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A LEVEL

Examiners' report

MATHEMATICS B (MEI)

H640

For first teaching in 2017

H640/03 Summer 2023 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper 3 series overview

Many candidates appeared well prepared and it was pleasing to see a number scoring very high marks across both sections. There was a minority of candidates that offered 'no response' across all the Section B comprehension questions (Questions 11 – 15) but this was less than in previous years.

Another ongoing problem was with the approach some candidates took with the '**In this question you must show detailed reasoning**' which will be a focus of the commentary of Questions 1, 3, 4 and 5.

Assessment for learning

Detailed reasoning questions

Probably the easiest way for candidates to think of DR questions is to put their calculator down (except for maybe doing some basic arithmetic) and do the relevant A Level technique themselves. Candidates are expected to know how to use their calculators efficiently but should avoid using them on DR questions except possibly for checking their steps of working and their final solution (or in more challenging situations perhaps as an initial check of the direction of travel to be taken). The examiners are well aware of the different features available on modern graphical calculators but in DR questions are often looking to see if you understand the mathematical techniques behind the calculator functions.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• showed their working clearly • could use exact forms where necessary • could apply the correct method with little procrastination • would label aspects of their solution making it easier for examiners to award marks • did not rely on their calculators in DR questions • could manipulate surds • could solve trig equations to find all solutions • understood definite integration.	• appeared to have gaps in knowledge of A Level techniques • spoiled DR solutions by getting answers from their calculators without clearly justifying the values • did not draw sufficient sketches • confused vectors and coordinates • felt they *improved* surd answers by writing as a decimal.

Section A overview

This section had the usual mix of pure maths questions ranging from some of the easiest A Level topics (overlapping with GCSE higher tier) through to some harder problem solving or 'Detailed reasoning' (DR) questions. There seemed to be plenty for all candidates to make at least a start on each question.

Questions 3 and 7 provided at least two ways to approach the question, one relatively straightforward and the other (or others) more complex and prone to errors. It would be worth pointing out to candidates that if a particular method is becoming difficult to apply, it is always worth considering if there could be an easier approach. However, where multiple attempts have been made, it is in the candidates' best interest to make a decision as to which of the solution attempts is the best and clearly cross out what should not be marked.

Question 1

1 **In this question you must show detailed reasoning.**

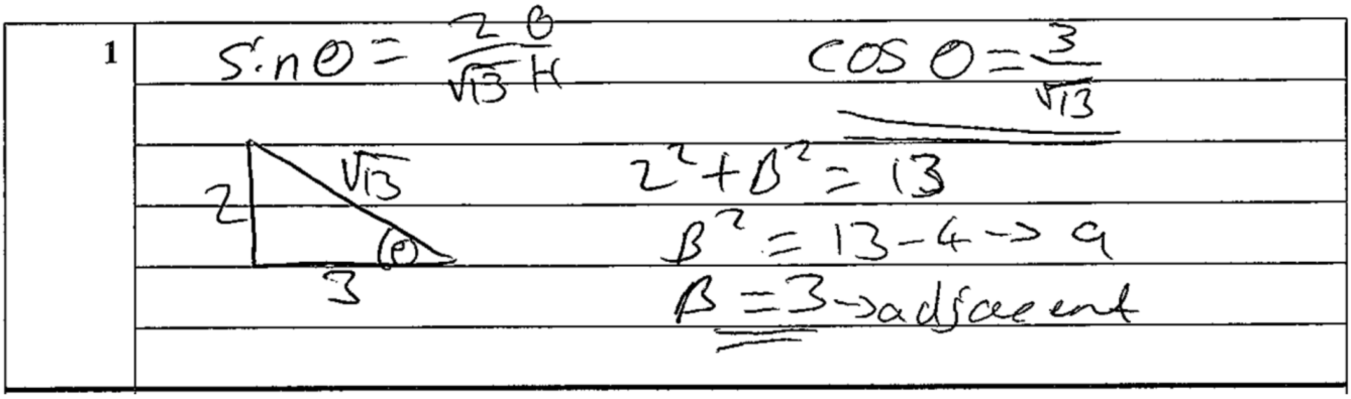
The obtuse angle θ is such that $\sin\theta = \dfrac{2}{\sqrt{13}}$.

Find the exact value of $\cos\theta$. **[3]**

Most candidates picked up a couple of marks here but would often lose the last one if they did not consider that if *θ* is obtuse, then cos *θ* must be negative. Moreover as the question asks for an 'exact value', then candidates should not attempt finding values of *θ* on their calculator which require rounding; such an approach scored 0. As discussed in the overview, avoid using a calculator until checking at the end.

Generally the most successful method for solving this question was to sketch a right-angled triangle and label the angle *θ* and hypotenuse √13. Candidates could then use Pythagoras Theorem to find the other short side and their usual definition of cos to find a positive value for cos*θ*. The harder bit was to then use either the CAST diagram or a sketch of $y = \cos x$ to determine whether the answer is positive or negative.

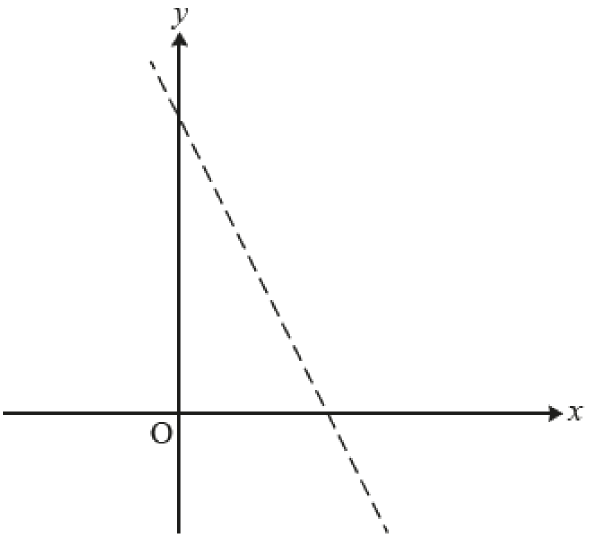
Exemplar 1



This was probably the most common response to this question clearly showing their use of a right-angled triangle and Pythagoras but not considering that the angle is obtuse.

Question 2 (a)

2 The straight line $y = 5 - 2x$ is shown in the diagram.



(a) On the copy of the diagram in the Printed Answer Booklet, sketch the graph of $y = |5 - 2x|$. **[1]**

In part (a) most candidates recognised that the section of the given graph which is below the x axis had to be reflected in the x axis. A few did not continue the line to the left of the y-axis thus losing the mark as it was suggesting a reduced domain.

Question 2 (b)

(b) Solve the inequality $|5 - 2x| < 3$. **[3]**

Part (b) was generally well answered, however the final mark was often lost if candidates did not write the answer as a 'double inequality', instead leaving two separate inequalities. Those that did not combine the inequalities in the usual way often struggled whether to use 'and', 'or' or simply a comma.

Question 3

3 In this question you must show detailed reasoning.

Find the value of k such that $\dfrac{1}{\sqrt{5}+\sqrt{6}}+\dfrac{1}{\sqrt{6}+\sqrt{7}}=\dfrac{k}{\sqrt{5}+\sqrt{7}}$. **[3]**

This question was not answered successfully despite the fact that this topic is regularly assessed with the '**In this question you must show detailed reasoning'** request. Many candidates struggled with 'rationalising the denominator'. What should have been a straightforward 4 or 5 line solution often ended up as a full page of surds with some candidates trying to combine the LHS into a single fraction or trying to clear fractions by multiplying throughout by (√5 + √6), (√6 + √7) and (√5 + √7). This approach rarely succeeded though a few candidates did get there having spent a lot longer than they should have on this 3 mark question.

Candidates also need to consider the advice on Detailed reasoning questions given at the beginning. Candidates might not gain full credit if enough working is not shown. So, 1/(√5 + √6) = √6 − √5, etc. with no working shown was often seen, yes a calculator could be used, but the DR is to show why the calculator gives the required result. Others attempted too many steps in one line of working, either resulting in insufficient evidence of all the work, or introducing arithmetic mistakes. Some candidates did not show convincingly that (√7 - √5) was equal to 2/(√5 + √7) and so lost the final mark.

Question 4

4 In this question you must show detailed reasoning.

Find the coordinates of the points where the curve $y=x^3-2x^2-5x+6$ crosses the x-axis. **[4]**

Again candidates lost marks here for not showing detailed reasoning. Often seen in the first line or two of the candidate's answer was 'x = 1, x = 3, x = -2' or '(x – 1)(x – 3)(x + 2)' clearly having been arrived at using a calculator. The most common error by far was in not being detailed enough at the start. Most candidates realised that x = 1 or x = -2 or, less commonly x = 3 were roots of the equations but many just stated this fact without showing it using factor theorem. Others knew the factor theorem but merely stated, e.g. f(1) = 0 without showing the calculation and then those that did show the calculation did not go on and state that this meant that, e.g. (x – 1) was a factor. The result was that 2 marks out of 4 was by far the most common score on this question. The long division and factorising of the subsequent quadratic was done well by candidates.

Exemplar 2

4

$y = x^3 - 2x^2 - 5x + 6$

crosses the x axis when $y = 0$

$x^3 - 2x^2 - 5x + 6 = 0$

when $x = 1$

$(1)^3 - 2(1)^2 - 5(1) + 6 = 0$

so $(x-1)$ is a factor.

$$\begin{array}{r} x^2 - x - 6 \\ x-1 \overline{) x^3 - 2x^2 - 5x + 6} \\ -\ \underline{x^3 - x^2} \quad\downarrow\quad\downarrow \\ 0 \ -x^2 - 5x + 6 \\ -\ \underline{-x^2 + x} \quad\downarrow \\ 0 \ -6x + 6 \\ -\ \underline{-6x + 6} \\ 0 \quad 0 \end{array}$$

$y = (x-1)(x^2 - x - 6)$

$y = (x-1)(x-3)(x+2)$

so crosses x axis at $x = 1$, $x = 3$ and $x = -2$

coordinates:

$(-2, 0)$ $(1, 0)$ $(3, 0)$

This candidate demonstrates a 'perfect' solution to this question. They used factor theorem showing their working and, importantly, concluded that (x - 1) is a factor to get the first method mark. They carried out an algebraic long division to get a quadratic factor and hence the second method mark. They proceeded to factorise their quadratic and get the second and third factors and the final method mark. The final accuracy mark relied on them giving the coordinates of the x-intercept points.

Question 5 (a)

5 In this question you must show detailed reasoning.

This question is about the curve $y = x^3 - 5x^2 + 6x$.

(a) Find the equation of the tangent, T, to the curve at the point (0, 0). **[3]**

Part (a) was answered very successfully and the vast majority of candidates were able to score all 3 marks. Those that didn't usually lost a mark by not showing their reasoning in going from a gradient of 6 to the equation y = 6x. A small number of candidates scored less than 2 marks in this question.

Question 5 (b)

(b) Find the equation of the normal, N, to the curve at the point (1, 2). **[3]**

Part (b) was also answered successfully. A few candidates did not read the question carefully and went on to find the equation of the normal at the point (0, 0) otherwise most errors were just arithmetical slips.

Question 5 (c)

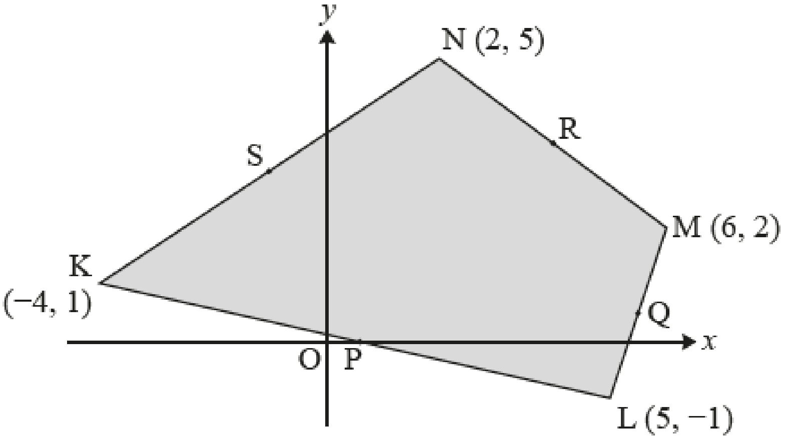
(c) Find the coordinates of the point of intersection of T and N. **[2]**

Candidates who scored full marks in parts (a) and (b) nearly all scored full marks in part (c). There were some however who, having correctly found the *x*-value 1.2, then substituted this value into the equation of the curve.

Question 6 (a) (i) and (ii)

6 **(a)** Quadrilateral KLMN has vertices K $(-4, 1)$, L $(5, -1)$, M $(6, 2)$ and N $(2, 5)$, as shown in **Fig. 6.1**.

Fig. 6.1



(i) Find the coordinates of the following midpoints.

- P, the midpoint of KL
- Q, the midpoint of LM
- R, the midpoint of MN
- S, the midpoint of NK **[2]**

(ii) Verify that PQRS is a parallelogram. **[3]**

Most candidates scored full marks in part (a) (i) of the question.

In part (a) (ii) most candidates recognised what they had to do to show that PQRS is a parallelogram and it was pleasing to see the many different methods used for example column vectors, vectors using **i** and **j**, lengths of line segments or gradients. Given the large choice of relevant methods examiners had little sympathy with candidates who invented their own notations. Candidates are reminded that if asked to 'verify' something, full working should be shown. Some candidates lost the final mark as they did not conclude their argument at the end of the question by stating that the shape is a parallelogram.

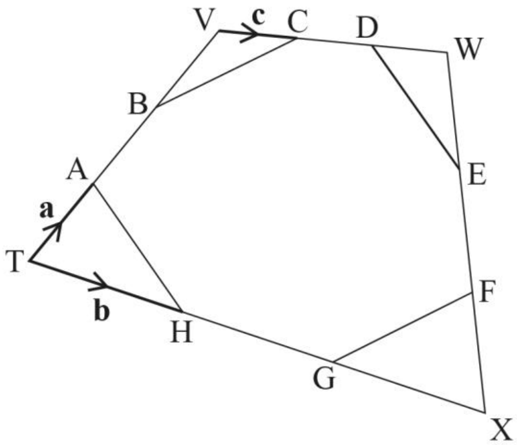
Question 6 (b) (i), (ii) and (iii)

(b) TVWX is a quadrilateral as shown in **Fig. 6.2**.

Points A and B divide side TV into 3 equal parts. Points C and D divide side VW into 3 equal parts. Points E and F divide side WX into 3 equal parts. Points G and H divide side TX into 3 equal parts.

$\overrightarrow{TA} = \mathbf{a}, \quad \overrightarrow{TH} = \mathbf{b}, \quad \overrightarrow{VC} = \mathbf{c}.$

Fig. 6.2



(i) Show that $\overrightarrow{WX} = k(-\mathbf{a}+\mathbf{b}-\mathbf{c})$, where k is a constant to be determined. **[1]**

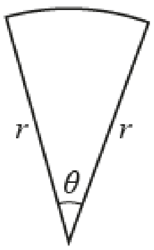
(ii) Verify that AH is parallel to DE. **[2]**

(iii) Verify that BC is parallel to GF. **[2]**

Most candidates had a good understanding of vectors and how they may be used to verify that two lines are parallel. Again, final marks were sometimes lost by not drawing a conclusion at the end of their working.

Question 7

7 A wire, 10 cm long, is bent to form the perimeter of a sector of a circle, as shown in the diagram. The radius is r cm and the angle at the centre is θ radians.



Determine the maximum possible area of the sector, showing that it is a maximum. **[6]**

There were two routes that candidates could use. The most common (and the most difficult) was to rewrite the area A in terms of θ and generally candidates struggled when they chose this method. The differentiation caused problems and many gave up. Those that used the less common route by finding A in terms of r found the going much easier and most were able to go on and earn most of the marks. There were some candidates that did not use the A Level formulae for arc length and sector area. There was also some very poor algebraic manipulation (e.g. $10/(2 + \theta)$ becoming $5 + 10/\theta$ or $r(2 + \theta) = 10$ becoming $2\theta = 10/r$.

Those that managed to get correct expressions for A in terms of r were generally able to earn all 6 marks but occasionally some forgot to give the maximum area or forgot to show that it was maximal.

Question 8

8 A circle with centre A and radius 8 cm and a circle with centre C and radius 12 cm intersect at points B and D.

Quadrilateral ABCD has area 60 cm^2.

Determine the two possible values for the length AC. **[7]**

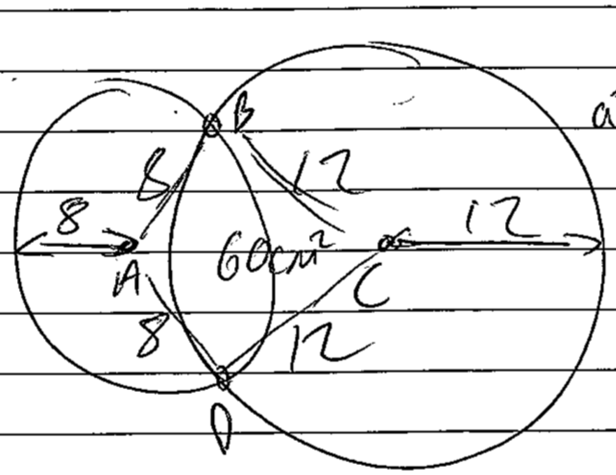
This was possibly the most challenging question on the paper with many candidates struggling to make any headway into the question. Most were able to gain a mark by giving a diagram though some of the diagrams did not allow the candidates to realise that ABCD was a kite and hence had a line of symmetry. Some tried using Pythagoras on invented right-angled triangles without much success. Others tried to use the cosine rule (finding the length BD in terms of the angle BAD or BCD) but again they did not get very far. There was also lots of fairly random algebra relating to equations of circles. Only those that realised $\frac{1}{2}ab\sin C$ was needed to use the area information from the question went on to get all 7 marks – if they overlooked the second possible value for the angle B then they may only have scored 5 marks.

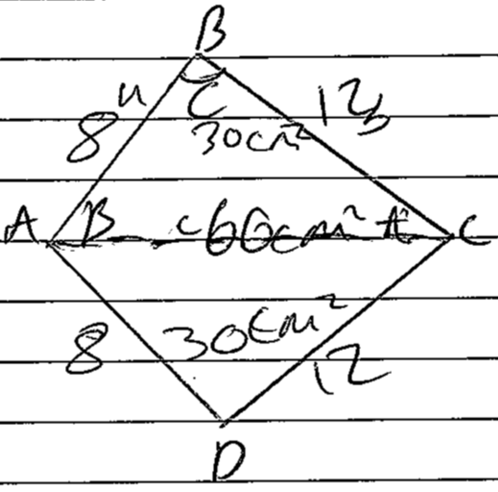
Exemplar 3

8

$\frac{1}{2}ab\sin C$

$a^2 = b^2 + c^2 - 2bc\cos A$





ABCD = 60cm²

$\vec{AB}$ = 8cm

$\vec{CD}$ = 12cm

$\vec{AD}$ = 8cm

$\vec{BC}$ = 12cm

area of $\overrightarrow{aBC}$ is now 30cm²
halfed

area = $\frac{1}{2}ab\sin C$

$30 = \frac{1}{2} \times 8 \times 12 \sin C$

~~48 = sin C~~

$30 = 48\sin C$

$\sin^{-1}\left(\frac{30}{48}\right) = C$

$C = 38.6822°$

$a^2 = \sqrt{12^2 + 8^2 - 2 \times 12 \times 8 \times \cos(38.6822)}$

$a = \vec{AC} = 7.6237$ cm

(answer space continued on next page)

8	**(continued)**
	so two values are, for $\vec{AC}$
	± 7.6237 cm

Although this candidate’s diagram was not particularly neat it was good enough for them to identify the kite and appreciate its symmetry. B1 was awarded for the quadrilateral but could also have been given for the triangle. M1 for use of area formula which led to A1 for angle 38.6..., no obtuse angle so A0. Cosine rule for M1, A1 for 7.62. Negative length is their second value so scores A0. To score final A1 they would have to reject any negative/extra values as well as finding 18.9.

Question 9 (a) (i), (ii), (b), (c), (d) (i), (ii), (iii) and (e)

9 A small country started using solar panels to produce electrical energy in the year 2000. Electricity production is measured in megawatt hours (MWh).

For the period from 2000 to 2009, the annual electrical energy produced using solar panels can be modelled by the equation $P = 0.3e^{0.5t}$, where P is the annual amount of electricity produced in MWh and t is the time in years after the year 2000.

(a) According to this model, find the amount of electricity produced using solar panels in each of the following years.

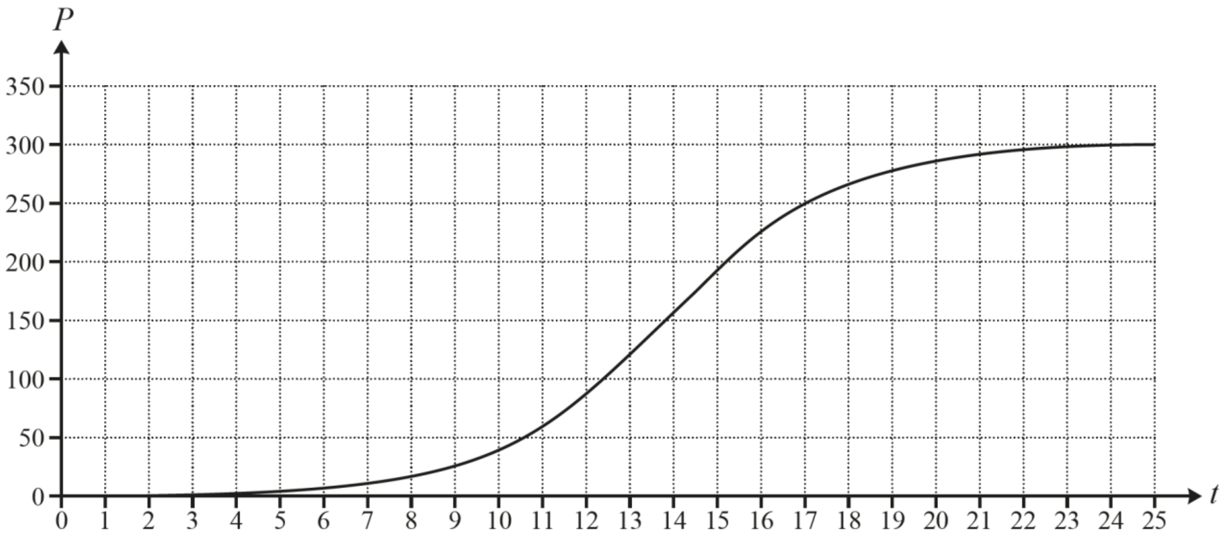
(i) 2000 **[1]**

(ii) 2009 **[1]**

(b) Give a reason why the model is unlikely to be suitable for predicting the annual amount of electricity produced using solar panels in the year 2025. **[1]**

An alternative model is suggested; the curve representing this model is shown in **Fig. 9**.

Fig. 9



(c) Explain how the graph shows that the alternative model gives a value for the amount of electricity produced in 2009 that is consistent with the original model. **[1]**

(d) **(i)** On the axes given in the Printed Answer Booklet, sketch the gradient function of the model shown in **Fig. 9**. **[2]**

(ii) State approximately the value of t at the point of inflection in **Fig. 9**. **[1]**

(iii) Interpret the significance of the point of inflection in the context of the model. **[1]**

(e) State approximately the long term value of the annual amount of electricity produced using solar panels according to the model represented in **Fig. 9**. **[1]**

Parts (a) and (b) were well answered. In part (a) (i) only a small number of candidates did not score. Some used the value t = 2000, some used t = 1.

In part (a) (ii) t = 2009 or t = 10 were occasionally used.

Part (c) was generally well answered though some candidates seemed to think that just stating 'extrapolation is not suitable' was sufficient for the mark. There were a lot of candidates who talked about the improvements in technology, the production of more efficient solar panels, etc.

Parts (d) (i) and (d) (ii) were well answered. In (d) (i) only the less successful responses saw candidates sketch an increasing function and in (d) (ii) most candidates gave an answer in the range 13 – 15.

Part (d) (iii) caused candidates a lot of trouble with many talking about the amount produced reaching a maximum or starting to decline, not realising that it was the rate of increase that was doing these things.

Part (e) was answered very well.

Question 10 (a) and (b)

10 **(a)** You are given that $(x^2+y^2)^3 = x^6+3x^4y^2+3x^2y^4+y^6$.

Hence, or otherwise, prove that $\sin^6\theta+\cos^6\theta = 1-\frac{3}{4}\sin^2 2\theta$ for all values of θ. **[4]**

(b) Use the result from part **(a)** to determine the minimum value of $\sin^6\theta+\cos^6\theta$. **[2]**

In part (a) very few candidates did the whole question correctly. The few that did showed high quality step by step reasoning. Many thought that sin^6 + cos^6 = 1 as if it's the same as sin^2 + cos^2 = 1. Use of double angle formula was not hugely frequent, and then used often squared incorrectly to 2sin^2cos^2 instead of 4sin^2cos^2. They tended to earn the method marks individually rather than systematically, with the double angle formula being gained the least. However convincing completion proved difficult for most, but a few did obtain full marks.

Part (b) was left blank fairly often. When answered it was 1 – ¾ = ¼ fairly often without explaining why using sin^2 (2θ)=1 was needed to give the minimum value.

Section B overview

This comprehension generally seemed quite accessible with fewer candidates than usual leaving lots of parts blank. The integrals in the formulas did not seem to cause undue concern to candidates.

Question 11 (a)

11 **(a)** Evaluate $\sum_{r=1}^{5} r^2$. **[1]**

For part (a) most candidates got 55. Some used summing the squares, others used the formula given in the article.

Question 11 (b)

(b) Show that Euler's approximate formula, as given in line 13, gives the exact value of $\sum_{r=1}^{5} r^2$. **[2]**

In part (b) the most common error was not doing any working between substitution and the final answer.

Some candidates got a decimal answer and said it is approximately 55.

Question 12

12 With the aid of a suitable diagram, show that the three triangles referred to in line 26 have the areas given in line 27. **[3]**

Realising that a 'show that' question required a thorough justification of the three area formulas was not widely appreciated. The most successful solutions gave three clearly annotated triangles showing the height and width and referencing these in an explicit statement of 0.5 base x height. Candidates whose responses were less successful ignored the instruction in the question to draw a diagram or else identified the incorrect triangles either triangles below the curve or 4 triangles.

Question 13

13 Prove that Euler's approximate formula, as given in line 13, when applied to $\sum_{r=1}^{n} r^2$ gives exactly $\frac{n(n+1)(2n+1)}{6}$. **[4]**

This was a proof question, and again some candidates did not show sufficient steps in their working to form a convincing proof. The majority of candidates showed the correct substitution into the formula and candidates who did well in this question showed successful manipulation of the fractions to reach the given result. Candidates who gave less successful responses often tried to skip steps and made sign errors and other algebraic slips.

Question 14

14 Show that the expression given in line 33 simplifies to $\sum_{r=1}^{n} \frac{1}{r} \approx \ln n + \frac{13}{24} + \frac{6n+5}{12n(n+1)}$, as given in line 34. **[3]**

This was a proof question, and again some candidates did not show sufficient steps in their working to form a convincing proof. Successful responses showed the substitution into the formula and the substitution of the limits into the integration. Some candidates struggled with the necessary algebraic manipulation of the fractions to reach the required result.

Question 15

15 The expression given in line 34 is used to calculate $\sum_{r=1}^{6} \frac{1}{r}$.

Show that the error in the result is less than 1.5% of the true value. **[2]**

Some candidates did not know the formula for percentage error (difference/true value x 100%) and used the approximate value in the denominator. Some candidates found the parentage error with acceptable accuracy but did not make the comparison with 1.5%. Candidates must form a clear conclusion in 'show that' questions.

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