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Examiners' report

FREE STANDING MATHEMATICS QUALIFICATION: ADDITIONAL MATHS

6993

For first teaching in 2018

6993/01 Summer 2022 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

Paper 1 series overview

There were some accessible questions in this paper for all candidates, meaning that it was slightly easier than in previous years. The topics tested ranged across the whole syllabus.

The most challenging questions were on logarithms and trigonometry.

Candidates who did well on this paper generally did the following:	Candidates who did less well on this paper generally did the following:
• were able to set their work out well and understand the requirements of the question • understood command words such as "find" or "determine" and their working was clear for examiners to see and mark.	• felt unable to tackle some questions although they were strong in other areas • did not answer "detailed reasoning" questions well as they did not show full, correct workings, particularly where they used their calculators • did not answer questions where they were required to "determine" well as, again, they did not show sufficient, correct workings.

Question 1

1 Solve the inequality $-3 < 2(x+1) < 7$. **[3]**

Candidates who did well in this question took the inequality as a whole and added, subtracted or divided across the three parts. Some candidates took the two inequalities separately which was acceptable providing the solutions were put together again. Replacing the inequalities by an equals sign can be a dangerous process, particularly when dividing by numbers. In this case there were no problems, however, so this was acceptable providing again that candidates replaced the equals signs by the appropriate inequality.

Question 2 (a)

2 A passenger train is 175 m long and is stationary in a station. As it leaves the station it accelerates with uniform acceleration. When the front of the train reaches the end of the platform it is travelling at a speed of $3\,\text{m}\,\text{s}^{-1}$ and when the rear of the train reaches the end of the platform it is travelling at a speed of $18\,\text{m}\,\text{s}^{-1}$.

(a) Determine the uniform acceleration of the train. **[2]**

This is a typical question involving constant acceleration where the use of calculus would be a very long-winded response, since *t* is not involved. The "suvat" formulae are in the formulae sheet and in this case should be used. The vast majority of candidates did and obtained the correct answer.

Question 2 (b)

(b) Determine the time taken from when the train starts to move until it reaches a speed of $18\,\text{m}\,\text{s}^{-1}$. **[2]**

This part has nothing to do with either the length of the train or the speed at the end of the platform. Some candidates did not read the question properly and so did not understand what "starts to move" meant. Consequently using $s = 175$ and/or $u = 3$ caused errors.

Question 3

3 A photograph is to be taken of 9 students who have represented their college in sailing this year. The students are to be arranged in a line. The captain is to stand in the middle with the vice-captain standing beside the captain.

How many ways are there of arranging the students? **[3]**

There were a number of correct answers for this question but also a significant number who were unable to enumerate the number of ways. Although the vice-captain only has two positions (on the left or the right of the captain), candidates multiplied the arrangements of the other students by a variety of numbers.

Question 4 (a)

4 In this question you must show detailed reasoning.

You are given the cubic polynomial $f(x) = x^3 + 6x^2 + 5x - 12$.

(a) Show that $f(1) = 0$. **[1]**

There were no problems with this part.

Question 4 (b)

(b) Hence solve the equation $f(x) = 0$. **[4]**

Most candidates took the result of part (a) to mean that (x – 1) was a factor and proceeded to divide the function by the factor to obtain the quadratic.

This was the first of the "detailed reasoning" questions and we needed to see the method of factorising the quadratic, given that it was possible to find the roots of the equation from a calculator.

Exemplar 1

4(b)

$$x^2+7x+12$$

$$x-1\,\overline{)\,x^3+6x^2+5x-12}$$

$$-\;x^3-x^2$$

$$0+7x^2$$

$$-\;7x^2-7x$$

$$0+12x$$

$$-\;12x-12$$

$$0+0$$

If $f(1)=0$ then $(x-1)$ must be a factor of $x^3+6x^2+5x-12$

$$x^2+7x+12 = x^2+4x+3x+12$$

$$(x+4)(x+3)$$

$$(x-1)(x+4)(x+3)$$

~~f(x)~~ $x=1 \quad x=-4 \quad x=-3$

All the working is seen here.

Question 5 (a)

5 In this question you must show detailed reasoning.

Find all the values of θ in the range $0° < \theta < 360°$ that satisfy the following equations, giving your answers correct to **1** decimal place.

(a) $\cos 2\theta = 0.6$ **[3]**

A number of candidates made an incorrect first step here by asserting that $\cos 2\theta = 0.6$ led to $\cos\theta = 0.3$. Others lost marks by failing to realise that there were 4 roots to the equation.

Question 5 (b)

(b) $12\cos^2\theta + \sin\theta = 11$ **[5]**

Most candidates were able to use Pythagoras to obtain a quadratic equation in sin θ. The same sort of errors were made, by not showing the working of the factorisation and not realising that roots to trigonometrical equations will be in pairs (so 3 roots are not possible) in the given range. Some candidates also did not read the question and gave their roots to more than 1 decimal place.

Question 6 (a)

6 Layla drives to work along a road which has three sets of traffic lights. The lights work independently of each other. Experience indicates that the probability that Layla has to stop at each set of lights is 0.5, 0.6 and 0.7 respectively.

(a) Draw a tree diagram to illustrate the probabilities of Layla having to stop at each set of lights on a particular journey to work. **[3]**

Most candidates were able to draw out the correct tree diagram, although some lost a mark for not labelling the tree with the branches and associated probabilities.

Question 6 (b)

(b) Calculate the probability that Layla has to stop exactly once on a particular journey to work. **[3]**

The sum of the probabilities was well done.

Question 7 (a)

7 A local council investigated the pattern of travel to a nearby town in one month. Residents were asked whether they drove, cycled or walked into the town. They had an opportunity to state if they used more than one mode of transport during the course of the month.

Some of the results are summarised in the table below.

Mode of travel	Frequency
Only walked	15
Only cycled	9
Only drove	12
Used all three modes of transport	4
Walked and cycled but did not drive	6

The total number of residents surveyed was 60.

(a) Draw a Venn diagram to illustrate this incomplete set of data. **[3]**

This question was a source of good marks and most candidates drew a Venn diagram well.

Question 7 (b)

The number that said they drove and walked but did not cycle was the same as the number that said they drove and cycled but did not walk. All those surveyed said that they had travelled to the nearby town by one of these three modes of transport at least once.

(b) Determine how many of these residents drove and walked but did not cycle. **[3]**

The command word "determine" is used when we want to see justification for the answer, usually meaning working. The answer 7 on its own is not sufficient here.

Exemplar 2

7(b)

$15+9+12+4+6 = 46 \quad 60-46=14$

$14 \div 2 = 7$

7 residents drove and walked but did not cycle.

The justification for the answer of 7 is clearly given.

Question 8

8 The gradient function of a curve is given by $\frac{dy}{dx} = 2 + 4x - 3x^2$ and the curve passes through the point (1, 2).

Determine the equation of the curve. **[4]**

Most candidates were able to find the equation of the curve by integration and the substitution of initial values to find the constant. Given that the gradient function was given as $\frac{dy}{dx}$ it was expected that the equation of the curve would be given as *y* = ... and so some lost the last mark by not doing so.

Question 9 (a)

9 In this question you must show detailed reasoning.

(a) You are given that $y = 4\log_3 x$.

Rewrite this equation with *x* as the subject. **[2]**

The three parts to this question proved to be the most challenging. The laws of logarithms were usually not applied correctly.

Question 9 (b)

(b) Write $2\log_{10}5 + \frac{1}{2}\log_{10}16$ as a single number. **[3]**

Both laws were needed in this part leading to $\log_{10}100 = 2$.

Some candidates were not aware that the wording of the question meant that the answer had to be exact so the use of calculators to find numeric values was not appropriate.

Question 9 (c)

(c) The equation $a^x = 17$ has the solution $x = 2.58$, correct to 3 significant figures. Given that *a* is an integer, determine the value of *a*. **[2]**

The simplest method of taking logs of both sides to give the equation $\log_{10} a = \frac{\log_{10} 17}{2.58}$ was not seen often.

Question 10 (a)

10 In this question you must show detailed reasoning.

Two curves have the following equations.

C_1: $y = x^2 - 4x + 4$
C_2: $y = -x^2 + 8x - 6$

(a) Find the coordinates of the **two** points of intersection of these curves. **[4]**

This part was done well to give the coordinates of the points of intersection of the curves.

Question 10 (b)

(b) Find the area of the region enclosed by these two curves. **[5]**

Most candidates knew that they had to find the area under each curve and subtract. The subtraction could be done at the beginning before the integration, at the end by the subtraction of the two areas found separately or after integration but before the application of the limits. Some were unable to understand which curve had the greater area and so produced a negative value. This was acceptable if they then discarded the negative sign as areas are positive.

Question 11 (a)

11 Nina records the speed, $v\,\mathrm{m\,s^{-1}}$, at which she is travelling in her car t seconds after accelerating from rest. The results are shown in the table below.

Time (t seconds)	0	2	4	6	8	10
Speed ($v\,\mathrm{m\,s^{-1}}$)	0	6.6	9.6	11.7	13.2	14.4

(a) Use these results and the axes below to draw a curve to show how her speed varies with time during these 10 seconds. **[2]**

The points were plotted well and smooth curves drawn. However, a few drew the first section, from t = 0 to 2 as a straight line while others drew the curve from right to left and took the curve on, leading to an intersection of about v= 2, thus ignoring the point (0,0).

Question 11 (b)

(b) By constructing 5 rectangles of equal width **above** your curve, estimate the distance she has travelled during the first 10 seconds. **[4]**

There are three ways of drawing rectangles in order to estimate the area under the curve. The one that produces rectangles below the curve is to take the height to be the left hand value of v. Taking the height of the rectangles at the midpoint (i.e. when $t = 1, 3, 5, 7$ and 9) is another method. Neither of these methods were indicated but were used by some candidates. The way intended was to take the heights of the rectangle to be the point on the curve at the right hand side, thus giving rectangles whose "top" was above the curve and which would give an overestimate. Some interpreted the question as meaning that the rectangle had to be totally above the curve and so took a top point, either 16 (because that was the top of most candidates' scales), or 14.4. This would give the correct answer, albeit by a longer way.

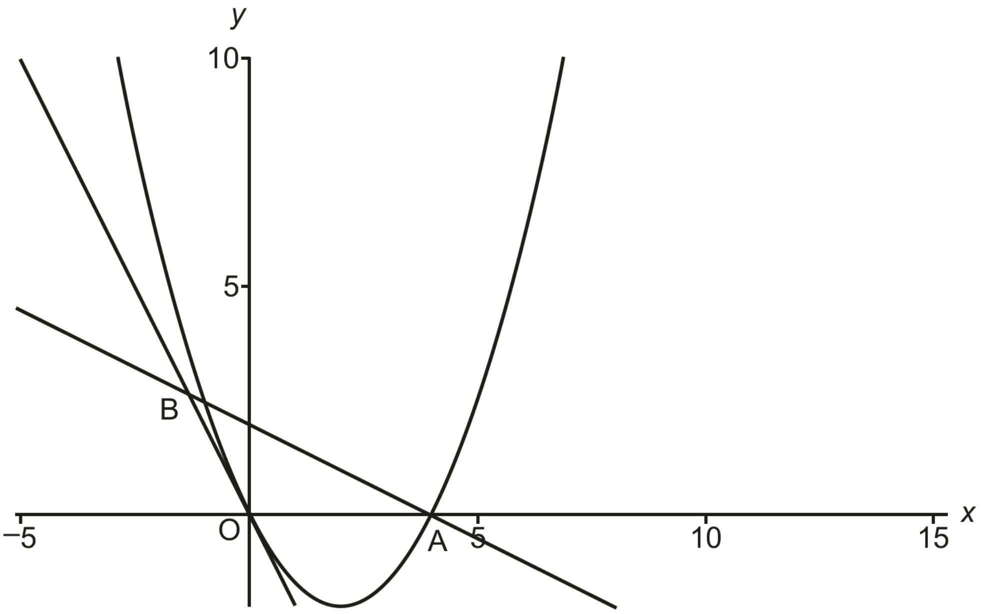
Question 11 (c)

(c) Without doing any further calculations, explain how she could obtain a better estimate of the distance she has travelled. **[1]**

One way of improving the estimate mentioned by some candidates was to take the rectangle below the curve to give an underestimate and then take the average. Others suggested that the trapezium rule would give a better estimate and an equal number suggested making the rectangles of smaller width, i.e. using $t = 1, 3, 5$, etc. and having more of them. Those that said that the equation should be found and integrated missed the point of the estimation.

Question 12 (a)

12



In the diagram the curve with equation $y = \frac{1}{2}x^2 - 2x$ crosses the x-axis at the origin, O, and the point A. The tangent to this curve at O and the normal to this curve at A intersect at the point B.

(a) Determine the equation of the line OB. **[3]**

The gradient function was needed by differentiation and the substitution of $x = 0$. However an incorrect differentiation would still give the correct equation for the tangent. It was for this reason that the question command was "determine".

Question 12 (b)

(b) Determine the equation of the line AB. **[6]**

The coordinates of point A were required which most candidates achieved. The process of finding the gradient of the normal followed for most candidates.

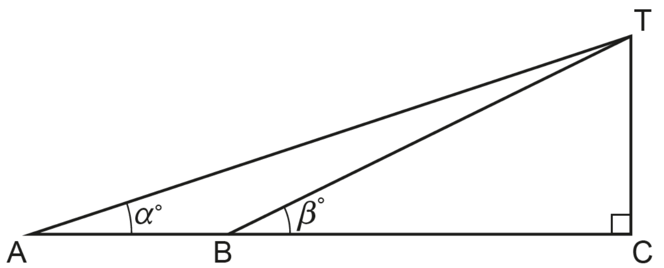
Question 12 (c)

(c) Hence determine the coordinates of the point B. **[3]**

Marks were available even if one of the equations was wrong but for most, the solution of the simultaneous equations was straightforward.

Question 13 (a)

13 A vertical tower CT stands with its base, C, on horizontal ground.
Amir stands at a point A and observes that the angle of elevation of the top of the tower, T, is $\alpha°$.
He then walks directly towards the base of the tower to a point B where he observes that the angle of elevation of the top of the tower is $\beta°$.



Not to scale

(a) Show that $BC = \dfrac{AB\tan\alpha}{\tan\beta - \tan\alpha}$. **[3]**

The trigonometry required to obtain the given result proved to be challenging. Most managed to write down two ratios involving AC and BC and even equating them but the conversion AC = BC + AB was usually missed.

Question 13 (b) (i)

(b) You are given that $AB = 25\,\text{m}$, $\alpha = 15°$ and $\beta = 20°$.

Find

(i) BC, **[2]**

Most candidates took the formula given in part (a) even if they had not managed to derive it and found BC easily. Others took the triangle TAB and used the sine rule, having found either TB or TA. This was acceptable and many got the correct answer, but it was a great deal of work for only 2 marks.

Question 13 (b) (ii)

(ii) the height of the tower. **[2]**

This also followed relatively easily.

Question 14 (a) (i)

14 (a) Ben wishes to estimate the gradient of a curve at the point where $x = 1.4$.

He calculates points on the curve which are given in the table below.

x	1.2	1.4	1.8
y	1.0732	1.5358	3.1447

(i) Explain why he should **not** use the coordinates at $x = 1.2$ and $x = 1.8$ to obtain a reasonable estimate for the gradient of the curve at $x = 1.4$. **[1]**

A majority of candidates knew that the points either side of the middle point had to be an equal distance.

Question 14 (a) (ii)

(ii) Calculate an estimate for the gradient of the curve at $x = 1.4$ by using the coordinates at $x = 1.4$ and $x = 1.8$. **[2]**

This calculation was usually done well.

Question 14 (a) (iii)

(iii) Calculate an estimate for the gradient of the curve at $x = 1.4$ by using the coordinates at $x = 1.2$ and $x = 1.4$. **[1]**

The correct value was also usually seen.

Question 14 (b)

(b) Mia wishes to estimate the gradient of another curve at the point where $x = 1.4$.

She calculates points on this curve which are given in the table below.

x	1.2	1.4	1.6
y	0.6899	0.9518	1.3132

Calculate a reasonable estimate of the gradient of this curve when $x = 1.4$. **[3]**

Most candidates knew that this was a situation where the outside values could be used and did so effectively. Others took the lower difference and the upper difference and took an average while a few took all three and found their average.

Question 15 (a) (i)

15 The points A and B have coordinates (3, 3) and (5, 7) respectively.

(a) On the grid below plot

(i) the points A and B, **[1]**

All candidates were able to plot the two points correctly.

Question 15 (a) (ii)

(ii) the line l with equation $x + 2y = 14$. **[2]**

Likewise, the line was usually well drawn. A few left a series of points rather than drawing the line.

Question 15 (b)

(b) Verify that the line l is the perpendicular bisector of AB. **[4]**

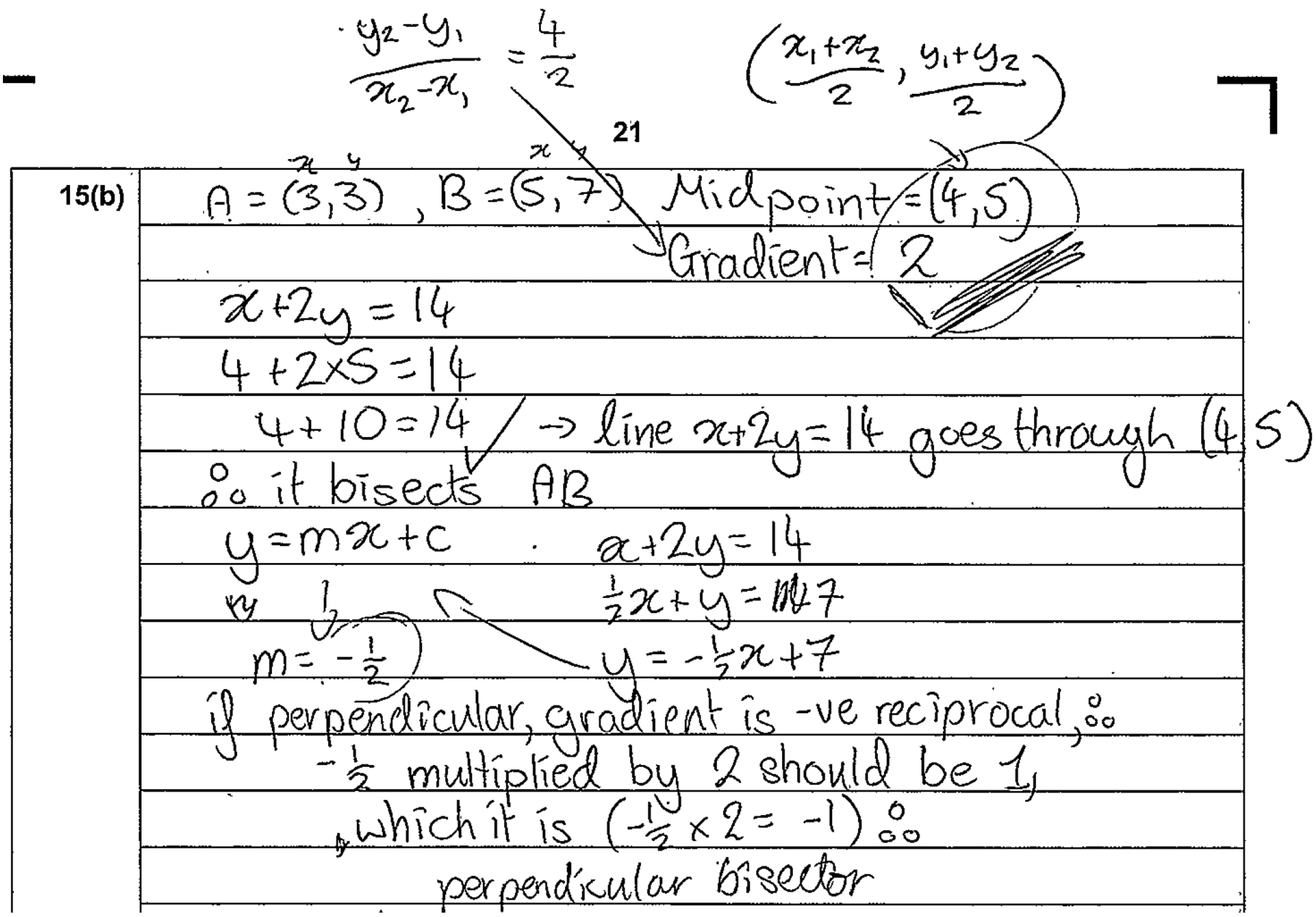
Candidates found the gradient of the line AB and referred to the fact that it was perpendicular to the line l without giving any justification.

A significant majority ignored the need to show that the line also bisected AB. Those who did do this part again did not justify the conclusion. A typical answer was that the point (4, 5) lay on l without stating that the point (4, 5) was the midpoint of the line AB.

Assessment for learning

This is a good example of the need to read the question carefully. It is good practice to underline the key words in the stem. Here the key words are 'perpendicular' and 'bisector'.

Exemplar 3


21

15(b)

$\frac{y_2-y_1}{x_2-x_1} = \frac{4}{2}$ $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

A = (3,3) , B = (5,7) Midpoint = (4,5)

Gradient = 2

$x+2y=14$

$4+2\times5=14$

$4+10=14$ → line $x+2y=14$ goes through (4,5)

∴ it bisects AB

$y=mx+c$ $x+2y=14$

$\frac{1}{2}x+y=7$

$m=-\frac{1}{2}$ $y=-\frac{1}{2}x+7$

if perpendicular, gradient is -ve reciprocal, ∴ $-\frac{1}{2}$ multiplied by 2 should be -1, which it is $(-\frac{1}{2}\times2=-1)$ ∴ perpendicular bisector

In this response both parts are clearly seen.

Question 15 (c)

AB is a diameter of the circle *C*.

(c) Find the equation of the circle *C*. **[2]**

The vast majority of candidates found the equation of the circle using the midpoint as the centre and calculating the radius. A neat way was to use the angle in a semicircle property but it was rarely seen.

Question 15 (d)

The line *l* cuts the circle *C* in two points, P and Q.

(d) Determine the coordinates of P and Q. **[3]**

As with 15 (c) a neat way is to use vector methods but this was rarely seen with candidates substituting the equation of the line into the equation of the circle to obtain a quadratic in either *x* or *y*.

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