omitted the context from their hypotheses. Very few mentioned correlation or tried to use parameters in their hypotheses. Most candidates obtained the correct X^2 value and provided a table of individual contributions as requested. Most candidates obtained the last 4 marks for carrying out the test using their X^2 value.

(ii) Quite well answered. However, it was unclear in many cases whether candidates appreciated the connection between the size of the individual contributions and the strength of the association. Simple comments could score 3 of the 5 available marks quite easily. To gain full credit, candidates needed to display a deeper understanding by interpreting the contributions to the test statistic.

4768 Statistics 3

General Comments

There were 371 candidates from 77 centres (June 2008: 348 from 72) for this sitting of the paper. The overall standard of the scripts seen suggested reasonable understanding of most, but by no means all, of the content of this module. However, Question 3 parts (i) (systematic sampling) and (ii) (reasons for the use of the Wilcoxon test) were conspicuously badly answered, with even good candidates appearing to have little, if any, feel for either of the two issues.

In a number of places it seemed that candidates had not read the question carefully before starting to answer it. Also, candidates continue to show poor regard for clear and accurate notation in their work, and for the need for accurate computation. On a number of occasions the work contained glaring errors of a kind that one simply would not normally expect to see at this level. Furthermore, despite the remarks made in last June's report concerning the quality of the language used in the conclusions to hypothesis tests, the deterioration in respect of this has continued.

Invariably all four questions were attempted. With very few exceptions there was no evidence to suggest that candidates found themselves unable to complete the paper in the available time.

Comments on Individual Questions

1) **Combinations of Normal distributions. CD units.**

   (i) In this part, just about all candidates managed to make a good start to the question and the paper as a whole.

   (ii) This part was well answered too. Only occasionally was there an issue with the wrong variance.

   (iii) The first stage of this part was answered correctly by the majority of candidates, but a substantial minority made a fundamental mistake when calculating the variance. The second stage, which required a binomial probability, was usually recognised as such but the calculation was frequently limited to the probability of just 3 out of 4, not "at least 3 out of 4".

   (iv) Fully correct answers to this part were rarely seen. Many, but by no means all, candidates gave the correct mean and variance for the difference between two CD units. However, in almost all cases the requirement was interpreted as one- rather than two-sided.

2) **The *t* distribution: test and confidence interval for a population mean. Use of a confidence interval for a population mean from a large sample to find the sample size. The weights of Pat & Tony's cakes.**

(i) Most candidates appeared to have learnt good habits when stating the hypotheses for this kind of test; weaker ones still neglected to define their symbol "μ". It was noticeable that many candidates did not seem to be making the best use of their calculators to find the mean and sample standard deviation; a range of values for these and the subsequent test statistic was seen, all the consequence of different levels of rounding at different stages of the calculations. The last stage, completion of the test, was usually well done except that the wording of the final conclusion often lacked any reference to the *average* weight of the cakes and/or was considered to be too assertive.
A noticeable minority of candidates elected to test the *difference* between the weights of Pat's cakes and the advertised average, 1 kg. This created extra, unnecessary work for them, and usually they were unable to express their hypotheses in a clear and coherent manner.

(ii) The confidence interval proved to be a straightforward task for many candidates. Among weaker candidates there was a costly tendency to use the Normal distribution.

(iii) In this part the large sample size meant that the Normal distribution should be used (Central Limit Theorem); that was not a problem for most. However, many candidates used the value of the variance given in the question as the standard deviation. This had a significant implication for part (iv).

(iv) It seemed that many candidates knew what to do here, and most remembered to include a factor of 2 for the total width of the interval. However, if the variance was used as the standard deviation (see part (iii)) then the final answer became rather unrealistic. Candidates seemed to accept this without any apparent concern. Furthermore, most candidates undertook to answer this part by setting up and attempting to solve an equation instead of an inequation, and many let themselves down by their poor facility with some fairly basic algebra.

3) **Sampling; Wilcoxon paired sample test. Employees' attitudes to new uniforms.**

(i) This part was very badly answered. Two things were quite clear: that candidates did not read the question and that they had little, if any, understanding of "systematic sampling" and "simple random sampling". This part of the specification (Sampling) continues to be conspicuously badly understood. Candidates suggested a variety of strategies, including the random ordering of the list of employees, intended to ensure that all would be equally likely to be selected, thereby guaranteeing, in their eyes, that a "simple random sample" would be obtained. Time and again candidates wrote that, in order to obtain a systematic sample of 10% of 600, one should select every 60th employee.

(ii) This was another part that was badly answered. While candidates would go on to carry out successfully the Wilcoxon test using paired data in part (iii), their responses here suggested that they could neither explain why they were doing it nor state (through their hypotheses) precisely and clearly what they would be testing. The word "mean" and/or the symbol "μ" were not uncommon. Frequently any symbol that was conveniently to hand was used to represent the median in the hypotheses ("*m*" would seem to be an obvious choice), and usually the word "population" was missing from the definition of it. In a number of cases the null hypothesis was given as "median of A – median of B = 0"; candidates should be aware that this is *not* equivalent to "median of (A – B) = 0".

(iii) Most candidates answered this part well, showing that they could reproduce the algorithm of the Wilcoxon test easily and reliably. Furthermore, for this test the final conclusion was usually well expressed in non-assertive terms.

4) **Continuous random variables; Chi-squared test of goodness of fit. The depth of space left in the top of a jar of jam.**

(i) Most candidates realised that they needed to show the condition that the integral of a p.d.f. over the domain equals 1. However, very few remembered to check (or even just state) that the given function was required to be non-negative in the domain, and those that did remember usually did not address the first condition.

(ii) Many candidates integrated successfully to find the mean of the distribution. Not as many managed to show satisfactorily that the probability connected with it was independent of λ; some worked out the correct probability but neglected to comment on it and some just made no attempt.

(iii) It was a little frustrating to see candidates working out by integration the value of $E(X^2)$, even though it was given to them in the question. These candidates then often seemed to think that the given expression was $\{E(X)\}^2$ which, of course, would make the variance equal to 0.

(iv) There were many good answers to this part of the question. A number of candidates believed, incorrectly, that they should combine the first two classes. There were occasional errors concerning the number of degrees of freedom and/or the critical value, and there was, for some, the usual shortcomings in the final conclusion. The last point required candidates to notice that the test statistic actually fell between critical values given in the tables and to pass comment accordingly. It was pleasing to see that very many of them did precisely that.

4769 Statistics 4

General Comments

There were 35 candidates from 17 centres (plus one more centre whose candidate was absent). While obviously a small entry, it is a noticeable and welcome increase from last year. Many centres entered just one candidate, but that is unsurprising for this advanced module at the "top" of the statistics strand. Indeed, it is pleasing that centres are able to support single candidates. Perhaps the Further Mathematics Network is making an important contribution too. A particularly pleasing feature was that there were some centres which had had no candidates for this module (or its predecessors) for many years, and one or two centres that, it is thought, were entering for the first time.

As usual, the paper consisted of four questions, each within a defined "option" area of the specification. The rubric requires that three be attempted. All four questions received many attempts, which is encouraging as it indicates that centres and candidates are spreading their work over all the options. Overall, there was some very good work, but also some distinctly poorer work.

We are seeing too many cases of unsupported numerical answers that are clearly taken straight from calculators. Candidates must be made to realise that this is a high-risk strategy. If the numerical value is wrong (beyond whatever latitude is allowed for say the second or third decimal place), then *no marks at all* can be awarded for that section of the work, because there is no evidence that a correct method is being used. A particular illustration of this was provided in question 3, where the value of a pooled estimator of variance had to be found, and where there were a number of cases of unsupported numerically incorrect answers (often quite substantially incorrect). Was there an attempt to use the right method with just a keying error, or did the candidate not know what to do? With no evidence, it cannot be assumed that the correct method was being used.

There were many cases where the conclusions in context for hypothesis tests were too assertive. This was disappointing as it had appeared that this point had been successfully made over recent years.

Comments on Individual Questions

1) This was on the "estimation" option. It was based on maximum likelihood estimation and method of moments estimation. The latter term was of course not used by name. The general idea of "moments" estimation has appeared in many previous papers.

   First, there was some good work. Some candidates were able to complete the question, or at least very nearly do so, in a careful, efficient and insightful way.

   However, there were some candidates who clearly had no idea what a likelihood is. This is very poor as it is an explicit and central item in this section of the syllabus.

   Maximisation of the given expression for the likelihood was usually reasonably well done, but some candidates did it without first taking logarithms, which again indicates lack of understanding of the usual procedures in this work.

   The work to find E(X) in part (iii) was commonly very poorly done. The random variable is obviously discrete, so the expected value is a sum; whyever did some candidates think it was an integral? The sum is *not* that of a GP. More subtle methods are required to find it. "More subtle methods" do not include simply writing down the given answer – faking was especially prevalent here. The given answer, as for the likelihood itself, is there so that candidates may *use* it in subsequent work, and of course it is entirely legitimate to do that.

The "moments" estimation in part (iii) was also commonly poorly done. There was bad confusion between estimators and parameters (poor notation was often a particular drawback here), for example in claims such as $\bar{X} = (1-\theta)/\theta$.

Finally, the confidence interval in part (iv) was sometimes done well, but often the work here was very confused. Silly nonsenses of "$s/\sqrt{n}$" for the standard deviation turned up far too often.

After all the above criticisms, it is well to reiterate that there was some very good work throughout this question.

2) This was on the "generating functions" option and was mostly based on standard work for the Normal distribution.

Many candidates knew that "completing the square" (in the exponent) is the right method for obtaining the moment generating function of the N(0, 1) distribution, but not all could do it. The step that follows, where the integral of the pdf of N(t, 1) is created and used, was not always convincing. Other candidates got themselves into various severe difficulties (it is hopeless to try to do this integral by parts) and often faked the result.

The linear combination work in part (ii) was usually done well.

The "unstandardising" in part (iii) was also usually done well, though some faking also occurred here.

In part (iv), the previous results were applied to finding the mean and variance of the lognormal distribution. Only some of the candidates got the (actually rather easy) point here.

3) This question was on the "inference" option, exploring unpaired and paired tests. It was often done very well. The usual errors (e.g. wrong number of degrees of freedom, wrong critical point) sometimes appeared. Wrong critical points were strangely more common in part (ii), often despite previous success in part (i). As mentioned in the "general comments" section above, over-assertive conclusions were seen too often.

Many candidates simply failed to discuss the arguments for pairing that are asked for in part (ii).

There were even some candidates who did part (ii) as another unpaired test, an especially disappointing error.

Solutions to part (iii) were often somewhat muddled, not clearly distinguishing the cases of parts (i) and (ii). Several candidates appeared to think that the Wilcoxon rank sum test and the Mann-Whitney test are different!

4) This was on the "design and analysis of experiments" option.

The examples to demonstrate a Latin square were generally fairly good. The contexts chosen by the candidates were remarkably uniform. Several contexts appeared several times (not including the classical "stream down two sides of a field", either); perhaps these are discussed in popular text books. The contrast with a randomised blocks design was not always grasped, and many candidates simply omitted the comparison with a completely randomised design.

The modelling work in part (ii) and the analysis of variance in part (iii) were usually done well.

4771 Decision Mathematics 1

General Comments

This proved an accessible paper with a number of good candidates scoring highly.

A comment must be made about the marking process. Candidates' scripts are now scanned in, and the module is marked online. Most of the scanning is in black and white, so colour should not be used. Correcting work by rubbing out was not always done as well as it could be, sometimes leaving ambiguous answers. Correction fluid does not work well with scanning, and should not be used.

There was some evidence that a minority of candidates had difficulty finishing the paper in the time allowed, but in virtually all cases this was due to spending time unnecessarily on question 2 (verbose writing) and question 4 (using all 100 random numbers – twice). Correct answers required the use of about 20% of the numbers.

Comments on Individual Questions

1) **Graphs**

Most were able to collect the marks from parts (i) and (ii).

In part (iii) there was some misnaming of a tetrahedron, but most candidates were able to describe the shape adequately enough to gain the mark. Fewer were able to sketch it.

2) **Algorithms**

This question was answered well in all parts by the majority of candidates, although a small minority failed to grasp the essential point that the computer's algorithm was just that – there was no element of choice.
Most problems that occurred were in part (iii) when some candidates forgot that, although the computer did not have a choice, the other player did, and would exercise that choice to choose a strategy which would lead to a win.
Some candidates used efficient terminology, but others wasted much time writing copious paragraphs when much less wordy answers were preferable. In some cases this seemed to lead to the candidate being rushed to answer the final question.

3) **Linear Programming**

In part (i) most candidates were able to draw the inequalities, although a surprising number failed to identify the feasible region by appropriate shading. For full marks in identifying the optimal point, it was necessary to show working. This could be done either by drawing a profit line on the diagram, or by calculating the value of the objective function at each vertex on the edge of the feasible region. Many candidates ignored the point (5, 10), and so failed to gain maximum marks on this part.
Part (ii) was found to be difficult. Candidates needed to find the point (10, 8), to realise that the inequality would become redundant at that point, substitute the values into it and hence get 26. Again, some indication of methodology was required. Answers of 26 with no working were not adequate.

4) **Simulation**

When candidates interpreted this question correctly they answered it very well. In part (i) the creation of a simulation rule without omitting any numbers was handled well. The interpretation of the process in part (ii), in which rats ran or exited, was handled less well.

In part (iii) the process was widened to rats going to one of two alternatives or exiting. Most candidates realised a digit had to be omitted to create the simulation rule, but only some realised that the rule needed in some way to take account of which vertex was current. This led to many and varied answers for the simulation in part (iv).

5) **Networks**

Some candidates had problems interpreting what parts (i) and (ii) were asking, and hence dropped marks. The question was designed to allow those candidates to recover in parts (iii) and (iv), and most candidates scored highly on those parts. In part (i) candidates were required to compute show the network of direct connections, and in part (ii) they then had to find the minimum connector. It was not uncommon to see exactly the same answer given for parts (i) and (ii).

Part (iii) was answered well and in part (iv) most were able to give a plausible real-life reason why the minimum connector was not necessarily the best solution.

6) **CPA**

This question was pleasingly well done. Candidates often scored full marks in parts (i) and (ii) in which they had to construct the network, do a forward and backwards pass, and identify critical activities.

In part (iii) the number of workers was limited to two, and a work schedule had to be produced on a cascade chart. Most candidates failed to identify which tasks were to be undertaken by which worker, and many had schedules in which more than two tasks were being undertaken at the same time.

In the final explanation part, many seemed to regard having two workers in part (iii) as being better than the situation in parts (i) and (ii), whereas those parts assumed no resource constraints.

4772 Decision Mathematics 2

General Comments

This paper was accessible to most students but contained plenty of challenge to distinguish between top candidates and others. The tail of poor performers was less long than has sometimes been the case.

Comments on Individual Questions

1) **Logic**

(a) Very few scored both marks for this. A large number did get one mark by removing double negatives but did not get the exact meaning.

(b) A substantial number of candidates did not know what a combinatorial circuit was. Many drew switching circuits. Of those who drew the correct type of circuit, many did not get the two OR gates placed correctly.

(c) The majority did well on this part, and many scored the full 7 marks.

2) **Decision Analysis**

Performances on this question were rather patchy; very few candidates scored full marks.

(i) This part was generally well done, although a few made arithmetic errors!

(ii) Most could draw the decision tree but a substantial number of candidates failed to put the value in the decision node.

(iii) Many failed to construct the correct tree. Very few started with 'ask' and 'don't ask' options. The other main error was getting the 'Michael predicts' and the 'revise' branches in the wrong order, which gave the wrong solution. A number of candidates had only the 'predict' branches, and did not have the 'revise' branches.

3) **Linear Programming (Simplex)**

This question was done well, with a significant number of candidates getting full marks.

(i) There are still a number of candidates who define variables as 'a is crop A' etc, and several took the objective function as the first constraint. Many were able to explain adequately why the inequalities were as they were.

(ii) This was done well, with many getting the basic simplex all correct. Most even remembered to interpret the solution.

(iii) This was designed to test understanding – and it did so! While some got it right, many did not realise that the equality should be expressed as two inequalities, and so their tableau had too few constraints. Those who chose the big M method often had sign errors in their objective.
In spite of being told not to solve the problem, a small number did so!

4) **Networks**

(i) Quite a few were confused by there not being a vertex between 1 and 2 where the roads meet; having 1 to 5 as 20 rather than 15 was a common error. Most could set up the initial tableaus more or less correctly.

(ii) As with the simplex, a small number tired to work through the whole of the Floyd, in spite of the fact that the third iteration was given. Most successfully got the distance matrix, but a number did not know how to complete the route matrix correctly and had '4 2 2 2 4' in the third row.

(iii) In spite of being told there were no changes, a few wrote out the fifth iteration! Most were able to read the distance and route from their matrices but not all tired to explain how they had done it. A significant majority found the distance by adding the distances on the steps corresponding to the route matrix rather than simply reading from the distance matrix.

(v) While many got the Hamilton cycle correct, a number did not return to the start so did not have a cycle. Some failed to interpret their result in the light of the original network. The majority knew that this related to the travelling salesperson problem, and the best candidates stated that it was an upper bound or a feasible tour. Some stated that it gave the shortest tour around the vertices. A small number thought it was the route inspection problem, and some stated that they had found a Eulerian cycle, in spite of being told it was Hamiltonian!

4773 Decision Mathematics Computation

General Comments

There were fewer entrants for this paper than has been the case since it was first set. This is a shame. Responses to the paper over the years have been excellent. They have demonstrated that the concept is viable. The mathematics, and its development of mathematical modelling skills, has been exciting.

Comments on Individual Questions

Overall, performances were good. The comments below focus on problem issues.

1) **Recurrence Relations**
The Excel modelling in this question was disappointing. Many candidates contrived to get it completely wrong, despite the prompts within the question.
(Note that the question was identical in structure to the 2001 Chancellor of the Exchequer in Mathland question in paper 2622 in 2001.)

(i) This was quite difficult.
The markscheme says it all – "$B_{n+2} = B_{n+1} + (0 – B_n)$" – but that is a rather sophisticated response. Candidates needed to identify that the correction is – B_n, to be added on to B_{n+1}, and to take effect at B_{n+2}.

(ii) There was very little excuse for not getting this right. All that was required was given in the question.

(iii) One can understand that some candidates might get this wrong initially, and many did. They applied the factor of 0.5 to the whole of the RHS. However, one might have expected this to be corrected once the question for part (v) had been read.

2) **Network Flows**
Not all candidates seemed to be prepared for the question about flow labelling in part (i). Many were not able to "augment", and many attempts were seen in which flows were increased along several routes at the same time.
Attempts at the LP formulation were generally much better, although some candidates failed to capture all possible flow directions in the vertex constraints.

3) **Simulation**
This question turned into a test of candidates' organisational abilities. Most were able, to a greater or lesser extent, to handle the mathematics and the associated Excel work. Only the best candidates were able to keep a grip on all that was required. Typically, part (iv), the tennis game, had a clear need for the identification of a stopping criterion. Whether or not one was provided by the candidate was clearly discriminating.

4) **LP Modelling**
It was anticipated that less good candidates would be confused by the s*i*m*j* notation for the variables, and this proved to be the case. Fewer good solutions were seen than might have been expected, given student responses in the past.

4776 Numerical Methods (Written Examination)

General Comments

There were many good scripts seen, and some were excellent. However, as usual, there were some candidates who appeared to be quite unprepared for this paper. The best candidates presented their work clearly and compactly, with due regard for the algorithmic nature of the subject. At the other extreme, some candidates presented their work as a jumble of figures, difficult to follow and frequently riddled with errors. It is worth saying yet again that candidates who adopt the latter approach put themselves at a considerable disadvantage.

Comments on Individual Questions

1) **Lagrange interpolation**

Most candidates know what to do, though some confused the x and f(x) values. The algebra to simplify the polynomial defeated far too many. In part (ii) most knew that Newton's formula requires equally spaced x values.

2) **Newton-Raphson method**

Almost everyone established the existence of the root correctly by using change of sign. The Newton-Raphson method is well understood and most could set up the iteration correctly and find the root to the required accuracy. A small number presented an answer without any working. As the rubric to the paper makes clear, this cannot be rewarded.

3) **Absolute and relative errors**

Though this is very elementary material, most candidates scored half marks or less and it was extremely rare for anyone to get full marks. The maximum possible errors are 0.0005, 0.001, 0.001, 0.015. It was quite common to see these values doubled. The maximum possible relative errors in X and Y were often found correctly. Some knew that the maximum possible relative error in XY will be the sum of the maximum possible relative errors in X and Y. Almost nobody knew that the maximum possible relative error in X/Y will *also* be the sum of the maximum possible relative errors in X and Y.

4) **Errors in evaluating a formula**

It was surprising that quite a few candidates failed to follow relatively simple instructions to work to 6 decimal places. Inevitably, some worked in degrees. The attempts at parts (ii) and (iii) were often poor. In part (ii) candidates might have said that the equality is only approximate, that the left side involves subtraction of nearly equal quantities, or that the left side involves two trigonometric evaluations while the right involves one. In part (iii) the point is that the problem of subtracting nearly equal quantities gets steadily worse on the left but there is no such problem on the right. This is a well worn topic on this paper, but many seemed not to know it.

5) **Fixed point iteration**

This question was frequently answered well. The graphs were not all objects of beauty, but they were used successfully by many to illustrate the divergence by means of a cobweb diagram. The error of going 'up to the line and across to the curve' was seen from time to time.

6) Numerical integration

Most candidates scored well on this question. The values of M, T and S were usually correct. Showing that the differences in M values reduce by a factor of 4 as h is halved was well done. Most could then show that the corresponding factor for S is about 16. Some candidates, despite having analysed the convergence of the S values, stated the final answer without any attempt to justify the number of figures given.

7) Newton's forward difference formula

Newton's formula was handled well by many, though the fact that $h = 0.2$ led to errors. In part (ii) the two estimates were often found correctly but the comments on the fact that they are equal were sometimes rather feeble: 'the central difference formula is quite accurate'. The best answer, building on what candidates should have learned, is that the central difference formula is exact for quadratics. In part (iii) the best conclusion is that the forward difference formula is not exact for quadratics.

4777 Numerical Computation

General Comments

Once again, the candidature for this paper was small. However, candidates were mostly well prepared and there were some high scores.

Comments on Individual Questions

1) **(Solution of an equation; relaxation)**

   The algebra to set up relaxation was not generally well handled, but the numerical solution in part (ii) was better.

2) **(Gaussian integration)**

   By contrast with question 1, the algebra to set up the Gaussian 3-point rule was handled well. In the numerical part, some candidates struggled with the fact that the required integral is not centred on zero.

3) **(Second order differential equation; finite difference method)**

   The algebra at the start of the question proved tricky for some, but the numerical work was carried out successfully.

4) **(Gauss-Seidel and Gauss-Jacobi methods)**

   Those candidates who attempted the question knew what to do and scored highly on it.

Coursework

Administration

It was helpful to Moderators to receive the vast majority of Authentication forms CCS160 with the MS1 and/ or scripts.
Centres will be aware that the software now used by OCR for the moderation process chooses the sample. This is an automatic process on the loading of centre marks. This can take a few days to complete and so in many cases, where there are 10 candidates or fewer, the work of all candidates had been sent off to the Moderator by the time the “sample request” arrived. The receipt of the automatically generated in these cases caused a little confusion with some centres.
Most centres adhered to the deadline set by OCR very well and if the first despatch was only the MS1 then they responded rapidly to the sample request.

The marks of most centres were appropriate and acknowledgement is made of the amount of work that this involves to mark and internally moderate. The unit specific comments are offered for the sake of centres that have had their marks adjusted for some reason.

Assessors are asked to ensure that they adjust the criteria marks in such a way that the final mark on the cover sheet agrees with the submitted mark on the MS1 and is the sum of criteria marks.
Additionally, some assessors only give domain marks. This might be fine if the candidate deserves full marks (or zero!) for a domain, but it makes it very difficult for external Moderators to understand the marking if a mark has been withheld – in this case we do not know which of the criteria have, in the opinion of the assessor, not been met adequately.

Assessors are reminded that Moderators can only moderate what they are given, and failure to submit any part of the assessed work will result in a mismatch of marks between the assessor and the moderator. This includes any spreadsheet work. Assessors are also reminded that they should not award marks in the domains as a result of the oral communication as this is work that is not presented and therefore not available to the Moderator.

Teachers should note that all the comments offered have been made before. These reports should provide a valuable aid to the marking process and we would urge all Heads of Department to ensure that these reports are read by all those involved in the assessment of coursework.

Core, C3 – 4753/02

The marking scheme for this component is very prescriptive. However, there are a significant number of centres where so many of the points outlined below are not being penalised appropriately that the mark submitted is too generous.

The following points should typically be penalised by half a mark – failure to penalise four or more results in a mark outside tolerance.

Change of Sign

- Lack of a proper graphical illustration – graphs of the function being used do not constitute an illustration of the method.
- Equations which can be solved analytically. This includes cubic equations with one integer root.
- The answer given as an interval rather than the statement of the root.
- The use of trivial equations to demonstrate failure.
- Tables of values which actually find the root.
- Graphs which the candidate claims crosses the axis or just touch but don't.

Newton Raphson

- Equations with only one root. This fails to address the second criterion and so the whole mark should be lost.
- The root stated with no iterates given.
- Inadequate illustrations (for example, an "Autograph" generated tangent with no annotation or just a single tangent).
- Graphs not matching iterates.
- Error bounds not established by a change of sign.
- Starting values too far away from the root or too artificial.
- Failure examples lacking iterates.

Rearrangement

- Incorrect rearrangements not spotted and sometimes marked as correct.
- Graphs not matching iterates.
- Graphs not explained.
- Different equation used to demonstrate failure.
- Weak discussions of $g'(x)$. Candidates should not just quote the criterion without linking it to their function and its graph.

Comparison

- Different starting values.
- Sometimes different roots are found.
- Different degrees of accuracy.
- Not quoting number of steps to reach given accuracy.
- Thin discussions.

Notation

- Equations, functions, expressions still cause confusion to candidates and teachers! Candidates who assert that they are going solve $y = x^3 + 2x + 3$ or that they are going to solve $x^3 + 2x + 3$ should be penalised.

Oral

- The specification asks for a written report.

We experienced a centre using the out of date cover sheet which is in the original specification booklet. We have asked that all of these be destroyed on at least 6 occasions. It is disappointing to realise that more than one centre is not taking note of these reports.

Differential Equations - 4758/02

The essential function of the coursework element of this module is to test the candidate's ability to follow the modelling cycle. That is, setting up a model, testing it and then modifying the assumptions to improve the original model. If two or three models are suggested at the outset and tested, more or less simultaneously, and the best chosen, then the modelling cycle has not been followed.

Likewise, it is sometimes noted (correctly) that the original model does not provide a very good fit, and various amendments to the original assumptions are suggested in order to obtain a better fit. This process is one of curve fitting rather than following the modelling cycle

Similarly, choosing 'too good' a model in the first place, e.g. flow proportional tos $\sqrt{h}$ initially for 'Cascades', does not leave much room for improvement of the model. Consequently the marks in Domains 5 and 6 are compromised.

For 'Aeroplane Landing', (still the most popular task) marks often seem to be automatically allocated for Domain 3 (Collection of data) when there is little discussion of the source or potential accuracy of the data. Here, also, the model proposed is not tested. Many candidates are rewarded for testing a particular model for only part of the motion – it is expected that they will produce a set of predictions for the whole motion according to their assumptions before proposing any amendments.

Numerical Methods – 4776/02

There were several cases where incorrect work had been ticked. Assessors are requested not to tick work unless it has been checked thoroughly. There were also many cases where there was no annotation at all, leaving the Moderators unable to discern what work had been checked.

The most popular task is to find the value of an integral numerically. The following comments are offered – it is to be hoped that those teaching and assessing use these comments to inform their teaching and their assessment of the work.

Domain 1.

Not all candidates fulfil the basic requirement of a formal statement of the problem. Additionally, a teacher-prescribed task should result in not all marks in this domain being available.

There are a significant number of candidates who state an intention to find one integral but then find another.

Domain 2

Most candidates describe what method they are to use (and describe how it works) but fail to say why – this is part of the criteria for this domain.

Domain 3

Finding numerical values for the mid-point rule up to M_{16} is not deemed to be substantial.

Domain 4

It is not enough to state what software is being used. A clear description of how the algorithm has been implemented is required, usually by presenting an annotated spreadsheet printout. Sometimes there is no spreadsheet work presented at all.

Domain 5

It is in this domain that the greatest generosity of marking is seen by the Moderators. Assessors might like to take note of the following reasons for this.

It is not appropriate to compare values obtained with “the real value”. This might be π.
It is accepted that candidates will use a function that they are unable to integrate (because of where they are in the course) but which is integrable. However, it is not then appropriate to state a value found by direct integration.
As a result of their insubstantial application candidates will then produce an answer which is incorrect but for which they are given credit.
Some candidates use the “theoretical” value for the ratio of differences, regardless of the values they are getting. These might be values that are converging but are not yet close enough to justify using the theoretical value, or, if the function is not well-behaved, values that are converging to a completely different value. These errors are often credited, leading to some very generous marking.

Domain 6

Some candidates were given full marks for quoting a value to 2 significant figures! Most of the marks in this domain are dependent on work in the error analysis domain and so a generous assessment in that domain leads to further generosity here as well. Comments justifying the accuracy of the solution are appropriate here, but comments on the limitations of Excel are not usually creditworthy.

Grade Thresholds

Advanced GCE MEI Mathematics 7895-8 3895-8
June 2009 Examination Series

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	59	52	45	39	33	0
4752	Raw	72	51	44	38	32	26	0
4753	Raw	72	57	52	47	42	37	0
4753/02	Raw	18	15	13	11	9	8	0
4754	Raw	90	67	59	51	43	35	0
4755	Raw	72	53	45	37	30	23	0
4756	Raw	72	51	45	39	33	27	0
4757	Raw	72	60	51	42	34	26	0
4758	Raw	72	61	55	49	43	36	0
4758/02	Raw	18	15	13	11	9	8	0
4761	Raw	72	57	48	39	30	21	0
4762	Raw	72	47	40	33	26	20	0
4763	Raw	72	55	46	38	30	22	0
4764	Raw	72	61	52	43	34	26	0
4766/G241	Raw	72	60	53	46	40	34	0
4767	Raw	72	57	50	44	38	32	0
4768	Raw	72	55	48	41	34	28	0
4769	Raw	72	56	49	42	35	28	0
4771	Raw	72	63	56	49	42	36	0
4772	Raw	72	57	51	45	39	33	0
4773	Raw	72	51	44	37	30	24	0
4776	Raw	72	62	53	45	37	28	0
4776/02	Raw	18	14	12	10	8	7	0
4777	Raw	72	55	47	39	32	25	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895-7898	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	44.1	65.4	81.4	92.1	97.9	100	10375
7896	57.2	78.0	88.9	95.4	98.9	100	1807
7897	87.1	93.55	100	100	100	100	31
7898	0	0	100	100	100	100	1
3895	35.3	52.9	67.4	79.1	88.1	100	16238
3896	52.1	70.2	82.4	90.4	95.7	100	2888
3897	80.4	88.2	91.2	96.1	97.1	100	102
3898	6.3	12.5	18.8	25.0	68.8	100	16

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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