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Examiners' report

FURTHER MATHEMATICS B (MEI)

H645

For first teaching in 2017

Y431/01 Summer 2023 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y431/01 series overview

This is the minor mechanics component of the A Level specification H645: Further Mathematics B (MEI). It is a 1 hour 15 minute paper worth 60 marks.

The report focuses on areas of improvement based on responses given by this year's cohort. However, it should not be construed that these were areas of development for all. Quite the contrary, and most responses showed a high level of mathematical understanding and fluency with the skills developed during the course.

In some questions detailed working was explicitly required, as dictated by the 'show that' and 'determine' command words. Those who provided consistently complete solutions for all questions were ultimately most successful in understanding the problems and modelling them correctly. This was particularly noticeable for those candidates who produced and annotated their own diagrams and organised information in tables.

Candidates at all levels exhibited excellent algebraic skills but provided comparatively little written explanation to justify choices or show their understanding of physical situations. Not only would this secure extra marks it would enhance candidates' own understanding of problems and focus their reasoning.

Candidates used their calculators appropriately and the only significant opportunity, Question 6 (b), required detailed reasoning to be shown and this requirement was only ignored by a very small minority. However, there was evidence of unchecked solutions which could have been performed by a calculator and occasionally large numbers of written intermediate numerical results indicating that calculators had not been used in the most effective manner.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• produced clear diagrams to support their understanding of physical situations • used tables to organise the components of their calculations • provided written explanations to interpret the results they calculated • understood what was expected by each of the given command words in the questions • produced structured solutions • used calculators to support their solutions appropriately.	• used fewer diagrams and tables • did not produce sufficient working to support their solutions particularly where the 'show that' or 'determine' command words explicitly indicate written justification must be seen • made errors transcribing information from the questions • did not consider the relationship between parts of questions.

Question 1 (a)

1 (a) State the dimensions of the following quantities.

- Force
- Velocity
- Density **[3]**

Virtually all students were able to express the dimensions of the 3 quantities correctly.

Question 1 (b)

A student investigating the drag force F experienced by an object moving through air conjectures the formula

$$F = ku^2(\rho m^2)^{\frac{1}{3}},$$

where

- k is a dimensionless constant
- u is the air velocity relative to the moving object
- ρ is the air density
- m is the mass of the object.

(b) Show that the student's formula is dimensionally consistent. **[2]**

This was well answered by the majority with only a very small number of candidates not providing enough working to meet the requirements of the 'show that' command.

Question 1 (c)

The student carries out experiments in an airflow tunnel. When the air density is doubled, the drag force is found to double as well, with all other conditions remaining the same.

(c) Show that the student's formula is inconsistent with the experimental observation. **[1]**

A small number of candidates did not attempt this question but those who did were able to demonstrate the inconsistency by obtaining the factor of $\sqrt[3]{2}$.

Question 1 (d)

The student's teacher suggests revising the formula as

$$F = k\rho^{\alpha}u^{\beta}A^{\gamma}$$

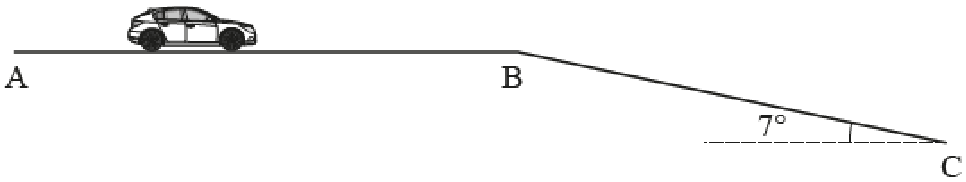
where m has been replaced by A, the cross-sectional area of the object. The constant k is still dimensionless.

(d) Use dimensional analysis to determine the values of α, β and γ. **[3]**

Most candidates were able to produce a set of equations and calculate all three values correctly. A noticeable number used M^2 as the dimension for A, despite not having made this error earlier in the question.

Question 2 (a)

2 A car of mass 1400 kg, travels along a straight horizontal road AB, after which it descends a hill BC inclined at a constant angle of 7° to the horizontal (see diagram). A, B and C all lie in the same vertical plane. Throughout the entire journey, the total resistance to the car's motion is constant.



Between A and B, the car moves at a constant speed of $12\,\text{m s}^{-1}$, and the power developed by the car is a constant P W. When the car reaches B, the engine is switched off and the car travels down a line of greatest slope from B to C with an acceleration of $0.8\,\text{m s}^{-2}$. The resistance to motion is unchanged.

(a) Determine the value of P. **[4]**

This was well answered; a small number omitted to include the weight component in their application of Newton's second law.

Question 2 (b)

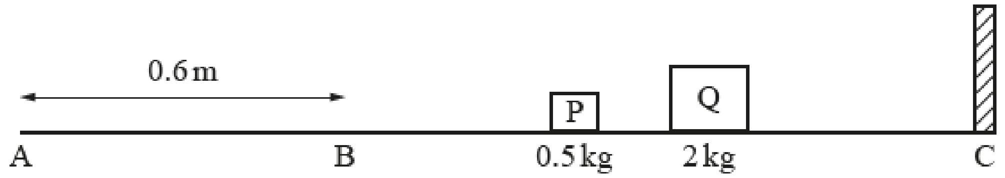
When the car reaches C it turns round and travels back up the hill towards B at a constant speed of $v\,\text{m s}^{-1}$. The power developed by the car between C and B is a constant 16 kW. The resistance to motion is unchanged.

(b) Determine the value of v. **[2]**

This was generally well answered apart from a larger proportion of candidates not including the weight component.

Question 3 (a)

3 The diagram shows two blocks P and Q of masses 0.5 kg and 2 kg respectively, on a horizontal surface. The points A, B and C lie on the surface in a straight line. There is a wall at C. The surface between B and C is smooth, and the surface between A and B is rough, such that the coefficient of friction between P and AB is $\frac{2}{3}$.



P is projected with a speed of $6\,\text{m s}^{-1}$ directly towards Q, which is at rest. As a result of the collision between P and Q, P changes direction and subsequently comes to rest at A. You may assume that P only collides with Q once.

(a) Determine the coefficient of restitution between P and Q. **[6]**

Marks were often lost as candidates did not consider the rough surface. Not signing the directions of motion correctly was another source of error. It was noticeable that responses where candidates attempted to calculate the coefficient of restitution by eliminating the velocities from the conservation of momentum and constant acceleration equations and not calculating their values were less successful.

Question 3 (b)

(b) Calculate the impulse exerted on P by Q during their collision. **[2]**

Very few candidates acknowledged that impulse is a vector quantity and did not clearly state the direction of the impulse. The change of direction P was not always considered in the calculation.

Question 3 (c)

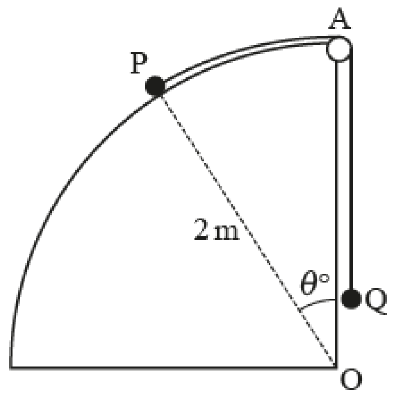
After colliding with P, Q strikes the wall, which is perpendicular to the direction of the motion of Q, and comes to rest exactly halfway between A and B. The collision between Q and the wall is perfectly elastic.

(c) Determine the coefficient of friction between Q and AB. **[3]**

This was generally answered well, and most candidates were aware that the speed of Q would be unchanged after the elastic collision.

Question 4 (a)

4 The diagram shows two particles P and Q, of masses 10 kg and 5 kg respectively, which are attached to the ends of a light inextensible string. The string is taut and passes over a small smooth pulley. The pulley is fixed at the highest point A on a smooth curved surface, the vertical cross-section of which is a quadrant of a circle with centre O and radius 2 m. Particle Q hangs vertically below the pulley and P is in contact with the surface, where the angle AOP is equal to $\theta°$. The pulley, P and Q all lie in the same vertical plane.



Throughout this question you may assume that there are no resistances to the motion of either P or Q and the force acting on P due to the tension in the string is tangential to the curved surface at P.

(a) Given that P is in equilibrium at the point where $\theta = \alpha$, determine the value of α. **[3]**

Most candidates understood the implication of equilibrium and were able to resolve correctly. Any errors were usually due to selecting the incorrect weight component of P.

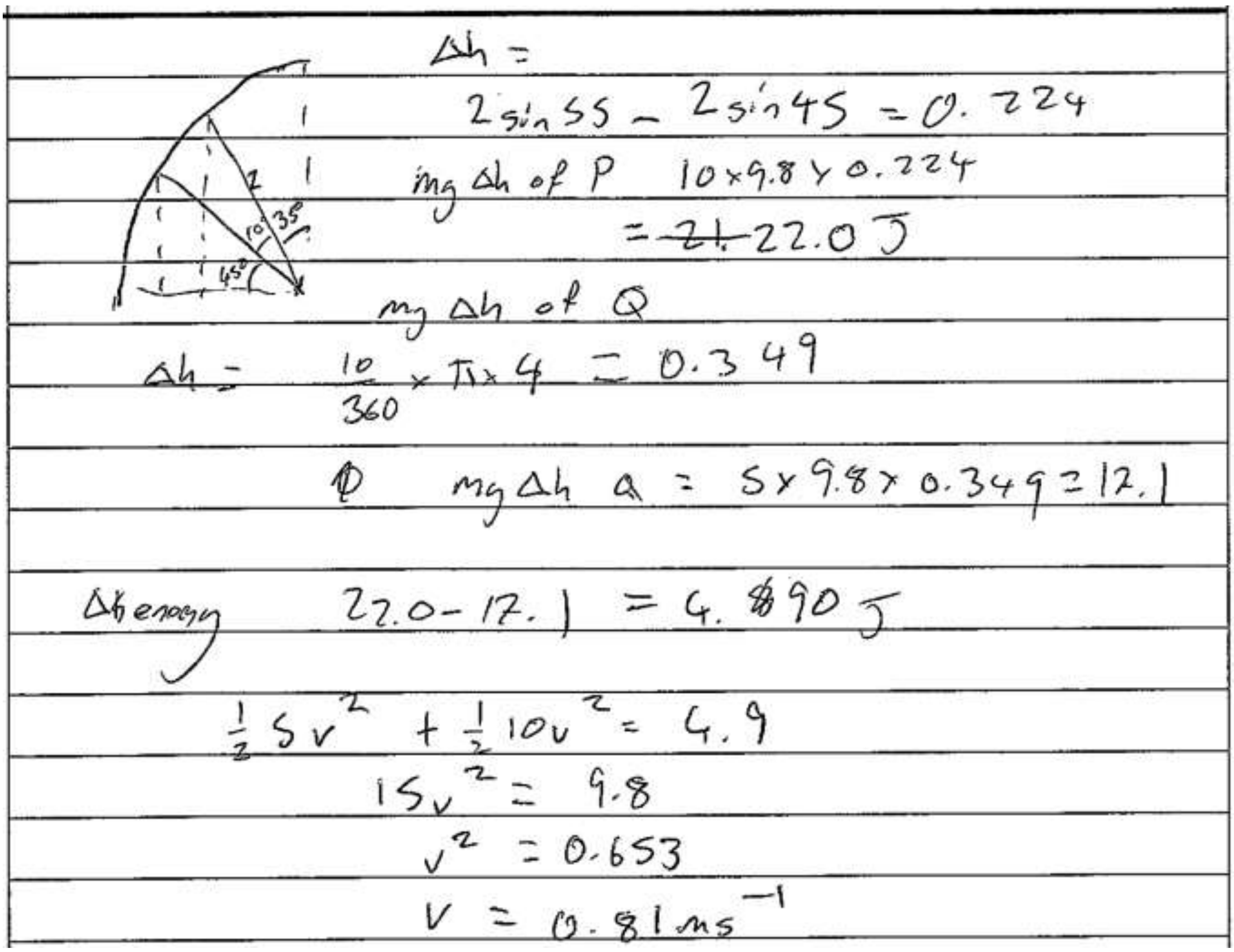
Question 4 (b)

Particle P is now released from rest at the point on the surface where $\theta = 35$, and starts to move downwards on the surface. In the subsequent motion it is given that P does not leave the surface.

(b) By considering energy, determine the speed of P at the instant when $\theta = 45$. **[4]**

Commonly only the change of potential and kinetic energy for P was considered throughout this question. A significant group of solutions only included the change in potential energy for Q. This left only a minority of candidates producing fully correct solutions such as the following exemplar.

Exemplar 1



This is a clear complete solution considering the changes in potential and kinetic energy for both P and Q.

Question 4 (c)

(c) State one modelling assumption you have made in determining the answer to part **(b)**. **[1]**

A majority of candidates gave an assumption stated in the text or a generic assumption; only a small number noted that it is required that Q does not reach the pulley before $\theta = 45°$.

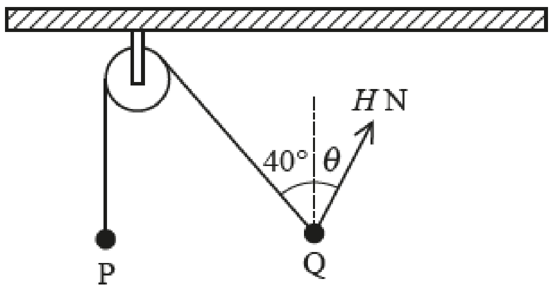
Assessment for learning

Candidates should consider the specific assumptions made in the question to give an individual rather than a generic response when commenting on modelling assumptions and consider the geometrical or physical assumptions made in the solution of the question.

Question 5 (a)

5 **Fig. 5.1** shows a particle P, of mass 5 kg, and a particle Q, of mass 11 kg, which are attached to the ends of a light, inextensible string. The string is taut and passes over a small smooth pulley fixed to the ceiling.

Fig. 5.1



When a force of magnitude H N, acting at an angle θ to the upward vertical, is applied to Q the particles hang in equilibrium, with the part of the string connecting the pulley to Q making an angle of 40° with the upward vertical. It is given that the force acts in the same vertical plane in which the string lies.

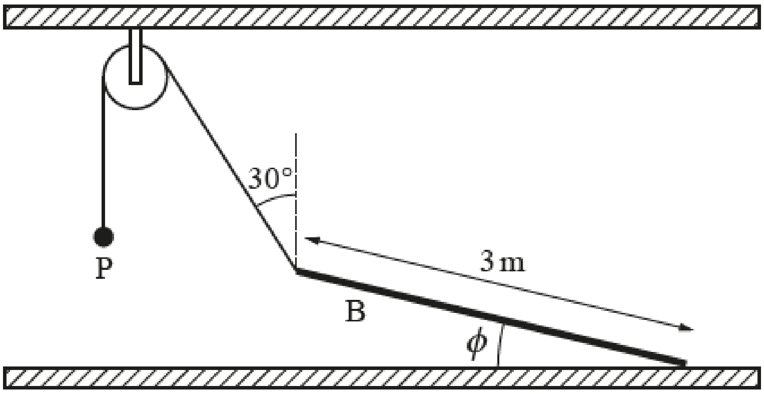
(a) Determine the values of H and θ. **[4]**

This question was answered very well by many candidates. The only errors arose from muddling the vertical and horizontal components of forces.

Question 5 (b)

Particle Q is now removed. The string is instead attached to one end of a uniform beam B of length 3 m and mass 7 kg. The other end of B is in contact with a rough horizontal floor. The situation is shown in **Fig. 5.2**.

Fig. 5.2



With B in equilibrium, at an angle ϕ to the horizontal, the part of the string connecting the pulley to B makes an angle of 30° with the upward vertical.

It is given that the string and B lie in the same vertical plane.

(b) Determine the smallest possible value for the coefficient of friction between B and the floor. **[3]**

This question was well answered with most errors caused when the vertical component of the tension in the string was not included in the resolution of the reaction.

Question 5 (c)

(c) Determine the value of ϕ. **[5]**

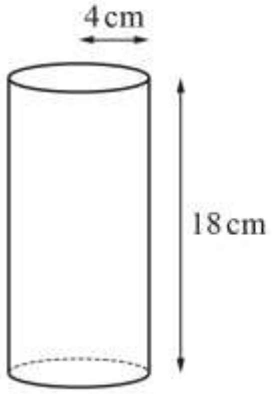
This question was well answered with the most common error being to omit the moment due to one of the components of the tension in the string.

Question 6 (a)

6 In this question you may use the fact that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Fig. 6.1 shows a container in the shape of an open-topped cylinder. The cylinder has height 18 cm and radius 4 cm. The curved surface and the base can be modelled as uniform laminae with the same mass per unit area. The container rests on a horizontal surface.

Fig. 6.1



(a) Show that the centre of mass of the container lies 8.1 cm above its base. **[3]**

The question was answered completely correctly by the majority of candidates with only a small number attempting to use volume rather than surface area in their calculations.

Question 6 (b)

The mass of the container is 400 grams. Water is poured into the container to reach a height of h cm above the base. The centre of mass of the combined container and water lies y cm above the base. Water has a density of 1 gram per cm^3.

(b) **In this question you must show detailed reasoning.**

By formulating an expression for y in terms of h, determine the value of h for which y is lowest. **[6]**

Most candidates were able to produce a correct expression. Those who did not continued to use surface area and not the given mass. Some attempted to use this expression directly to find a value for *h*, and most went on to differentiate and solve the subsequent quadratic equation correctly. It is worth noting that a handful of candidates did not follow the "**In this question you must show detailed reasoning**" instruction and gave an unsupported answer possibly using a calculator. The most successful responses were clear in combining all the components as in the following exemplar.

Exemplar 2

~~Container~~	mass	x	y	$\pi r^2 h$
Containe	400	4	8.1	
Water	$16\pi h$	4	$\frac{h}{2}$	
whole	$400 + 16\pi h$	$\bar{x}$	y	

$(400 \times 8.1) + 8\pi h^2 = (400 + 16\pi h)y$

$8\pi h^2 + 3240 = (400 + 16\pi h)y$

$y = \dfrac{8\pi h^2 + 3240}{16\pi h + 400}$ 5.907361956

$y = \dfrac{\pi h^2 + 405}{2\pi h + 50}$

$u = \pi h^2 + 405 \quad v = 2\pi h + 50$

$u' = 2\pi h \quad v' = 2\pi$

$\dfrac{dy}{dx} = \dfrac{2\pi h(2\pi h + 50) - 2\pi(\pi h^2 + 405)}{(2\pi h + 50)^2} = 0$

$4\pi^2 h^2 + 100\pi h - 2\pi^2 h^2 - 810\pi = 0$

$2\pi^2 h + 100\pi h - 810\pi = 0$

$\pi h + 50h - 405 = 0$

$h = \dfrac{-50 \pm \sqrt{2500 + 1620\pi}}{2\pi}$

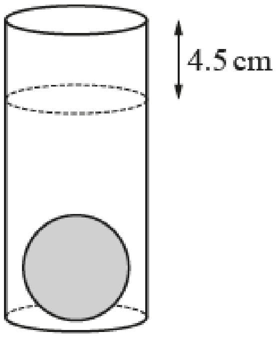
$h = 5.91$m when y is lowest

This is a clearly laid out response, combining all components using a table to support this solution.

Question 6 (c)

More water is now poured into the container. A sphere of radius 3 cm is placed into the container, where it sinks to the bottom. The surface of the water is now 4.5 cm from the top of the container, as shown in **Fig. 6.2**.

Fig. 6.2



(c) Show that the centre of mass of the water in the container lies 7.5 cm above the base of the container. **[2]**

Those who attempted this question answered it very well. Most by removing the sphere from the cylinder but a number produced elegant solutions splitting the water up to the height of the sphere and considered the cylinder above that separately.

Question 6 (d)

The sphere has a density of 4 grams per cm^3.

The centre of mass of the combined container, water and sphere lies z cm above the base.

(d) Determine the value of z. **[3]**

Errors arose when candidates reverted back to the total surface area of the container or did not use the earlier result that the centre of mass was at a height of 8.1cm. The most successful solutions used a table with masses and centres of mass heights clearly listed. It was also apparent that those successful on the question had considered the whole question and had a clear overview of the problem.

Assessment for learning

While questions are broken into sub-parts, candidates need to make sure they have an overview of the whole question and especially notice when the answers to earlier parts can be referenced.

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