Modified Enlarged 24pt

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Wednesday 13 October 2021 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes plus your additional time allowance

YOU MUST HAVE:
the Printed Answer Booklet sent with the standard paper or any suitable paper provided by the centre. The centre may enlarge the Printed Answer Booklet.
a scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

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INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

If you use the Printed Answer Booklet write your answer to each question in the space provided. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

If you use the Printed Answer Booklet fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.

The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total mark for this paper is 75.

The marks for each question are shown in brackets [ ].

ADVICE

Read each question carefully before you start your answer.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1} b + {}^n\mathrm{C}_2 a^{n-2} b^2 + \ldots$$
$$+ {}^n\mathrm{C}_r a^{n-r} b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \quad \text{or}$$

$$\sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then

$$\mathrm{P}(X = x) = \binom{n}{x} p^x (1 - p)^{n-x},$$

mean of X is np, variance of X is $np(1 - p)$

Kinematics

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

SECTION A: Pure Mathematics

Answer ALL the questions.

1 Given that $(x - 2)$ is a factor of $2x^3 + kx - 4$, find the value of the constant k. [2]

2

The diagram shows the line $y = -2x + 4$ and the curve $y = x^2 - 4$. The region R is the unshaded region together with its boundaries.

Write down the inequalities that define R. [3]

3 Sam invested in a shares scheme. The value, £V, of Sam's shares was reported t months after investment.

Exactly 6 months after investment, the value of Sam's shares was £2375.

Exactly 1 year after investment, the value of Sam's shares was £2825.

(a) Using a straight-line model, determine an equation for V in terms of t. [3]

Sam's original investment in the scheme was £1900.

(b) Explain whether or not this fact supports the use of the straight-line model in part (a). [2]

4 The quadratic polynomial $2x^2 - 3$ is denoted by $\mathrm{f}(x)$.

Use differentiation from first principles to determine the value of $\mathrm{f}'(2)$. [5]

5 (a) Show that the equation $2\cos x \tan^2 x = 3(1 + \cos x)$ can be expressed in the form

$$5\cos^2 x + 3\cos x - 2 = 0.$$ [3]

(b) In this question you must show detailed reasoning.

Hence solve the equation

$$2\cos 3\theta \tan^2 3\theta = 3(1 + \cos 3\theta),$$

giving all values of θ between 0° and 120°, correct to 1 decimal place where appropriate. [6]

6 A curve C has an equation which satisfies $\frac{d^2y}{dx^2} = 3x^2 + 2$, for all values of x.

(a) It is given that C has a single stationary point. Determine the nature of this stationary point. [1]

The diagram shows the graph of the GRADIENT FUNCTION for C.

(b) Given that C passes through the point $(-1, \frac{1}{4})$, find the equation of C in the form $y = f(x)$. [5]

7

The diagram shows the circle with equation $x^2 + y^2 - 6x + 9y + 19 = 0$ and centre C.

(a) Find the following.

The coordinates of C.

The exact radius of the circle. [3]

The tangent to the circle at D meets the x-axis at the point A $(\frac{55}{4}, 0)$ and the y-axis at the point B $(0, -11)$.

(b) Determine the area of triangle OBD. [6]

8

The diagram shows the curve $y = 1 - x + \frac{6}{\sqrt{x}}$ and the line l, which is the normal to the curve at the point (1, 6).

(a) Determine the equation of l in the form

$$ax + by = c$$

where a, b and c are integers whose values are to be stated. [5]

(b) Verify that the curve intersects the x-axis at the point where $x = 4$. [1]

(c) In this question you must show detailed reasoning.

Determine the exact area of the shaded region enclosed between l, the curve, the x-axis and the y-axis. [5]

SECTION B: Mechanics

Answer ALL the questions.

9

A body remains at rest when subjected to the horizontal and vertical forces shown in the diagram.

Determine the value of F_1 and the value of F_2. [3]

10 A cyclist starts from rest and moves with constant acceleration along a straight horizontal road. The cyclist reaches a speed of $6\,\mathrm{m\,s^{-1}}$ in 25 seconds. The cyclist then moves with constant acceleration $0.05\,\mathrm{m\,s^{-2}}$ until the speed is $10\,\mathrm{m\,s^{-1}}$. The cyclist then moves with constant deceleration until coming to rest. The total time for the cyclist's journey is 150 seconds.

(a) Sketch a velocity-time graph to represent the cyclist's motion. [2]

(b) Find the acceleration during the first 25 seconds of the cyclist's motion. [1]

The cyclist takes T seconds to decelerate from $10\,\mathrm{m\,s^{-1}}$ until coming to rest.

(c) Determine the value of T. [2]

(d) Determine the average speed for the cyclist's journey. [3]

11

A train consists of an engine A of mass 50 000 kg and a carriage B of mass 20 000 kg. The engine and carriage are connected by a rigid coupling. The coupling is modelled as light and horizontal.

The resistances to motion acting on A and B are 9000 N and 1250 N respectively (see diagram).

The train passes through station P with speed $15\,\mathrm{m\,s^{-1}}$ and moves along a straight horizontal track with constant acceleration $0.01\,\mathrm{m\,s^{-2}}$ towards station Q. The distance between P and Q is 12.95 km.

(a) Determine the time, in minutes, to travel between P and Q. [3]

For the train's motion between P and Q, determine the following.

(b) The driving force of the engine. [2]

(c) The tension in the coupling between A and B. [2]

12

A particle P moves in a straight line. At time t seconds, where $t \geqslant 0$, the velocity of P is v m s^{-1}. It is given that $v = -3t^2 + 24t + k$, where k is a positive constant.

The diagram shows the velocity-time graph for the motion of P.

P attains its maximum velocity at time T seconds. Given that the distance travelled by P between times $t = 1$ and $t = T$ is 297 m, determine the time when P is instantaneously at rest. [7]

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