Qualification Accredited

Oxford Cambridge and RSA

A LEVEL

Examiners' report

FURTHER MATHEMATICS B (MEI)

H645

For first teaching in 2017

Y434/01 Summer 2023 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y434/01 series overview

This optional minor paper counts for 16⅔ % of the qualification OCR A Level Further Mathematics B (MEI) (H645). There is one examination paper which lasts 1 hour 15 minutes. Candidates are expected to know the content of A Level Mathematics and the Core Pure mandatory paper (Y420).

Candidates should be able to use the iterative capability of a calculator in the examination. No credit is given for writing down solutions generated by equation solvers.

It is expected that candidates will have routinely used a spreadsheet throughout the course. In the examination candidates will be given output from spreadsheets and may be asked: what certain cells represent; to explain or give formulae for certain cells; to give solutions and justify their accuracy; to comment on errors, convergence or order of a method.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• understood that a request to 'Determine' entails providing some justification for their answer • made efficient use of the iterative capability of their calculator when implementing algorithms, and gave sufficient detail of their work • demonstrated an understanding of spreadsheet output and articulated this clearly and concisely • articulated their understanding of how different algorithms succeed or fail in the given context • used error analysis effectively to generate an improved solution, and interpreted their answers correctly • explained how computers and calculators may work with values that are stored to a different precision than they are displayed.	• did not provide any justification for their answers when faced with a request to 'Determine' • were only partly successful in using the iterative capability of their calculator, and did not provide sufficient detail of their work • demonstrated only limited understanding of spreadsheet output • did not seem to have a clear understanding of how different methods may succeed or fail, or were unable to articulate their understanding clearly • did not recognise situations in which extrapolation was appropriate • did not appreciate that stored values and displayed values in a spreadsheet and/or calculator may not be the same.

Question 1 (a), (b), (c) and (d)

1 You are given that $(x_1, y_1) = (0.9, 2.3)$ and $(x_2, y_2) = (1.1, 2.7)$.

The values of x_1 and x_2 have been **rounded** to **1** decimal place.

(a) Determine the range of possible values of $x_2 - x_1$. **[2]**

The values of y_1 and y_2 have been **chopped** to **1** decimal place.

(b) Determine the range of possible values of $y_2 - y_1$. **[2]**

You are given that $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

(c) Determine the range of possible values of m. **[2]**

(d) Explain why your answer to part **(c)** is much larger than your answer to part **(a)** and your answer to part **(b)**. **[1]**

The many candidates who did well in this question worked successfully with interval arithmetic to determine the correct intervals in parts (a) and (b), going on to successfully find the interval for *m* in part (c). They sometimes recognised that the seemingly large interval in part (c) was due to the division by the difference between nearly equal quantities, but were not always able to articulate this correctly.

In some cases candidates who did less well wrote down the correct answers to parts (a) and (b) with no justification. Some candidates answered part (a) correctly, but worked with 2.749 and 2.349 in part (b). The method mark in part (c) was usually earned, but a few candidates worked with $\frac{x_2 - x_1}{y_2 - y_1}$ and didn't score.

Question 2 (a), (b), (c) and (d)

2 A car tyre has a slow puncture. Initially the tyre is inflated to a pressure of 34.5 psi. The pressure is checked after 3 days and then again after 5 days. The time t in days and the pressure, P psi, are shown in the table below. You are given that the pressure in a car tyre is measured in pounds per square inch (psi).

t	0	3	5
P	34.5	29.4	27.0

The owner of the car believes the relationship between P and t may be modelled by a polynomial.

(a) Explain why it is not possible to use Newton's forward difference interpolation method for these data. **[1]**

(b) Use Lagrange's form of the interpolating polynomial to find an interpolating polynomial of degree 2 for these data. **[4]**

The car owner uses the polynomial found in part **(b)** to model the relationship between P and t.

Subsequently it is found that when $t = 6$, $P = 26.0$ and when $t = 10$, $P = 24.4$.

(c) Determine whether the owner's model is a good fit for these data. **[2]**

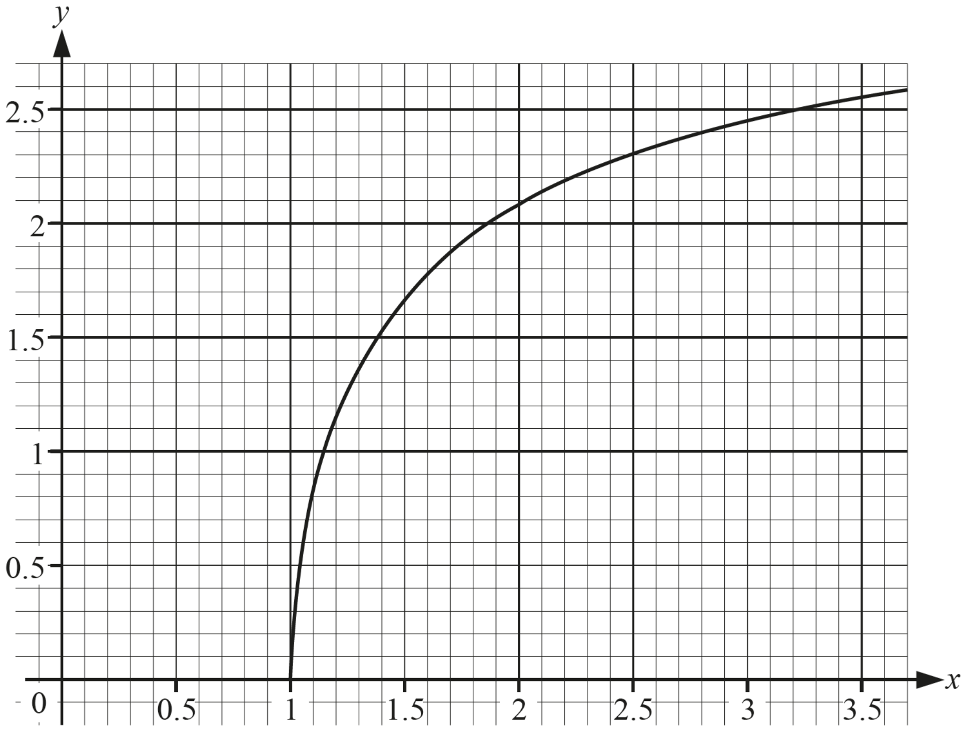
(d) Explain why the model would not be suitable in the long term. **[1]**

The many candidates who did well in this question answered part (a) correctly. In part (b) they successfully found the correct quadratic, but may have lost an accuracy mark by omitting P or expressing the equation in terms of x, rather than t. In part (c) the correct values were calculated, and usually compared with the given values to justify an appropriate comment. In part (d) they recognised that the model will eventually predict that the pressure in the tyre increases and is therefore inappropriate.

Candidates who did less well often understood what was required in part (a), but gave a vague response such as 'the difference between the values is different', or referred to x instead of t. In part (b) they may have made an arithmetical slip with one of the coefficients, or left their answer in terms of x, not t. In part (c) they did not compare their calculated values with the given values to justify the comment that the model is a good fit, and in part (d) some commented that the model predicts a negative value for P in the long run.

Question 3 (a), (b), (c) and (d)

3 The diagram shows the graph of $y = \mathrm{f}(x)$ for values of x from 1 to 3.5.



The table shows some values of x and the associated values of y.

x	1.5	2	2.5
y	1.682137	2.094395	2.318559

(a) Use the forward difference method to calculate an approximation to $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$. **[2]**

(b) Use the central difference method to calculate an approximation to $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$. **[2]**

(c) On the copy of the diagram in the Printed Answer Booklet, show how the central difference method gives the approximation to $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$ which was found in part **(b)**. **[1]**

(d) Explain whether your answer to part **(a)** or your answer to part **(b)** is likely to give a better approximation to $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$. **[1]**

Nearly all candidates answered the first two parts correctly, with just a few making an arithmetical slip or transcription error from their calculator. Most candidates drew the correct chord on the graph, but didn't add the tangent at $x = 2$ to show what was being approximated. Candidates who did well usually compared the order of the central difference method with the order of the forward difference method to explain why their answer to part (b) was more likely to give a better approximation.

Candidates who did less well either referred to just one method in their explanation, or gave an incomplete or incorrect justification in terms of straddling the point. A small number of candidates incorrectly thought that the forward difference method gave a better approximation.

Question 4 (a)

4 A spreadsheet is used to approximate $\int_a^b f(x)\,dx$ using the midpoint rule with 1 strip.

The output is shown in the table below.

	B	C	D
3	x	f(x)	M_1
4	1.5	1.3103707	0.65518535

The formula in cell C4 is `=B4^(1/B4)`.

The formula in cell D4 is `=0.5*C4`.

(a) Write the integral in standard mathematical notation. **[3]**

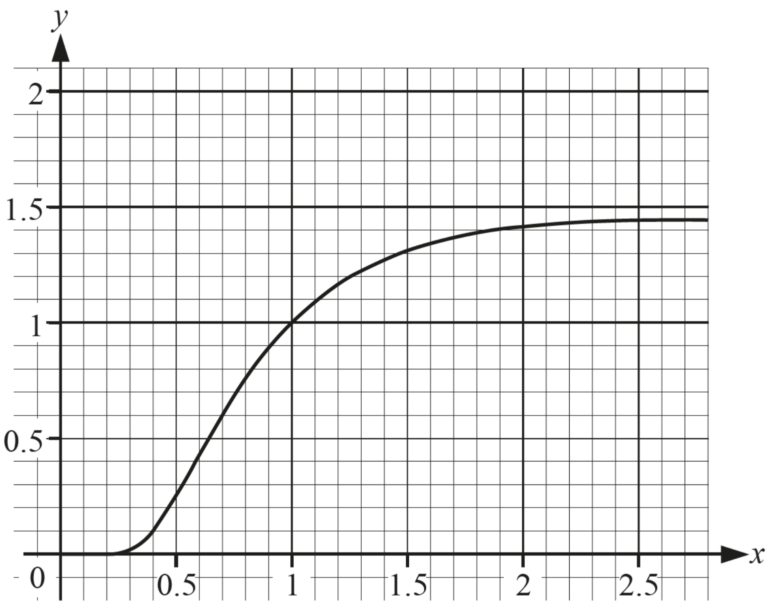
A graph of $y = f(x)$ is included in the diagram below.

Candidates who did well in this question identified the integrand and understood that the limits had to be symmetrical about 1.5. Many identified the correct limits and expressed the integral correctly. A significant minority gave the wrong limits but earned SC1.

Candidates who did less well identified $x^{\frac{1}{x}}$ correctly, but gave incorrect limits which did not earn the special case mark. Limits from 0 to 1.5 was a common wrong answer.

Question 4 (b)

A graph of $y = \mathrm{f}(x)$ is included in the diagram below.



(b) Explain whether 0.65518535 is an over-estimate or an under-estimate of $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$. **[1]**

Candidates who did well in this question stated the correct answer and cited that the curve is concave down over the interval.

Candidates who did not do well described the curve simply as concave or convex, which is insufficient for the convention used in this specification (concave downwards is the preferred terminology here but convex upwards would be equivalent).

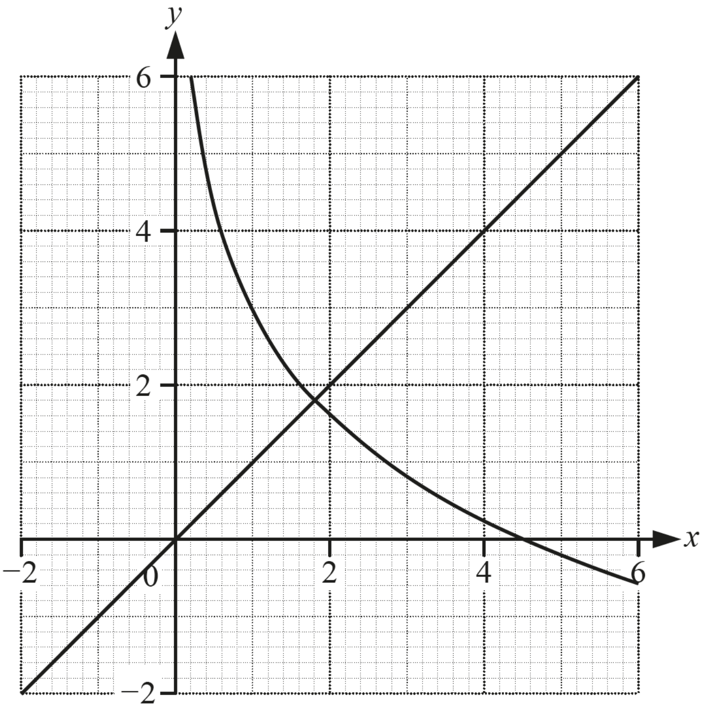
Candidates who attempted an explanation framed in more general terms were almost always unsuccessful.

Question 5 (a)

5 The equation $3-2\ln x-x=0$ has a root near $x=1.8$.

A student proposes to use the iterative formula $x_{n+1}=\mathrm{g}(x_n)=3-2\ln x_n$ to find this root.

The diagram shows the graphs of $y=x$ and $y=\mathrm{g}(x)$ for values of x from -2 to 6.



(a) With reference to the graph, explain why it might not be possible to use the student's iterative formula to find the root near $x=1.8$. **[1]**

Candidates who did well made reference to the gradient of g(x) at x = 1.8 and the criterion for convergence of the method of fixed point iteration.

Candidates who did not do well missed one or more of the key elements of the explanation, or did not understand the criterion for convergence.

Question 5 (b)

(b) Use the relaxed iteration $x_{n+1}=\lambda\mathrm{g}(x_n)+(1-\lambda)x_n$, with $\lambda=0.475$ and $x_0=2$, to determine the root correct to **6** decimal places. **[3]**

Candidates who did well understood that the request to 'determine' the root correct to six decimal places necessitated showing sufficient level of detail in their working, presenting the iterates to 7 decimal places at each stage. A few candidates went even further and confirmed the precision with a change of sign.

Candidates who did less well often just presented iterates to 6 decimal places, or omitted some iterates altogether.

Question 5 (c)

A student uses the same relaxed iteration with the same starting value. Some analysis of the iterates is carried out using a spreadsheet, which is shown in the table below.

r	difference	ratio
0		
1	−0.1834898	
2	−0.0049137	0.02678
3	−6.44E-06	0.00131
4	−3.862E-09	0.0006
5	−2.313E-12	0.0006

(c) Explain what the analysis tells you about the order of convergence of this sequence of approximations. **[2]**

Candidates who did well commented that the ratio appears to be converging to 0.0006 and interpreted this as suggesting first order convergence.

Candidates who did less well commented that the ratio is equal to 0.0006, which did not score, but went on to correctly suggest first order convergence.

Candidates who did not do well generally commented that the ratio was continuing to decrease and suggested a higher order of convergence.

Assessment for learning

When commenting on the ratio of differences, candidates need to decide whether the values are converging and comment accordingly. Statements such as 'the ratio of differences is equal to…' do not usually score. This issue was also seen with Question 7 (b) (i) and Question 9 (c).

Question 6 (a) (i)

6 (a) (i) Calculate the relative error when π is **chopped** to **2** decimal places in approximating $\pi^2 + 2$. **[2]**

Candidates who did well used the correct formula and gave their answer to a suitable precision.

Candidates who did less well calculated the magnitude of the relative error or made an arithmetical slip.

The small number of candidates reversed the terms in the numerator, and occasionally candidates appeared to completely misunderstand the request and calculated something else entirely.

Question 6 (a) (ii)

(ii) **Without** doing any calculation, explain whether the relative error would be the same when π is **chopped** to **2** decimal places when approximating $(\pi+2)^2$. **[1]**

Candidates who did well commented that that the relative error would be different because the order of operations is different.

Candidates who did not do well were often on the right lines but gave incomplete reasoning.

Question 6 (b) (i)

The table shows some spreadsheet output. The values of x in column A are exact.

	A	B	C
1	x	10^x	$\log_{10} 10^x$
2	1E-12	1	1.00001E-12
3	1E-11	1	9.99998E-12

The formula in cell B2 is =10^A2 .

This has been copied down to cell B3.

The formula in cell C2 is =LOG(B2) .

This formula has been copied down to cell C3.

(b) **(i)** Write the value displayed in cell C2 in standard mathematical notation. **[1]**

Nearly all candidates wrote down the correct answer.

Occasionally the negative sign in the exponent was missed, or they wrote $\log_{10} 1.00001 \times 10^{-12}$, or less frequently 9.99998×10^{-12}

Question 6 (b) (ii)

(ii) Explain why the values in cells C2 and C3 are neither zero nor the same as the values in cells A2 and A3 respectively. **[2]**

Candidates who did well in this question wrote one sentence explaining why the values in cells C2 and C3 are not zero, and a separate sentence explaining why the values in C2 and C3 are different from each other.

Candidates who did less well tended to over-complicate their explanations with references to binary code and errors in calculations.

Candidates who did not do well wrote in general terms and did not relate their comments to either non-zero values or not equal values.

Question 7 (a)

7 The value of a function, $y = \mathrm{f}(x)$, and its gradient function, $\frac{\mathrm{d}y}{\mathrm{d}x}$, when $x = 2$, is given in **Table 7.1**.

Table 7.1

x	$\mathrm{f}(x)$	$\frac{\mathrm{d}y}{\mathrm{d}x}$
2	6	-2.8

(a) Determine the approximate value of the error when $\mathrm{f}(2)$ is used to estimate $\mathrm{f}(2.03)$. **[2]**

Many candidates did well in this question with the straightforward calculation error ≈ 0.03 × (–2.8).

Candidates who did less well had often over-complicated the situation by first evaluating the approximate value of f(2.03), before making a slip and losing the accuracy mark.

Question 7 (b) (i)

The Newton-Raphson method is used to find a sequence of approximations to a root, α, of the equation $f(x) = 0$. The spreadsheet output showing the iterates, together with some further analysis, is shown in **Table 7.2**.

Table 7.2

	A	B	C	D
1	r	x_r	difference	ratio
2	0	12		
3	1	−13.1165572	−25.1165572	
4	2	1.76283279	14.87939004	−0.5924136
5	3	2.18052157	0.41768878	0.02807163
6	4	2.182419024	0.001897454	0.00454275
7	5	2.182419066	4.13985E-08	2.1818E-05

(b) **(i)** Explain what the values in column D tell you about the order of convergence of this sequence of approximations. **[2]**

Candidates who did well in this question noted that the ratio of differences is decreasing rapidly, and so the convergence is faster than first order.

Candidates who did less well usually made the correct observation regarding the ratio, but concluded that the convergence was second order. Some candidates noted that the ratio is not converging, so the convergence is not first order to earn the special case mark.

See also the Assessment for learning note for Question 5 part (c).

Exemplar 1

7(b)(i)

~~would be different relative error due to the operations being different $(\pi+2)^2 = \pi^2+2\pi+4 \neq \pi^2+2$~~ ~~[illegible]~~

ratios rapidly ~~[illegible]~~ decreasing so much faster than 1st order, likely ~~[illegible] second order~~ 2nd order.

This candidate made two correct statements and earned both marks. The following statement that the convergence is 'likely second order' does not spoil the response, as candidates are expected to know that the order of convergence of the Newton-Raphson method is generally second order. Had the candidate simply stated, for example 'the ratio of differences is decreasing, so the convergence is likely second order', the response would have earned B1B0.

Question 7 (b) (ii)

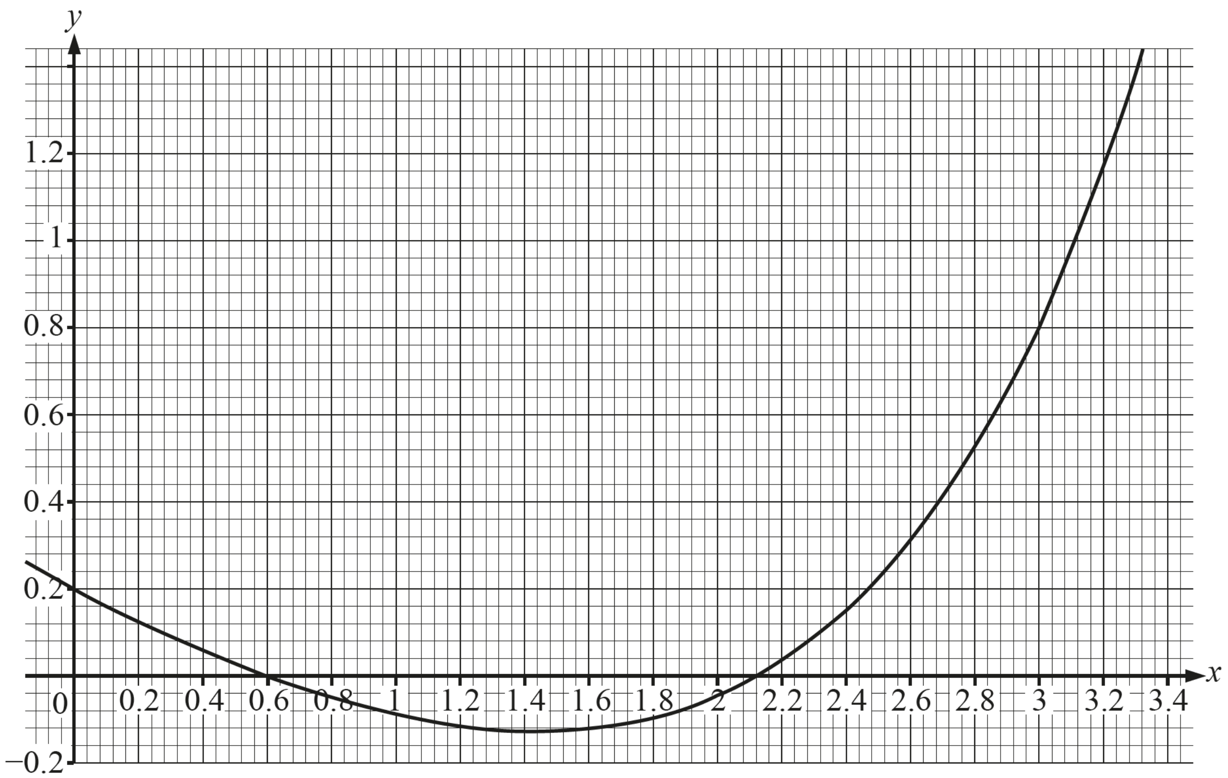
(ii) **Without** doing any further calculation, state the value of α as accurately as you can, justifying the precision quoted. **[2]**

Most candidates earned both marks via one of the routes identified in the mark scheme, with most opting for the more cautious approach of 6 decimal place precision.

Candidates who did less well did not explain the precision quoted adequately, or did not explain it at all.

Question 8 (a)

8 The graph of $y = 0.2\cosh x - 0.4x$ for values of x from 0 to 3.32 is shown on the graph below.



The equation $0.2\cosh x - 0.4x = 0$ has two roots, α and β where $\alpha < \beta$, in the interval $0 < x < 3$. The secant method with $x_0 = 1$ and $x_1 = 2$ is to be used to find β.

(a) On the copy of the graph in the Printed Answer Booklet, show how the secant method works with these two values of x to obtain an improved approximation to β. **[1]**

Candidates who did well drew the appropriate chord, and showed the intersection with the *x*-axis. Sometimes they needlessly showed subsequent iterations of the method.

Candidates who did not do well either drew a tangent, or simply drew a chord between the two correct points, with no extension to the *x*-intercept.

Question 8 (b)

The spreadsheet output in the table below shows the result of applying the secant method with $x_0 = 1$ and $x_1 = 2$.

	I	J	K	L	M
2	r	x_r	$f(x_r)$	x_{r+1}	$f(x_{r+1})$
3	0	1	−0.0914	2	−0.0476
4	1	2	−0.0476	3.08529	0.95784
5	2	3.08529	0.95784	2.05134	−0.0298
6	3	2.05134	−0.0298	2.08259	−0.0181
7	4	2.08259	−0.0181	2.13042	0.00155
8	5	2.13042	0.00155	2.12664	−7E-05

(b) Write down a suitable cell formula for cell J4. **[1]**

Some candidates incorrectly wrote J4 = L3 in part (b), but many successfully answered parts (b) and (c) for all 3 marks.

Question 8 (c)

(c) Write down a suitable cell formula for cell L4. **[2]**

Some candidates gained full credit for part (c) even if they did not get the mark on part (b). Some candidates spoiled their answer to part (c) by omitting brackets or using × instead of * to denote multiplication in the cell formula.

Question 8 (d)

(d) Write down the most accurate approximation to β which is displayed in the table. **[1]**

Many candidates gave the correct answer for part (d) but struggled to use this correctly for parts (e) and (f). Those candidates who did not do well on these final parts of question 8 often compared the two most accurate approximations in the table and gave their answer for (d) as 2.13.

Question 8 (e)

(e) Determine whether your answer to part **(d)** is correct to **5** decimal places. You should **not** calculate any more iterates. **[2]**

Not all candidates realised they needed to carry out a change of sign test in part (e). Some candidates gave the answer to part (d) as 2.13042 and could earned FT marks in part (e).

Those candidates that compared the two most accurate approximations in the table and gave the answer to part (d) as 2.13 could not then gain marks in part (e), although a few of these candidates were given the mark for part (f) if they used the information that $a > 1$.

Question 8 (f)

(f) It is decided to use the secant method with starting values $x_0 = 1$ and $x_1 = a$, where $a > 1$, to find α. State a suitable value for a. **[1]**

Candidates who did well in parts (d) and (e) sometimes, but not always, gave an acceptable value for a in part (f), although many ignored the information that $a > 1$.

Question 9 (a)

9 The trapezium rule is used to calculate 3 approximations to $\int_0^1 \sqrt[3]{\sinh(x)}\,dx$ with 1, 2 and 4 strips respectively. The results are shown in **Table 9.1**.

Table 9.1

n	T_n
1	0.52764369
2	0.66617652
4	0.72534275

(a) **Use these results** to determine **two** approximations to $\int_0^1 \sqrt[3]{\sinh(x)}\,dx$ using Simpson's rule. **[2]**

Candidates who did well in this question gave full details of their working and presented their answers to a suitable degree of precision.

Candidates who did less well wrote down the values from their calculator and usually earned the special case.

Question 9 (b)

(b) Use your answers to part **(a)** to state the value of $\int_0^1 \sqrt[3]{\sinh(x)}\,dx$ as accurately as you can, justifying the precision quoted. **[1]**

Candidates who did well in this question quoted a suitable degree of precision and justified this with reference to the two Simpson's rule estimates calculated in part (a).

Candidates who did not do well did not explicitly use the answers from part (a) to justify one of the acceptable answers.

Question 9 (c)

Table 9.2 shows some further approximations found using the trapezium rule, together with some analysis of these approximations.

Table 9.2

n	T_n	difference	ratio
1	0.5276437		
2	0.6661765	0.138533	
4	0.7253427	0.059166	0.42709
8	0.7498821	0.024539	0.41475
16	0.7598858	0.010004	0.40766
32	0.7639221	0.004036	0.40348
64	0.7655404	0.001618	0.40095

(c) Explain what can be deduced about the order of the method in this case. **[2]**

Candidates who did well in this question commented on the convergence of the ratio of differences and compared an appropriate value (usually 0.4) with 0.25 and 0.5 to identify the order of the method.

Candidates who did less well often did not comment on the convergence of the ratio, but correctly deduced the order of the method. A few candidates did comment that the ratio appears to be converging to 0.4, and deduced first order convergence.

See also the Assessment for learning note for Question 5 part (c).

Exemplar 2

9(c)	between 0·25 and 0·5 so
	between 1st and 2nd order method.

This candidate understood what was being asked here, but gave insufficient detail to earn any marks. There was no mention of the convergence of the ratio of differences, and '0.4' was not explicitly compared to 0.25 and 0.5.

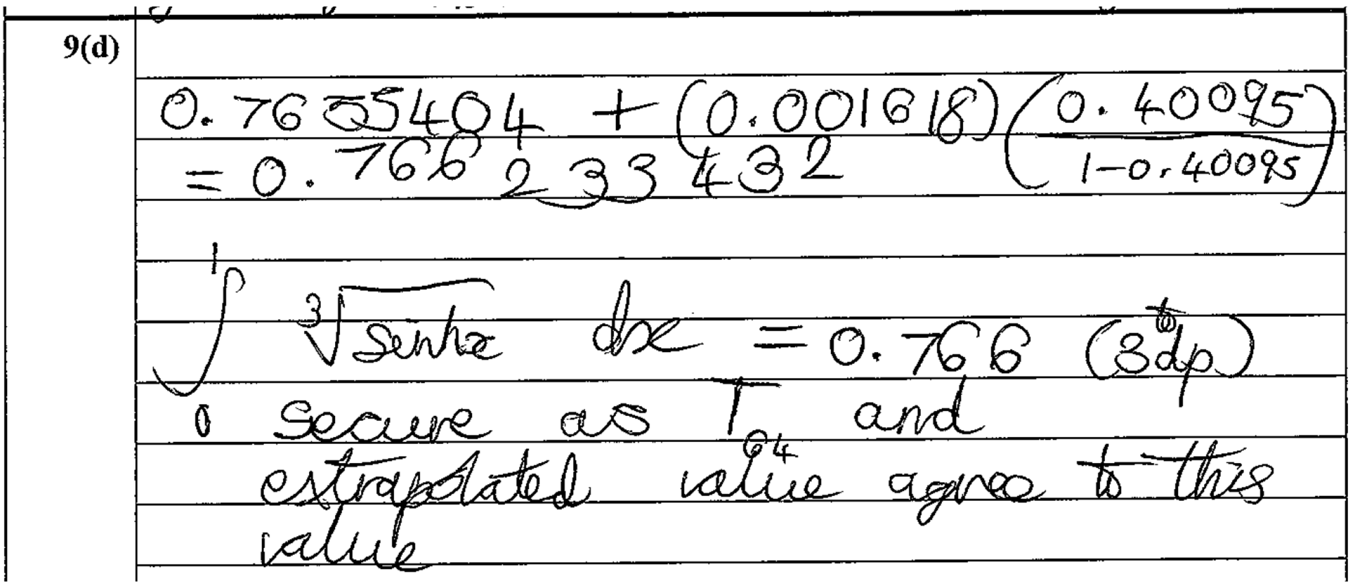
Question 9 (d)

(d) Use extrapolation to obtain the value of $\int_0^1 \sqrt[3]{\sinh(x)}\,\mathrm{d}x$ as accurately as you can, justifying the precision quoted. **[4]**

Many candidates did well in this question. They knew how to extrapolate to infinity and identified the correct values to use from Table 9.2. They usually were able to interpret their final answer correctly via one of the routes outlined in the mark scheme.

Candidates who did less well made either substitution errors in the formula or slips in arithmetic or in transcription from their calculator. A small number of candidates extrapolated to infinity using r = 0.25, and some only partially extrapolated with correct values.

Exemplar 3



This candidate adopted the correct method and substituted the correct numbers to earn M1A1. Unfortunately there is a slip in the result – probably a transcription error – as there is a 6 missing in the decimal. The final accuracy mark is then unavailable as the precision quoted is bound to be wrong.

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