OCR
Oxford Cambridge and RSA

Wednesday 6 October 2021 – Afternoon

Level 3 Certificate Core Maths A (MEI)

H868/01 Introduction to Quantitative Reasoning

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Time allowed: 2 hours

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INSTRUCTIONS
- Do **not** send this Insert for marking. Keep it in the centre or recycle it.

INFORMATION
- This Insert contains the pre-release material that you have already seen.
- This document has **8** pages.

A The mark and recapture method

Many organisations and people need to estimate the sizes of certain populations. Health workers need to know numbers of people with a particular disability. Ecologists are interested in the numbers of bees. Marine biologists investigate the number of whales to see if it is increasing or decreasing and by how much. In a few cases it is possible to count the actual population, for example, the number of families with three children can be found from the Census. Finding, say, the rat population in a particular area by counting all the rats is impossible!

One method which is used when it is impossible to count every individual in the population is the **mark and recapture method**. It is commonly used to estimate animal populations.

- m animals are captured, **marked** or identified in a harmless way and released.
- Some time later c animals are captured. The number of them marked from the first capture is counted and found to be r: they have been **recaptured**.

After the first capture, m animals are marked out of a population of N. So the proportion of marked animals is $\frac{m}{N}$.

In the second sample, r out of c animals are already marked, so the proportion marked is $\frac{r}{c}$.

The calculation that follows is based on two assumptions.

(1) There is no change in the population as a whole.

(2) The population is thoroughly mixed up after the m animals first captured are put back.

The proportion of marked animals found in the second sample will be approximately the same as the proportion of the population marked following the first sample.

So $\frac{m}{N} \approx \frac{r}{c} \Rightarrow Nr \approx m \times c$

This gives the estimated value of N as $N = \frac{m \times c}{r}$.

Marine biologists used this method to estimate the number of humpback whales off the US West Coast in 2008. In 2007, a total of 297 whales were photographed, i.e. marked, as individual whales can be identified visually. A year later, at about the same time and area, 438 whales were photographed. Of these, 63 were identified, i.e. recaptured, from the previous year.

This gives the estimated number of whales, N, as $\frac{297 \times 438}{63}$ which rounds to 2065 whales.

Mark and recapture can be used in many different situations, for example, estimating the total number of errors in a computer program. Two software engineers, working independently, check the program for errors.

The first software engineer finds (or captures) m errors and notes (marks) these. The second software engineer does the same, finding (or capturing) c errors.

Of these errors, r were ones which the first software engineer also found (they have been recaptured). Using the mark and recapture formula above, the estimated total number of errors in the program, N, is given by $\frac{m \times c}{r}$.

Some people find it helpful to remember the mark and recapture formula

$$N = \frac{m \times c}{r}$$

in words as

$$\text{Estimate of total population} = \frac{(\text{Number in first sample}) \times (\text{Number in second sample})}{(\text{Number in both samples})}.$$

B Crowd size measurement

Reliable estimation of crowd sizes, such as those shown in **Fig. B.1**, is important because of safety considerations. However, some people who report crowd sizes may want to exaggerate the number; others may want to give the impression that fewer people were there.

Fig. B.1

Organisers of a recent demonstration estimated that 150 000 people took part. The area of the demonstration was 30 000 m^2. This meant that there were, on average, 5 people per square metre. This would be very dense, as can be seen from these diagrams in **Fig. B.2**.

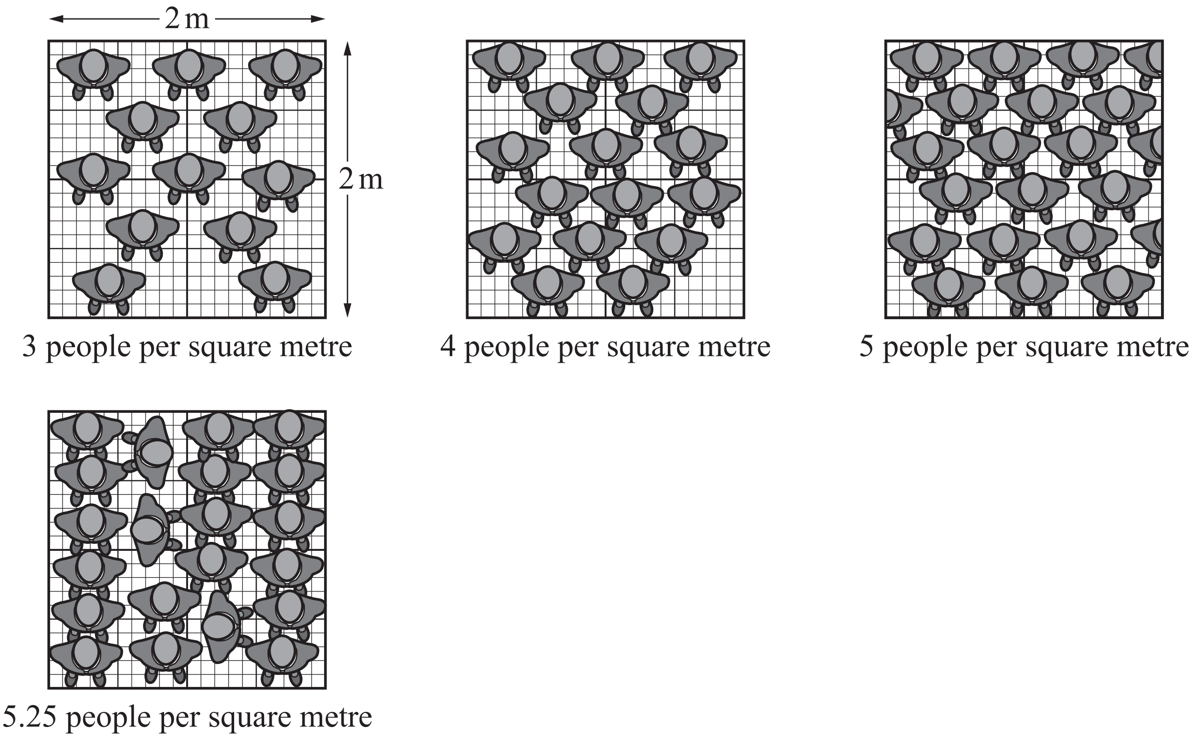

Fig. B.2

5 people per square metre is usually taken as the maximum realistic figure for a crowd.

The most common technique for counting crowds is Jacobs's Method. It involves using a photograph of the crowd, dividing it up into sections, then calculating an average number of people in each section and multiplying by the number of sections occupied.

In a variation of Jacobs's Method, people are given a photograph of the crowd divided up into areas. They must assign a number from 1 to 5 to each area to match reference pictures of different crowd densities. After some practice, people tend to become quite good at estimating the number of people in a crowd visually on a scale 1 to 5. The numbers 1 to 5 roughly correspond to 1 to 5 people per square metre. In some cases, the final figure is the mean of 20 or more people's values.

A recent method that has been investigated involves monitoring social media activity. The chart in **Fig. B.3** shows the mobile phone activity around the San Siro Stadium, Milan, from November 9th to November 23rd, 2013. Each 1 cm interval represents the 24 hours of the given day. It also shows the increased mobile phone activity on the three match days, November 9th, 15th and 23rd.

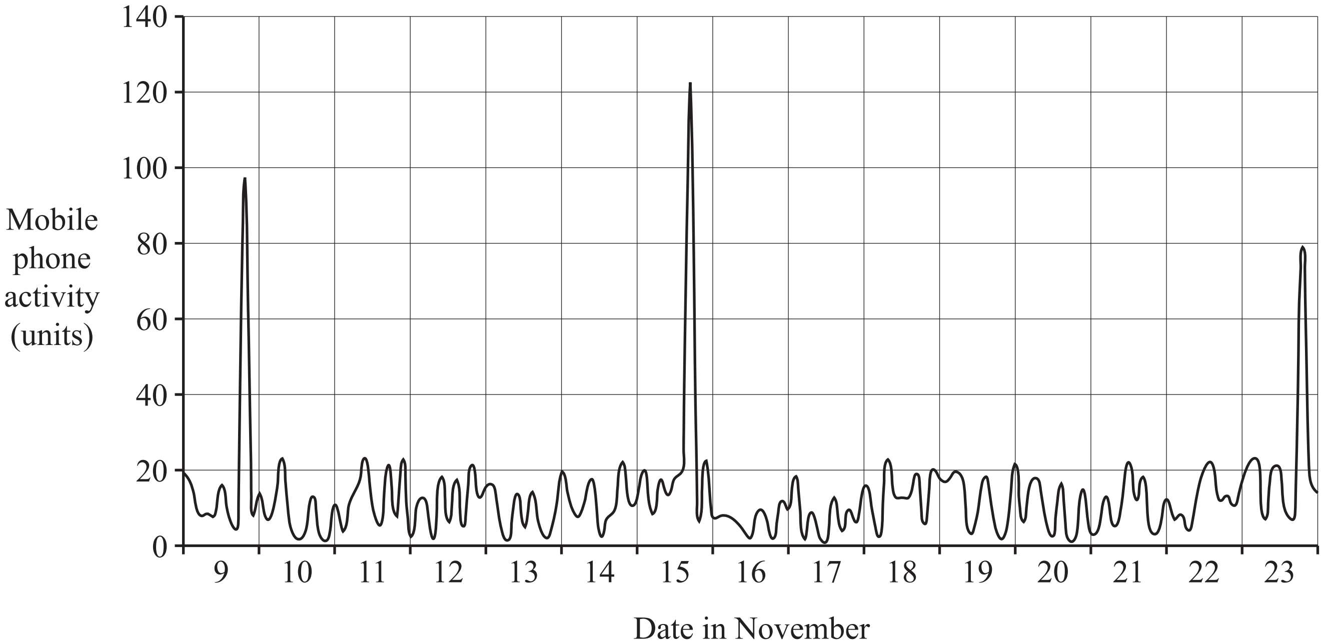

Fig. B.3

The level of mobile phone activity should be approximately proportional to the number of people in the crowd. On the November 9th match mobile phone activity was about 100 units. There were 40 000 at the match. So, for example, an activity of 50 units would indicate a match crowd of about 20 000.

A football stadium was chosen for the investigation as it was a definite location for which to monitor mobile phone activity. In addition, the exact number of spectators in the stadium was known.

An advantage of the method is that these data are available very quickly. This would be useful for safety in situations involving large crowds.

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