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AS LEVEL

Examiners' report

FURTHER MATHEMATICS A

H235

For first teaching in 2017

Y531/01 Summer 2022 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers are also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

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Paper Y531/01 series overview

Y531 is the mandatory paper for AS Further Mathematics. It is taken alongside two other papers which can be freely chosen from a choice of 4. It tests knowledge of proof, complex numbers, matrices, vectors and further algebra as well as testing the understanding of the overarching themes of mathematical argument, problem solving and modelling. To do well on this paper candidates need a thorough understanding of the techniques covered and they need to support their answers with detailed working. They also need to have good algebraic and numerical manipulation skills.

Candidates generally seemed well prepared for this paper, and all but a small number had sufficient time to attempt all the questions. A handful of candidates seemed to spend a lot of time on Question 7 and then omitted Question 8. Presentation was generally good with working well laid out, although there were a few candidates who could have made their solutions clearer. Labelling equations so that they can be referred to and underlining/putting a box around final answers are two methods which candidates can use to make their work easier to follow.

Candidates did well on the routine application of methods, such as in Questions 1, 2 parts (a) and (b) (i) and 5 part (a). They found questions which asked them to explain (such as Questions 6 (a) and 8 (a)) or to apply their knowledge in slightly more unfamiliar situations more difficult. Question 6 (d) – finding gradient of the invariant lines in a reflection – had a particularly high omit rate.

Candidates must take careful note of any "command words" given in the question. In particular, if a question asks for "detailed reasoning", candidates must show all of their working. Calculators can be used as a check, but the solutions must be fully detailed in order to gain full credit.

OCR support

You can download our poster guide to the "command words" used in OCR A Level Maths exams.

There were only a handful of cases of candidates writing solutions in the wrong answer box, usually affecting Question 2. Those who did so clearly indicated which question they were actually answering. It is very important that candidates who mistakenly write their solutions in the wrong space clearly identity this to the examiner.

Multiple solutions to questions were rare, with candidates usually deleting any work they did not want marked leaving one solution. However, candidates should note that in the situation when 2 or more solutions are made then it is the last solution that will be marked, even if this results in the candidate gaining little credit.

The "Additional Answer Space" at the end of the answer booklet was used fairly frequently. Mostly this was for a continuation of attempts at Question 6 (d) when candidates were using a general method for finding invariant lines rather than considering the geometry of the situation. Candidates should take note of the instruction at the top of this page which says "The question number(s) must be clearly shown in the margin(s)" so that examiners can identify which question the working should be attached to.

Candidates who did well on this paper generally did the following:	Candidates who did less well on this paper generally did the following:
• applied routine methods accurately • provided clear explanations • thought about the context of the problem (in particular the geometrical situations in Questions 6 and 8).	• made arithmetic mistakes when applying routine methods, often involving negative numbers • omitted steps in working in Detailed Reasoning questions (Questions 3, 5 and 7) • confused the concepts of a Line of Invariant Points and an Invariant Line.

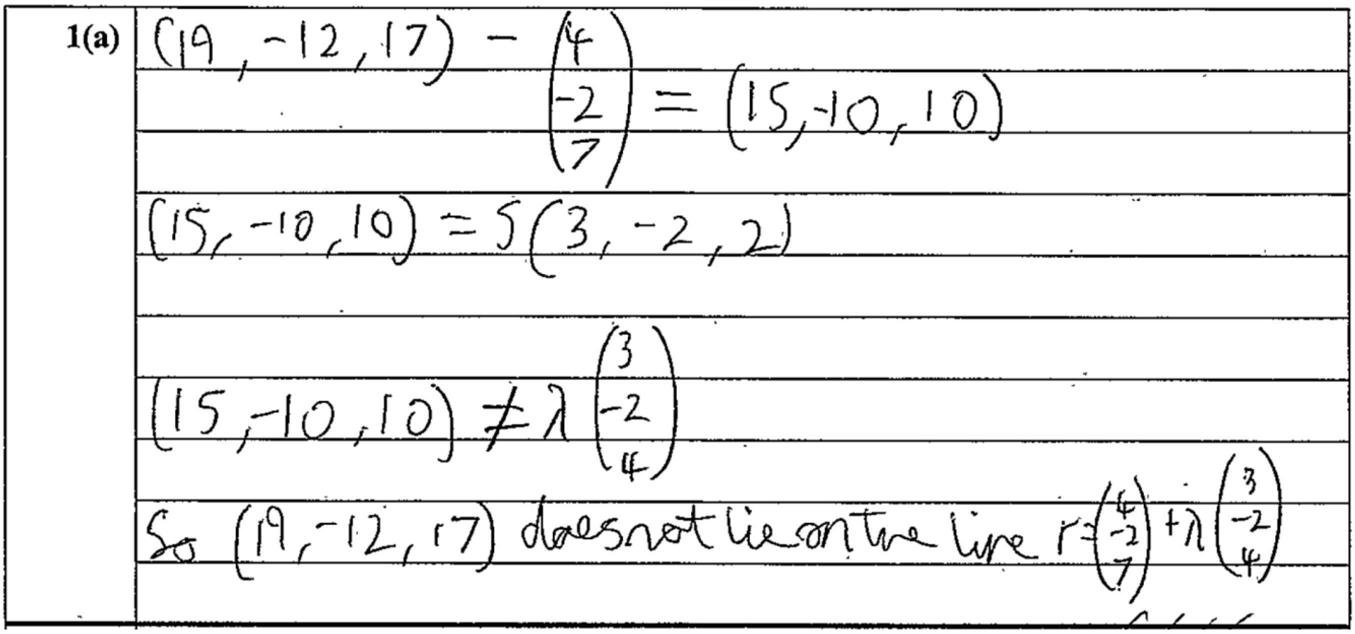
Question 1 (a)

1 **(a)** Determine whether the point (19, −12, 17) lies on the line $\mathbf{r} = \begin{pmatrix} 4 \\ -2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$. **[3]**

Candidates who started by finding the value of λ which solved the x coordinate equation generally went on to gain full credit for this part. These candidates were generally equally distributed between those who showed that the z coordinate equation needed a different value of λ, or that the x value led to an inconsistency in z.

A few candidates rearranged the vector equation to give $\begin{pmatrix} 15 \\ -10 \\ 10 \end{pmatrix} = \lambda \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$. These candidates only gained credit if they went on to explain why there wasn't a single value of λ that satisfied this. The most common way for candidates to do this was to explain that the x coordinate needed $\lambda = 5$, but the z coordinate needed $\lambda = 2.5$.

Exemplar 1


1(a) $(19, -12, 17) - \begin{pmatrix} 4 \\ -2 \\ 7 \end{pmatrix} = (15, -10, 10)$

$(15, -10, 10) = 5(3, -2, 2)$

$(15, -10, 10) \neq \lambda \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$

So (19, −12, 17) doesnot lie on the line $r = \begin{pmatrix} 4 \\ -2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$

This candidate took a slightly different approach than most. They started by finding $\begin{pmatrix} 15 \\ -10 \\ 10 \end{pmatrix}$ and then pulled out a factor of 5 to show that it couldn't be written as a multiple of $\begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$. Benefit of the doubt was given for the mixture of row and column vectors shown.

Question 1 (b) (i)

Vectors **a** and **b** are given by $\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 6 \\ 2 \end{pmatrix}$.

(b) **(i)** Find, in degrees, the angle between **a** and **b**. **[3]**

Most candidates were successful with this part. A few went on to give a final answer of 58.4°, not realising that the angle between two vectors depends on the direction of the vectors. Some candidates used sin θ in their attempt at the scalar product.

Misconception

? Some candidates thought that the angle between two vectors had to be acute.

Question 1 (b) (ii)

(ii) Find a vector which is perpendicular to both **a** and **b**. **[2]**

Almost all candidates knew what was required here, although some made arithmetical mistakes – particularly when calculating the *z* component.

Question 2 (a)

2 Matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} a & 1 \\ -1 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 5 \\ -1 & 0 \end{pmatrix}$ where a is a constant.

(a) Find the following matrices.

- $\mathbf{A} + \mathbf{B}$
- $\mathbf{AB}$
- $\mathbf{A}^2$ **[3]**

Candidates generally answered this question well. A few made arithmetical mistakes. Some candidates wrote the answers to the three matrices in the spaces for 2 (a), 2 (b) (i) and 2 (b) (ii), these candidates then answered the 2 (b) parts on the additional answer paper.

Question 2 (b) (i)

(b) **(i)** Given that the determinant of **A** is 25 find the value of a. **[2]**

Almost all candidates gained full credit for this part.

Question 2 (b) (ii)

(ii) You are given instead that the following system of equations does **not** have a unique solution.

$$ax + y = -2$$
$$-x + 3y = -6$$

Determine the value of a. **[2]**

Candidates who realised that this meant that the matrix was singular (and so the determinant is equal to 0) almost always gained full credit for this part (a few made sign errors). Some candidates tried to show that the two equations were the same, which was generally a less successful method although it was applied well by some candidates.

Question 3

3 **In this question you must show detailed reasoning.**

The roots of the equation $5x^3 - 3x^2 - 2x + 9 = 0$ are α, β and γ.

Find a cubic equation with integer coefficients whose roots are $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$. **[6]**

Most candidates knew what was expected here, and many did very well. The hardest part was found to be rearranging $(\alpha\beta)(\beta\gamma)+(\beta\gamma)(\gamma\alpha)+(\gamma\alpha)(\alpha\beta)$ into a symmetric form with some candidates not recognising the common factor of $\alpha\beta\gamma$.

A few candidates did not gain the final mark through making a sign error when transcribing coefficients, or for not including the "= 0" and so giving their answer as an expression rather than an equation.

Question 4

4 Prove that $3^n > 10n$ for all integers $n \geqslant 4$. **[5]**

Some candidates correctly assumed that $3^k > 10^k$ but then stated that as the left hand side is multiplied by three and the right hand side has 10 added the left hand side remains greater. This step needed more justification if it was to be given the second method mark (and so be eligible for the answer marks).

Many candidates wrote down the two lines:

$$3^k \times 3 > 10^k + 1$$
$$10^k \times 3 > 10^k + 1$$

This was not enough to be given the second method mark as there was no evidence of the assumption step being used, i.e. there was no indication of $3^k \times 3 > 10k \times 3$ being used.

The candidates who were successful tended to split $30k$ up into $10k + 20k$, and then used the fact that $k \geq 4$ to explain why $20k > 10$.

Exemplar 2

4 Propisition $3^n > 10n$ for integer $n \geq 4$

when $n = 4$ LHS RHS

~~KHS~~ $3^4 = 81$ $10(4)$

$= 40$

LHS > RHS

true for $n = 4$.

Assum $3^k > 10k$ for integer $n \geq 4$

when $n = k+1$ RHS =

$10(k+1)$

$3^k \times 3 > 10k \times 3$ $10k + 10$

$3^{k+1} > 10k \times 3$

~~$> 10k \times 10k \times$~~

~~$k \times 3 > 10k \geq 4$~~

~~$10k \geq 40 > 10$~~

~~$10(3k) \geq 120$~~

~~$3^{k+1} > 10$~~

$3^{k+1} > 10k + 10k + 10k$

~~10k~~ $k \geq 4$

$10k \geq 40$

$3^{k+1} \geq 10k + 10k + 40 > 10k + 10$

$3^{k+1} \geq 10k + 10$

$3^{k+1} \geq 10(k+1)$

~~When n=k+1 is~~

$n=4$ is true, Assume true for $n=k+1$

$3^k > 10k$, proven statement true

By mathematical induction proposition is always true for ~~n≥~~ integer $n \geq 4$

This candidate has successfully used the assumption case and clearly argued why $30k > 10(k + 1)$. The last mark was not given as the final conclusion contains an inaccuracy as they state that it was assumed true for $n = k + 1$.

Question 5 (a)

5 In this question you must show detailed reasoning.

(a) Use an algebraic method to find the square roots of $-16 + 30\text{i}$. **[5]**

This part was answered well by candidates, with the majority gaining full credit. The most common mistake was to not reject the pair of imaginary values when solving the quartic. Some other candidates expanded $(a + \text{i}b)^2$ incorrectly.

Question 5 (b)

(b) By finding the cube of one of your answers to part **(a)** determine a cube root of $\frac{-99+5\text{i}}{4}$.

Give your answer in the form $a + b\text{i}$. **[2]**

The majority of candidates gained at least one mark here for cubing their answer to part (a), and of these over half were able to use this to find the required cube root.

Question 6 (a)

6 The matrix **A** is given by $\mathbf{A} = \frac{1}{13}\begin{pmatrix} 5 & 12 \\ 12 & -5 \end{pmatrix}$.

You are given that **A** represents the transformation T which is a reflection in a certain straight line. You are also given that this straight line, the mirror line, passes through the origin, O.

(a) Explain why there must be a line of invariant points for T. State the geometric significance of this line. **[2]**

Most candidates gained at least one of the 2 marks here, either for clearly stating that points on the mirror line stay in the same place, or for stating that the mirror line is the line of invariant points. Less than a third managed to gain credit for both statements as they had not clearly stated both aspects.

Question 6 (b)

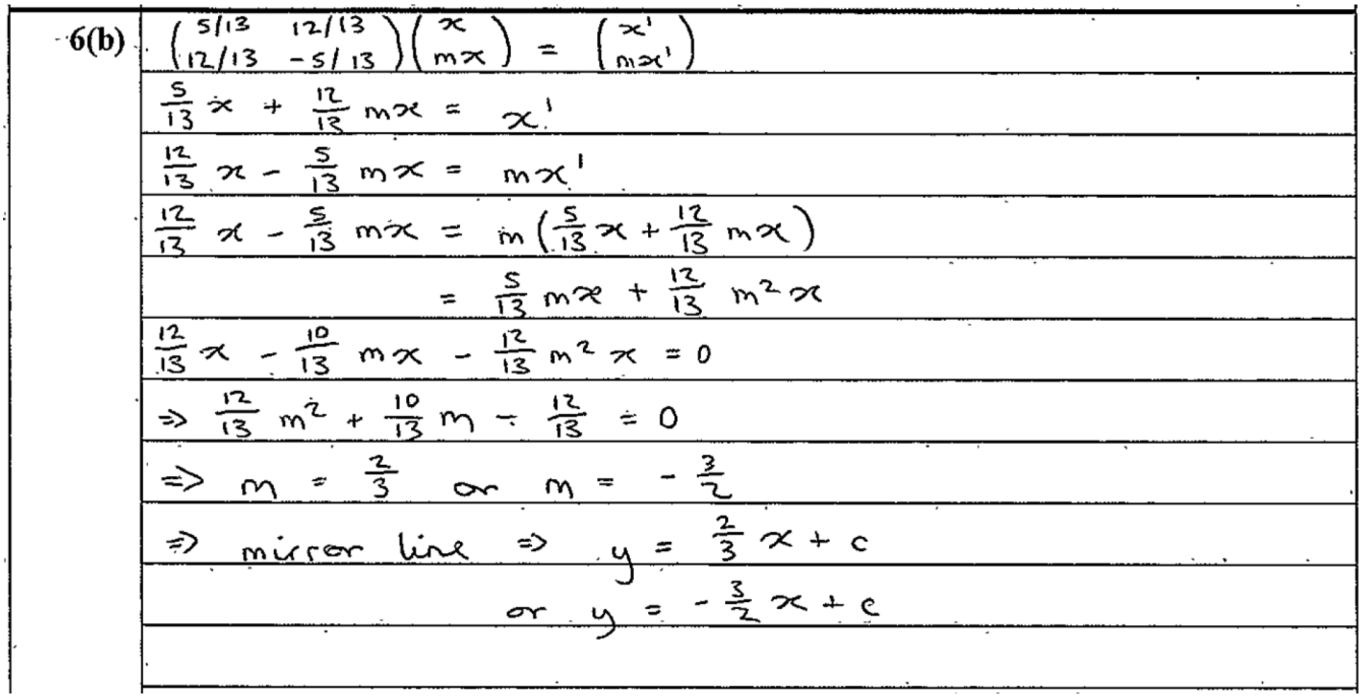
(b) By considering the line of invariant points for T, determine the equation of the mirror line. Give your answer in the form $y = mx + c$. **[4]**

Candidates using the **Ar** = **r** method needed to show that the equation $y = \frac{2}{3}x$ was satisfied by both coordinates in order to gain full credit. There were only a few candidates using this method who did not check both coordinates.

Misconception

Many candidates used a method for finding invariant lines here, rather than a method for finding a line of invariant points. Some candidates did not appear to understand the distinction between the two different types of invariance.

Exemplar 3



6(b)

$\begin{pmatrix} 5/13 & 12/13 \\ 12/13 & -5/13 \end{pmatrix}\begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} x' \\ mx' \end{pmatrix}$

$\frac{5}{13}x + \frac{12}{13}mx = x'$

$\frac{12}{13}x - \frac{5}{13}mx = mx'$

$\frac{12}{13}x - \frac{5}{13}mx = m\left(\frac{5}{13}x + \frac{12}{13}mx\right)$

$= \frac{5}{13}mx + \frac{12}{13}m^2x$

$\frac{12}{13}x - \frac{10}{13}mx - \frac{12}{13}m^2x = 0$

$\Rightarrow \frac{12}{13}m^2 + \frac{10}{13}m - \frac{12}{13} = 0$

$\Rightarrow m = \frac{2}{3}$ or $m = -\frac{3}{2}$

$\Rightarrow$ mirror line $\Rightarrow y = \frac{2}{3}x + c$

or $y = -\frac{3}{2}x + c$

This candidate has used a method for finding an invariant line. This method has been applied correctly, with the given restriction that $c = 0$. In order to gain full credit the candidate would have had to eliminate the $m = -\frac{3}{2}$ case with a reason for doing so.

This could have been done by taking a point on the $y = -\frac{3}{2}x$ line and showing that this point is not invariant. For example, if the point (2, -3) is substituted into **Ax** then the image is given by (-2, 3) and so this cannot be a line of invariant points.

Question 6 (c)

The coordinates of the point P are (1, 5).

(c) By considering the image of P under the transformation T, or otherwise, determine the coordinates of the point on the mirror line which is closest to P. **[3]**

Only a small number of candidates realised that the required point would be the midpoint of P and P'. The more inefficient method of finding a line perpendicular to the mirror line and then working out the intersection between these was far more popular and was often completed successfully.

Candidates found this question part difficult, and the majority of candidates made no attempt at it.

Diagrams and sketches

Candidates who drew a sketch of the situation tended to perform better on this question, and often those who drew a sketch realised that they could use the more efficient midpoint method to find the required coordinates.

Question 6 (d)

(d) The line with equation $y = ax + 2$ is an invariant line for T.

Determine the value of a. **[2]**

This question part was found to be the most difficult on the paper and a large number of candidates omitted it. Higher achieving candidates tended to consider the geometry of the situation and so realised that the invariant lines have to be perpendicular to the invariant points. Lower achieving candidates often used a full general method to try find all of the invariant lines, they often gained one mark but did not complete the argument.

Assessment for learning

The amount of space and number of marks available for a question can be used as a guide to how much working is necessary to solve the question part. In this case, candidates who attempt the general method of finding invariant lines should note that this is very unlikely to fit in the response space here and also involves a lot more work than 2 marks suggests.

In this sort of situation candidates might benefit from taking a moment to re-read the question, and possibly try to draw a sketch or diagram.

Question 7

7 In this question you must show detailed reasoning.

Two loci, C_1 and C_2, are defined as follows.

$C_1 = \left\{z : \arg(z + 2 - \mathrm{i}) = \frac{1}{4}\pi\right\}$ and $C_2 = \left\{z : \arg(z - 2 - \sqrt{3} - 2\mathrm{i}) = \frac{2}{3}\pi\right\}$

By considering the representations of C_1 and C_2 on an Argand diagram, determine the locus $C_1 \cap C_2$. **[7]**

The most common marks here were 0, or 1 (usually gained for an accurate argand diagram sketch). Some candidates found it difficult to find the gradients of the two lines.

Those candidates who managed to correctly find the equations of the two lines were usually successful in finding the point of intersection. However some wrote this as a pair of coordinates rather than as a complex number, and so did not gain the final mark.

Only a handful of candidates used set notation to describe the locus.

Question 8 (a)

8 The line segment AB is a diameter of a sphere, S. The point C is **any** point on the surface of S.

(a) Explain why $\overrightarrow{AC} \cdot \overrightarrow{BC} = 0$ for **all** possible positions of C. **[3]**

Most candidates realised that the vectors were perpendicular, but many struggled to explain why this was the case in a precise manner. Only a small number were able to quote "The angle in a semi-circle is a right angle" with many trying to explain this in other (often less precise) ways. Candidates who talked about triangles being formed with one side equal to the diameter had to make it clear that the third point was on the circumference to gain the first B mark.

No candidates gained full credit for this part as no candidates considered the edge cases when *C* is the same as either *A* or *B*.

Direction of implication

Some candidates stated "$\overrightarrow{AC} \cdot \overrightarrow{BC} = 0$ and so the vectors are perpendicular". This implication is in the wrong direction, but these candidates did not realise that this statement is different to the required statement "The vectors are perpendicular and so $\overrightarrow{AC} \cdot \overrightarrow{BC} = 0$".

Question 8 (b)

You are now given that A is the point $(11, 12, -14)$ and B is the point $(9, 13, 6)$.

(b) Given that the coordinates of C have the form $(2p, p, 1)$, where p is a constant, determine the coordinates of the possible positions of C. **[6]**

This question was done well by many candidates. The most common mistakes were sign errors when finding $\overrightarrow{AC}$ and/or $\overrightarrow{BC}$, usually in the *z* coordinate, or errors when calculating the scalar product. A reasonably large number of candidates completed the question correctly up to finding the correct two values of *p*, but then either stopped there and did not attempt to write down the coordinates of *C* or wrote down the position vectors of *C* rather than the coordinates.

Misconception

Some candidates seemed to be unclear on the distinction between *coordinates* of a point and the *position vector* of a point. If coordinates are required then these have to be written as a horizontal array of numbers, for example (8, 4, 1).

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