Qualification Accredited

Oxford Cambridge and RSA

A LEVEL

Examiners' report

MATHEMATICS B (MEI)

H640

For first teaching in 2017

H640/01 Summer 2023 series

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper 1 series overview

This paper had been written during the disruption to education due to the measures taken to deal with Covid-19. It is by design a paper on which candidates can show what they know and many very good responses were seen. Candidates could pick up almost a third of the marks on routine questions.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• attempted all questions • worked accurately with arithmetic and algebra • used correct trig identities • explained their answers so that the method was clear to the examiner • demonstrated an understanding of modelling and its relation to the real life situation.	• left blank spaces rather than trying something • made errors in algebraic manipulation and in the use of the laws of logarithms • used incorrect formulae and identities • worked in fragments rather than a coherent argument • did not draw conclusions from the evidence they had found • were unable to use vectors in a Mechanics context.

Section A overview

The questions in Section A are designed to be fairly routine procedural and require little reading or understanding of contexts. There is a steady gradient of difficulty through the section.

Question 1

1 A ball is thrown vertically upwards with a speed of $8\,\mathrm{m\,s^{-1}}$.

Find the times at which the ball is 3 m above the point of projection. **[2]**

Most candidates used a correct *suvat* equation to set up a quadratic equation for t which they solved successfully. Many then went on to give a range of times for which the ball was above 3m, but full credit was given for this where the boundary values were correct.

Question 2

2 Express $\dfrac{5x+1}{x^2-x-12}$ in partial fractions. **[4]**

This question was answered successfully by the majority of candidates, with errors mostly being around the use of +/- signs. Very few candidates did not realise the need to factorise the denominator.

Question 3

3 Find $\int (2x^4 - x\sqrt{x})\,\mathrm{d}x$. **[3]**

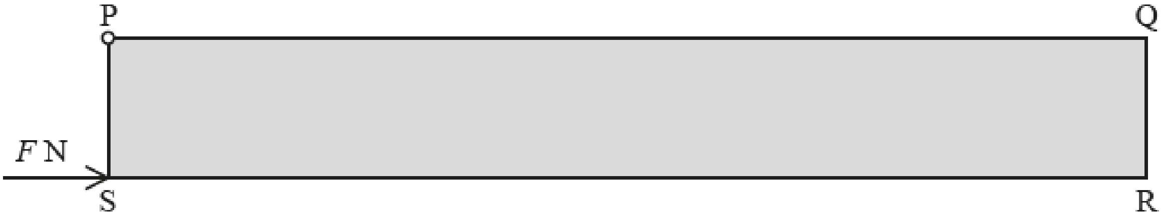
Many candidates used fractional powers and integrated this successfully. Some lost a mark for omitting the arbitrary constant.

Assessment for learning

Simple methods better – some candidates tried to use integration by parts for the $x\sqrt{x}$ term. As this is an unnecessarily complicated method, it had to be successful to get the method mark.

Question 4 (a)

4 A ruler PQRS is a uniform rectangular lamina with mass 20 grams. The length of PQ is 30 cm and the length of PS is 4 cm. The ruler is attached at P to a smooth hinge and held with S vertically below P by a horizontal force of magnitude F N as shown in the diagram.



(a) Calculate the value of F. **[3]**

This question was well answered with most candidates aware that the centre of mass was 15 cm from P horizontally. Some candidates made mistakes with units. Candidates who used mass instead of weight were only eligible for the B1 mark for 15 cm.

Question 4 (b)

(b) Explain what would happen to the lamina if the force at S were removed. **[1]**

The expected response was 'rotate clockwise', but we accepted variations on rotate provided 'clockwise' was seen. However it was common to see just 'fall', or a description of where the ruler would finish.

Question 5 (a)

5 **In this question you must show detailed reasoning.**

(a) Find the coordinates of the two stationary points on the graph of $y = 15 - x^2 - \frac{16}{x^2}$. **[3]**

Most candidates understood that they needed to differentiate and equate to zero, although some had difficulty with the negative index. There were many good solutions with both turning points found correctly, but some candidates took out a factor of x and were led to believe that there was a turning point at x = 0. Many of these did not notice that the function was undefined at x = 0.

Question 5 (b)

(b) Show that both these stationary points are maximum points. **[2]**

The majority of candidates used the second derivative and were able to show that this was negative at both turning points to show that these were local maxima. As in part (a), there were a few issues with the negative index. A minority looked at the gradient on each side of each turning point, which was a more laborious approach.

Question 6 (a)

6 (a) Show that the equation $\sin\left(x+\frac{1}{6}\pi\right)=\cos\left(x-\frac{1}{4}\pi\right)$ can be written in the form

$\tan x=\dfrac{\sqrt{2}-1}{\sqrt{3}-\sqrt{2}}$. **[4]**

Some candidates had learned $\tan=\frac{\sin}{\cos}$ without realising that the arguments of these functions have to match. They tried to divide the LHS by the RHS mistakenly thinking that gave a correct line of working.

Others just wrote $\sin\left(x+\frac{1}{6}\pi\right)=\sin x+\sin\frac{1}{6}\pi$ etc These responses could get the second M mark for using correct exact values for two trig terms.

There were some candidates who used almost correct identities but introduced a sign error in the cosine identity.

Where correct identities and exact values were used, the examiners were looking for correctly factorised work, and missing brackets could lose the final mark.

Question 6 (b)

(b) Hence solve the equation $\sin\left(x+\frac{1}{6}\pi\right)=\cos\left(x-\frac{1}{4}\pi\right)$ for $0\leqslant x\leqslant 2\pi$. **[1]**

Almost all candidates used the given expression and their calculator to get one root but some candidates did not get the second root in the range.

Occasionally the mark was not given when the handwriting meant that their written response read as $7/24\pi$ or even $\frac{7}{24\pi}$ which may not be what the candidate intended.

Section B overview

In Section B, there are more stretching questions involving more interpretation. Many very successful responses were seen.

Question 7

7 Determine the exact distance between the two points at which the line through (4, 5) and (6, −1) meets the curve $y = 2x^2 - 7x + 1$. **[7]**

This was generally well answered with all candidates able to pick up some of the method marks since these were not dependent. There were occasional mistakes on the gradient formula for the equation of a straight line. But those candidates who evidenced their working by showing substitution into the formula correctly before making a slip could still earn this method mark. Most did this successfully and went on to solve simultaneously forming and solving their quadratic. Occasionally candidates forgot the last part of the question or added coordinates in their calculation for length of line rather than subtracting before squaring, but many gained this M mark FT even if their coordinates found were incorrect.

Question 8 (a)

8 A bus is travelling along a straight road at $5.4\,\mathrm{m\,s^{-1}}$. At $t = 0$, as the bus passes a boy standing on the pavement, the boy starts running in the same direction as the bus, accelerating at $1.2\,\mathrm{m\,s^{-2}}$ from rest for 5 s. He then runs at constant speed until he catches up with the bus.

(a) The diagram in the Printed Answer Booklet shows the velocity-time graph for the bus.

Draw the velocity-time graph for the boy on this diagram. **[3]**

Many candidates had fully correct graphs with the change in gradient at (5, 6) clearly shown. Candidates who had not calculated the velocity of the boy at $t = 5$ did not always have the horizontal section above the given line representing the velocity of the bus and lost 2 marks.

Question 8 (b)

(b) Determine the time at which the boy is running at the same speed as the bus. **[2]**

This was well answered by equating an expression of the boy's velocity to 5.4.

Question 8 (c)

(c) Find the maximum distance between the bus and the boy. **[3]**

This question is straightforward once you recognise that the distance between the boy and the bus is growing while his speed is less than that of the bus, and reduces when he is quicker than the bus. Some candidates assumed that the maximum distance happened at $t = 5$ instead and some credit was given for calculating the distance travelled by the boy at any time up to and including 5.

This question was sometimes answered using calculus and an expression for the difference between the displacements of the bus and the boy.

The area between the two graphs up to $t = 4.5$ gives the maximum distance in one step and is equal to the displacement of the boy, which is structural to the solution as the boy starts from rest. Where candidates used an area calculation, examiners assumed they were attempting this method. Where it seemed that $suvat$ equations were used, they assumed that they were looking at the displacement of either the bus of the boy, So 12.15 m from a $suvat$ calculation on its own did not receive full credit.

Assessment for learning

It is important that candidates explain their methods in their response. Just a word or two next to the calculations can help the examiner recognise the method being used and award the method marks.

Question 8 (d)

(d) Find the distance the boy has run when he catches up with the bus. **[3]**

This was a more challenging question as the boy has two phases of motion to consider before catching up with the bus. Many candidates equated the expression for displacement valid during the acceleration phase to $5.4t$ without noticing that this was only valid for the first 5 seconds. Some muddled up the time from the start with the time after $t = 5$.

Candidates who realised that the boy was 12m behind the bus when he reached top speed and used relative speed to calculate an additional 20s were often successful, although some did not include the first 15m in their total distance.

Question 9 (a)

9 The gradient of a curve is given by $\frac{dy}{dx} = e^x - 4e^{-x}$.

(a) Show that the x-coordinate of any point on the curve at which the gradient is 3 satisfies the equation $(e^x)^2 - 3e^x - 4 = 0$. **[2]**

This question asked candidates to derive the equation by equating the expression for $\frac{dy}{dx}$ to 3 and rearranging. In the structure of the whole question, it should lead candidates to solve a quadratic in e^x in part (b). Some candidates tried to solve the equation in part (a) rather than rewriting.

Question 9 (b)

(b) Hence show that there is only one point on the curve at which the gradient is 3, stating the exact value of its x-coordinate. **[3]**

To obtain full marks here, candidates needed to solve the quadratic to give $e^x = -1, 4$ and then explain why $e^x = -1$ did not lead to a solution, leaving $x = \ln 4$ as the only root.

Misconception

Taking logs of terms which are added together does not give logs of each term added together.

So $e^x - 4e^{-x} = 3$ does not give $x - 4\ln e^{-x} = \ln 3$

Question 9 (c)

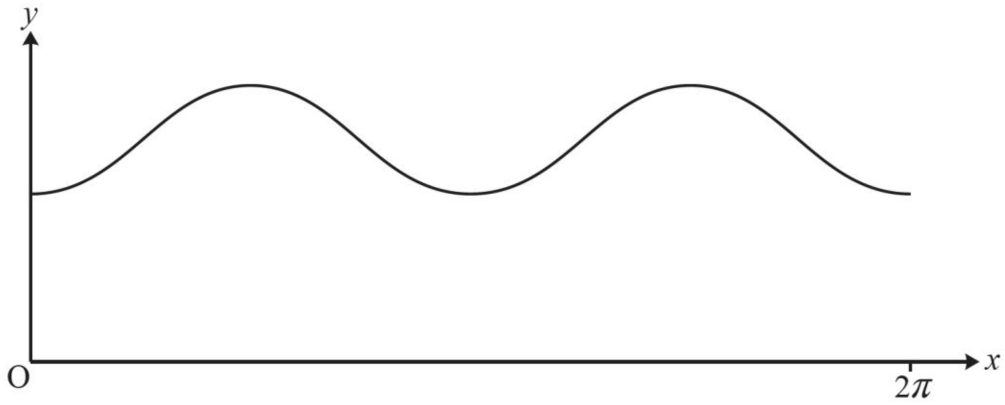
(c) The curve passes through the point $(0, 0)$.

Show that when $x = 1$ the curve is below the x-axis. **[5]**

Most candidates realised they had to integrate the expression for $\frac{dy}{dx}$ and evaluate the y-coordinate when $x = 1$. Some explanation was needed after the line $y = -0.81$ that this meant the curve was below the axis. A few omitted the constant so were unable to get a negative value.

Question 10 (a)

10 The diagram shows the graph of $y = 1.5 + \sin^2 x$ for $0 \leqslant x \leqslant 2\pi$.



(a) Show that the equation of the graph can be written in the form $y = a - b\cos 2x$ where a and b are constants to be determined. **[2]**

A number of candidates confused $\cos^2 x$ and $\cos 2x$, treating them interchangeable. The few who compared like terms to evaluate a and b usually made an error when dealing with the negative signs. A few candidates attempted alternative methods using graph transformations. Full credit was given for a correct expression even if $a = 2, b = -0.5$ as subsequently seen and marked ISW (Ignore Subsequent Working).

Assessment for learning

There are two formulae for $\sin^2 x$ namely $\sin^2 x = 1 - \cos^2 x$ which was not relevant here and $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ which is needed here.

Also emphasis that $\cos^2 x$ and $\cos 2x$ should not look the same when written.

Question 10 (b)

(b) Write down the period of the function $1.5 + \sin^2 x$. **[1]**

This was often well answered. No credit was given for 180° unless π was seen as well.

Question 10 (c)

(c) Determine the x-coordinates of the points of intersection of the graph of $y = 1.5 + \sin^2 x$ with the graph of $y = 1 + \cos 2x$ in the interval $0 \leqslant x \leqslant 2\pi$. **[3]**

Many candidates found a value for $\cos 2x$ and at least one root in the required interval. Not many candidates went on to find the other three roots. This was more of a problem for candidates who found an equation for $\sin^2 x$

Assessment for learning

The question was structured so that the expression found in (a) could be used for the simplest method to find the points of intersection. It was much more work to begin from scratch and find an equation for $\sin^2 x$ or even $\cos^2 x$.

Question 11 (a) (i)

11 The height h cm of a sunflower plant t days after planting the seed is modelled by $h = a + b\ln t$ for $t \geqslant 9$, where a and b are constants. The sunflower is 10 cm tall 10 days after planting and 200 cm tall 85 days after planting.

(a) (i) Show that the value of b which best models these values is 88.8 correct to **3** significant figures. **[2]**

Many realised the need to build and solve simultaneous equations and used an algebraic method to find the exact value for b which rounds to 88.8 to 3 s.f. Where a calculator was used to solve the equations, it was necessary to show a value for b to more than 3 s.f. and show it rounds to 88.8.

Question 11 (a) (ii)

(ii) Find the corresponding value of a. **[1]**

This was generally correct even where candidates had to use the given value for b. Some did not write the negative sign needed.

Question 11 (b) (i)

(b) (i) Explain why the model is not suitable for small positive values of t. **[1]**

Most candidates realised that the model predicted negative values for h which was not possible. Some made an attempt to explain away the negative sign by talking about the seed being below the ground, which was not necessary and does not explain the fact that the model gives $h \to -\infty$ as $t \to 0$.

Question 11 (b) (ii)

(ii) Explain why the model is not suitable for very large positive values of t. **[1]**

To obtain a mark here candidates had to state that the model predicts that the height tends to infinity and that this is not possible in reality. Some did not mention what the model predicts. Others stated that the model predicts unlimited growth but did not explain that there was a problem with that in reality. Some stated that the height grows exponentially or at an increasing rate which is not correct, so they also lost this mark.

Exemplar 1

Because the Sunflower will have a maximum height and wont just keep growing.

This exemplar shows a response where it is not clearly stated that the model predicts unlimited growth although it hints at it. This was a borderline case that would have had the mark had the response included 'for large values of t' or similar.

Question 11 (c)

(c) Show that the model indicates that the sunflower grows to 1 m in height in less than half the time it takes to grow to 2 m. **[2]**

Many candidates decided to calculate the time to grow to 1m and also to 2m, not realising that 85 days for 2m is given in the question. A clear statement comparing 27.4 days to a half of 85 (or their calculated value) was also needed to get the second mark.

There were quite a lot of errors with units – the given units in the question are cm.

Question 11 (d)

(d) Find the value of t for which the rate of growth is 3 cm per day. **[3]**

This was well answered but a few candidates did not think to use the derivative of the model in their response and so obtained no marks.

Question 12 (a)

12 In this question the unit vectors **i** and **j** are horizontal and vertically upwards respectively.

A particle has mass 2 kg.

(a) Write down its weight as a vector. **[1]**

In all parts of this question, examiners expected to see vector answers and it was clear that some candidates did not know that Newton's second law and the *suvat* equations are vector equations. Column vectors or vectors in terms of **i** and **j** were equally acceptable.

For this first part it was quite common to see a positive component for weight even though the question defined the **j** direction as vertically upwards. Some candidates omitted the g effectively giving mass acting in the **j** direction which makes no sense.

Question 12 (b)

A horizontal force of 3 N in the **i** direction and a force $\mathbf{F} = (-4\mathbf{i} + 12\mathbf{j})$ N act on the particle.

(b) Determine the acceleration of the particle. **[3]**

Most candidates wrote the 3N horizontal force correctly as a vector and many fully correct answers were seen. A large minority forgot to include the weight of the particle in their equation of motion, although part (a) was included to help them with that. Candidates who had a correct acceleration vector did not lose a mark for then finding its magnitude.

Misconception

? Some candidates thought that Newton's second law must be a scalar equation and found the magnitude of the resultant force to get the magnitude of the acceleration instead. This did not get the method mark.

Question 12 (c)

(c) The initial velocity of the particle is $5\mathbf{i}\,\mathrm{m\,s^{-1}}$.

Find the velocity of the particle after 4 s. **[2]**

Similar problems confusing vectors and scalars were frequently seen. Some candidates attempted to add vectors and scalars, which lost the method mark. Some candidates who had a correct velocity vector sometimes lost the final mark by giving the speed rather than the velocity as their final answer. The decision was made not to award the final mark where this was seen on this part of the question as the distinction between speed and velocity is a specification point to be tested.

Assessment for learning

Velocity is vector and must be the final answer. If you then find its magnitude, you are finding the speed and not the velocity. Students need to recognise the distinction between scalar speed and vector velocity..

Question 12 (d)

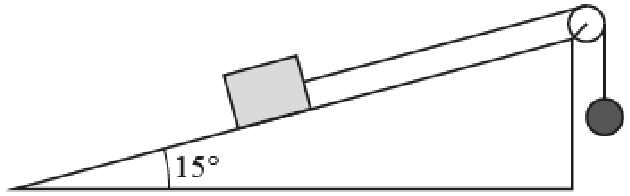
(d) Find the extra force that must be applied to the particle for it to move at constant velocity. **[1]**

Candidates who had included the weight earlier in the question typically got this correct, but those who had omitted weight earlier did so again here and were not given the mark.

Question 13 (a)

13 A block of mass 8 kg is placed on a rough plane inclined at 15° to the horizontal. The coefficient of friction between the block and the plane is 0.3.

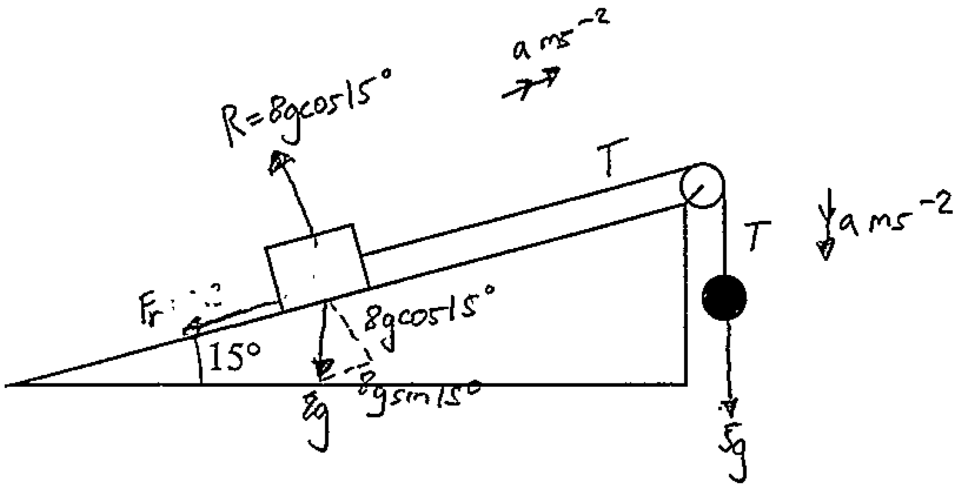
One end of a light rope is attached to the block. The rope passes over a smooth pulley fixed at the top of the plane, and a sphere of mass 5 kg, attached to the other end of the rope, hangs vertically below the pulley. The part of the rope between the block and the pulley is parallel to the plane. The system is released from rest, and as the sphere falls the block moves directly up the plane with acceleration $a\,\mathrm{m\,s^{-2}}$.



(a) On the diagram in the Printed Answer Booklet, show all the forces acting on the block and on the sphere. **[4]**

There were many fully correct diagrams with the direction of the forces clearly shown. Some candidates lost a mark for using T_1 and T_2 for the tensions. Some candidates lost the mark for the weights either by labelling them both W, or showing the weight of the block and its components as additional forces.

Exemplar 2



This response shows a good example of the components of the weight indicated with dotted lines without looking like additional forces. The label R for the normal reaction would be enough on its own and the friction is in the correct direction. There was no indication of the direction of the tension, so a mark was lost.

Question 13 (b)

(b) Write down the equation of motion for the sphere. **[2]**

This was usually well answered, but a few candidates had the direction of travel wrong despite it having been stated in the question. Some did not read the question correctly and tried to look at whole system equation. Others used 5g as mass in $F = ma$. A few candidates did not understand the term 'equation of motion' and tried to use one of the *suvat* equations.

Question 13 (c)

(c) Determine the value of a. **[6]**

Candidates who considered the 2 objects separately and then eliminated T in their simultaneous equations usually wrote more successful responses than those that attempted a 'round the corner' method trying to do the whole system = 13a in one go. Some used the value for T from the equilibrium case not realising the tension is not the same while the system is in motion.

Candidates should be encouraged to write down their methods and leave substituting in values for terms until the last stage. This usually means they get less confused with their method and force directions and it leads to more accurate answers.

Question 14 (a)

14 **(a)** Use the laws of logarithms to show that $\log_{10}200 - \log_{10}20$ is equal to 1. **[2]**

Most candidates used the laws of logs correctly and showed sufficient working to get full credit.

Question 14 (b)

The first three terms of a sequence are $\log_{10}20, \log_{10}200, \log_{10}2000$.

(b) Show that the sequence is arithmetic. **[2]**

It was very common for candidates just to state that the difference between the second and third terms was 1 without proving it, or by giving numerical values only.

Exemplar 3

$\log_{10} 2000 - \log_{10} 200 = 1$

$\log_{10} 200 - \log_{10} 20 = 1$

then $a_3 - a_2 = a_2 - a_1$

all share same common difference

$\therefore$ arithmetic

In this exemplar, the working was enough for the M1 but not the second mark for a full explanation. There is no evidence to support the statement in the second line.

Question 14 (c)

(c) Find the exact value of the sum of the first 50 terms of this sequence. **[2]**

This was successfully answered by candidates who used the formula with clear substitution. The A mark was given for a fully correct expression so candidates who made an error in simplifying were able to have 2 marks, ignoring subsequent working. Responses where candidates attempted to use the form involving the last term were often less successful as that term was tricky to get right. Candidates who used their calculator to sum the series to give a decimal answer did not get full credit here as an exact answer was needed.

Question 15 (a)

15 A projectile is launched from a point on level ground with an initial velocity u at an angle θ above the horizontal.

(a) Show that the range of the projectile is given by $\frac{2u^2 \sin\theta\cos\theta}{g}$. **[3]**

This question asked candidates to derive algebraically formulae that may be familiar to some. It was specifically set using this form as the better known version with $\sin 2\theta$ is not helpful in part (b).

Many fully correct solutions were seen where candidates found the time for the whole flight, or sometimes where candidates doubled the time needed to reach the highest point. Sometimes candidates did not introduce the factor of 2 until very late and without explanation, so the final E mark was lost, as the final result was a given answer.

Question 15 (b)

(b) Determine the set of values of θ for which the maximum height of the projectile is greater than the range, where θ is an acute angle. Give your answer in degrees. **[5]**

Candidates who used the equation $v^2 = u^2 + 2as$ with $v = 0$ often found a correct expression for the maximum height. Candidates who found the time to the top first and then tried to use it in $s = ut + \frac{1}{2}at^2$ often struggled to combine the terms to get the simplified answer.

A method mark was given for comparing their expression for maximum height with the given range as long as the expression for height did not contain t.

Some candidates struggled to simplify the resulting equation to a simple equation for $\tan\theta$ and occasionally the final mark was lost by candidates who reached $\tan\theta = 4$ where their final answer was not a correct inequality.

Supporting you

Teach Cambridge

Make sure you visit our secure website **Teach Cambridge** to find the full range of resources and support for the subjects you teach. This includes secure materials such as set assignments and exemplars, online and on-demand training.

Don't have access? If your school or college teaches any OCR qualifications, please contact your exams officer. You can forward them this link to help get you started.

Reviews of marking

If any of your students' results are not as expected, you may wish to consider one of our post-results services. For full information about the options available visit the OCR website.

Access to Scripts

For the June 2023 series, Exams Officers will be able to download copies of your candidates' completed papers or 'scripts' for all of our General Qualifications including Entry Level, GCSE and AS/A Level. Your centre can use these scripts to decide whether to request a review of marking and to support teaching and learning.

Our free, on-demand service, Access to Scripts is available via our single sign-on service, My Cambridge. Step-by-step instructions are on our website.

Keep up-to-date

We send a monthly bulletin to tell you about important updates. You can also sign up for your subject specific updates. If you haven't already, sign up here.

OCR Professional Development

Attend one of our popular CPD courses to hear directly from a senior assessor or drop in to a Q&A session. Most of our courses are delivered live via an online platform, so you can attend from any location.

Please find details for all our courses for your subject on **Teach Cambridge**. You'll also find links to our online courses on NEA marking and support.

Signed up for ExamBuilder?

ExamBuilder is the question builder platform for a range of our GCSE, A Level, Cambridge Nationals and Cambridge Technicals qualifications. Find out more.

ExamBuilder is **free for all OCR centres** with an Interchange account and gives you unlimited users per centre. We need an Interchange username to validate the identity of your centre's first user account for ExamBuilder.

If you do not have an Interchange account please contact your centre administrator (usually the Exams Officer) to request a username, or nominate an existing Interchange user in your department.

Active Results

Review students' exam performance with our free online results analysis tool. It is available for all GCSEs, AS and A Levels and Cambridge Nationals.

Find out more.

Need to get in touch?

If you ever have any questions about OCR qualifications or services (including administration, logistics and teaching) please feel free to get in touch with our customer support centre.

Call us on
01223 553998

Alternatively, you can email us on
support@ocr.org.uk

For more information visit
- **ocr.org.uk/qualifications/resource-finder**
- **ocr.org.uk**
- **facebook.com/ocrexams**
- **twitter.com/ocrexams**
- **instagram.com/ocrexaminations**
- **linkedin.com/company/ocr**
- **youtube.com/ocrexams**

We really value your feedback

Click to send us an autogenerated email about this resource. Add comments if you want to. Let us know how we can improve this resource or what else you need. Your email address will not be used or shared for any marketing purposes.

I like this **I dislike this**

Please note – web links are correct at date of publication but other websites may change over time. If you have any problems with a link you may want to navigate to that organisation's website for a direct search.