

**A LEVEL**

**Examiners' report**

**FURTHER  
MATHEMATICS B  
(MEI)**

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**H645**

For first teaching in 2017

**Y420/01 Summer 2023 series**

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## Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

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## Paper Y420/01 series overview

The Y420 paper is relatively long and is designed with an increasing gradient of demand to present an appropriately challenging test for all candidates. It was encouraging to see many responded with sustained, high quality work, providing detailed, clearly structured solutions.

More than half the candidates achieved over 100 marks and a high proportion of candidates were able to gain at least some credit on questions across the paper. The overall standard seemed to be equivalent to that seen in previous series.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
<ul style="list-style-type: none"> <li>• set out their working clearly and logically</li> <li>• showed all their working and each method they used</li> <li>• understood the requirements of the defined question command words</li> <li>• gave complete explanations for results when asked to 'show that' or 'prove'</li> <li>• attempted the majority of questions and gained partial credit even if a complete solution was not achieved.</li> </ul>	<ul style="list-style-type: none"> <li>• did not show their methods in questions asking for detailed reasoning and those which used the command words 'determine' and 'show that'</li> <li>• had difficulty applying formulae correctly in multi-step problems or were unclear on how to use their results</li> <li>• made arithmetic and sign errors and mistakes in algebraic manipulation</li> <li>• did not simplify their answers.</li> </ul>

### OCR support



[A guide to the command words used in OCR A Level Maths exams](#) can be found on Teach Cambridge.

## Section A overview

This section proved to be accessible to most candidates and many scored well here.

Candidates performed most successfully on Questions 1 and 3. Questions 4 (b) and 6 (d) were the least successfully answered.

### Question 1 (a) (i)

1 (a) The complex number  $a + ib$  is denoted by  $z$ .

(i) Write down  $z^*$ . [1]

This question was answered correctly by almost all candidates.

### Question 1 (a) (ii)

(ii) Find  $\operatorname{Re}(iz)$ . [2]

The vast majority of candidates answered this correctly although some found  $\operatorname{Re}(iz^*)$ .

### Question 1 (b) (i)

(b) The complex number  $w$  is given by  $w = \frac{5 + i\sqrt{3}}{2 - i\sqrt{3}}$ .

(i) In this question you must show detailed reasoning.

Express  $w$  in the form  $x + iy$ . [2]

This question was very well answered and most candidates followed the instruction to show detailed reasoning. Some who did not get full marks did not explicitly show their multiplication of both numerator and denominator by  $2 + i\sqrt{3}$ . Others wrote both in modulus-argument form and worked with those values. This method was not credited as this was invariably not followed up with a method to find the argument of the final answer.

## Question 1 (b) (ii)

(ii) Convert  $w$  to modulus-argument form.

[2]

This question was also very well answered. Some candidates found the modulus and argument correctly but did not gain full credit because they did not convert their answer into the correct form.

## Misconception



The form  $re^{i\theta}$  is distinct from the modulus-argument form  $r(\cos \theta + i \sin \theta)$ .

## Question 2

2 In this question you must show detailed reasoning.

Find the angle between the vector  $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$  and the plane  $-x + 3y + 2z = 8$ .

[5]

The majority of candidates were given full marks on this question. Almost all candidates used the scalar product to find the angle between the vector and the normal to the plane. A minority did not go on to use their result to find the angle between the vector and the plane itself.

## Question 3 (a)

3 (a) Using partial fractions and the method of differences, show that

$$\frac{1}{1 \times 3} + \frac{1}{2 \times 4} + \frac{1}{3 \times 5} + \dots + \frac{1}{n(n+2)} = \frac{3}{4} - \frac{an+b}{2(n+1)(n+2)},$$

where  $a$  and  $b$  are integers to be determined.

[5]

Most candidates were familiar with using partial fractions and the method of differences together. Having obtained the sum of four terms, a minority of candidates did not combine them correctly into the required form, typically making errors in algebraic manipulation.

## Question 3 (b)

(b) Deduce the sum to infinity of the series.

$$\frac{1}{1 \times 3} + \frac{1}{2 \times 4} + \frac{1}{3 \times 5} + \dots \quad [1]$$

Most candidates were able to deduce the correct answer. Some candidates assumed the phrase 'sum to infinity' implied they needed to find the sum of a geometric series.

## Question 4 (a) (i)

4 (a) (i) Given that  $f(x) = \sqrt{1+2x}$ , find  $f'(x)$  and  $f''(x)$ . [2]

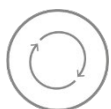
Almost all candidates found the derivatives correctly, although some errors with signs and indices were seen.

## Question 4 (a) (ii)

(ii) Hence, find the first three terms of the Maclaurin series for  $\sqrt{1+2x}$ . [2]

This was very well answered. Of those who had the correct derivatives in Question 4 (a) (i) but were not given both marks here, most did not show the substitution of their values for  $f'(0)$  and  $f''(0)$  into the Maclaurin series formula and so could not receive any credit if they did not get the correct answer. Some ignored the instruction 'hence' and used a binomial expansion.

### Assessment for learning



When a question uses the word 'hence' it is an indication that the next step should be based on what has gone before. Candidates should start from the given statement or result and may receive no credit if they use another method even if it results in the correct answer.

## Question 4 (b)

(b) Hence, using a suitable value for  $x$ , show that  $\sqrt{5} \approx \frac{143}{64}$ . [2]

This question proved to be one of the least successfully answered on the whole paper. A minority of candidates recognised the need to let  $x = \frac{1}{8}$  (to give  $\sqrt{1+2x} = \sqrt{\frac{5}{4}}$ ), with the majority using  $x = 2$  instead.



## Question 5 (a)

5 (a) **In this question you must show detailed reasoning.**

Determine the sixth roots of  $-64$ , expressed in  $re^{i\theta}$  form. [4]

Most candidates were able to find all the required roots. A significant minority of responses did not show a method for finding the sixth roots, by first finding the modulus and argument of  $-64$ , and so could receive at most 1 mark.

### Assessment for learning



**'In this question you must show detailed reasoning'** is used to indicate that a solution should be given which leads to a conclusion showing a detailed and complete analytical method. The solution should contain sufficient detail to allow the line of argument to be followed. Results obtained without any evidence of a clear mathematical method will not gain credit.

## Question 5 (b)

(b) Represent the roots on an Argand diagram. [3]

Most candidates were able to gain at least 2 marks for plotting their roots. More often than not the final mark could not be given because neither of the roots on the imaginary axis were labelled.

## Question 6 (a)

6 The matrices  $\mathbf{M}$  and  $\mathbf{N}$  are  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  and  $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$  respectively.

(a) **In this question you must show detailed reasoning.**

Determine whether  $\mathbf{M}$  and  $\mathbf{N}$  commute under matrix multiplication. [3]

The vast majority of candidates were able to find the matrices  $\mathbf{MN}$  and  $\mathbf{NM}$  although a small minority concluded incorrectly or did not conclude at all.

### Question 6 (b) (i)

(b) Specify the transformation of the plane associated with each of the following matrices.

(i)  $M$

[1]

This question was very well answered with a significant majority of candidates getting the mark. It was noted by examiners that candidates regularly used the incorrect preposition for reflection – e.g. reflection 'along' or 'on' the line – but credit was not withheld in this instance.

### Question 6 (b) (ii)

(ii)  $N$

[2]

Only a slight majority of candidates scored both marks on this question. The most common mistakes were to describe the transformation as a shear or enlargement or to confuse the descriptive requirements of a stretch with those of a shear. Examiners noted that 'stretch' was often spelt incorrectly.

#### Misconception



Enlargements are distinct from (one-way) stretches and the transformations are not generally equivalent.

### Question 6 (c)

(c) State the significance of the result in part (a) for the transformations associated with  $M$  and  $N$ . [1]

Candidates who identified that the question referred to transformations, rather than matrices, performed well on this question. Responses which referred exclusively to matrices received no credit.

Question 6 (d)

- (d) Use an algebraic method to show that all lines parallel to the  $x$ -axis are invariant lines of the transformation associated with  $N$ . [2]

A minority of candidates received both marks on this question. Most candidates considered the mapping of a general vector under the transformation but it was often assumed that this took the form  $\begin{pmatrix} 0 \\ y \end{pmatrix}$  or  $\begin{pmatrix} x \\ mx \end{pmatrix}$ . Another common error was to argue the converse of the given statement, that all invariant lines were parallel to the  $x$ -axis. In this instance the final mark was withheld as this conclusion is incorrect. Many candidates demonstrated solid algebra skills in this question but often did not support their findings with an explanation of what they had found.

Exemplar 1

6(d)	$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix} \quad y = mx + c \quad y' = mx' + c$
	$2x = x' \quad mx + c = mx' + c$
	$y = y' \quad \therefore mx + c = 2mx + c$
	$mx = 2mx$
	<del>either <math>x=0</math> or <math>x \neq 0</math></del> $m=0$
	$\therefore y = c \quad \nabla c$
	<p style="text-align: right;">for all <math>c</math></p>

Although this response contains all the required algebra to support a correct conclusion, the explanation offered is not sufficient for the accuracy mark. This scored 1 mark.

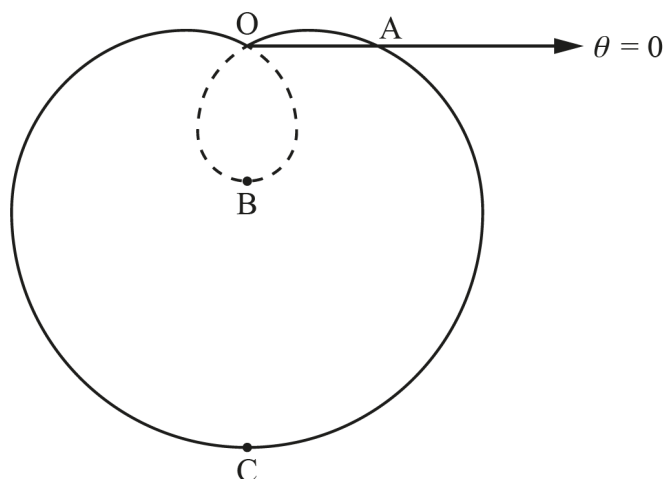
## Section B overview

Section B contained longer, less structured and less straightforward questions and, as a result, candidate performance was more variable. Many candidates were able to demonstrate a good understanding of the majority of the topics tested, however, and candidates found most question parts accessible, including in the final question.

The questions that were answered most successfully were Questions 10 and 15; the least successfully answered were Questions 11, 12 and the later parts of 17.

### Question 7 (a)

- 7 The diagram below shows the curve with polar equation  $r = a(1 - 2 \sin \theta)$  for  $0 \leq \theta \leq 2\pi$ , where  $a$  is a positive constant.



The curve crosses the initial line at A, and the points B and C are the lowest points on the two loops.

- (a) Find the values of  $r$  and  $\theta$  at the points A, B and C. [3]

This question was well answered. Most candidates found the correct values at the point A but some candidates confused points B and C or gave values of  $\theta$  outside the given range.

### Question 7 (b)

- (b) Find the set of values of  $\theta$  for the points on the inner loop (shown in the diagram with a broken line). [3]

This question was well answered and most candidates were given full marks.

## Question 8

8 Prove by mathematical induction that  $8^n - 3^n$  is divisible by 5 for all positive integers  $n$ . [5]

The structure of proof by induction appears to be well understood by the majority of candidates and most were able to establish divisibility by 5 when  $n = 1$ , to make the correct assumption about the case where  $n = k$  and to use this assumption in their proof. Many candidates did not go on to provide sufficient working to show that their result was divisible by 5 in the case where  $n = k + 1$ . Often candidates were not explicit in explaining that their sum was divisible by 5, instead explaining only that each of its summands were divisible by 5. Given the rigorous nature of proof by induction this could not be given credit. More successful candidates wrote their initial assumption in the form  $8^k - 3^k = 5m$ . A number of candidates did not have a clear conclusion; the most common error was to omit the conditional 'if'.

## Exemplar 2

8

$$8^n - 3^n$$

$$\text{Let } n=1, \quad 8^1 - 3^1 = 5 = 5(1)$$

so true for  $n=1$

Assume true for  $n=k$ ,

so  $8^k - 3^k$  is divisible by 5

Prove true that

$8^{k+1} - 3^{k+1}$  is divisible by 5

$$= 8(8^k) - 3^{k+1}$$

$$\left| \begin{array}{l} 8^k = 5p + 3^k \\ \text{where } p \text{ is any} \\ \text{positive integer} \end{array} \right.$$

$$8(\cancel{5}5p + 3^k)$$

$$5(8p) + 8(3^k) - 3^k(3)$$

$$5(8p) + 3^k(8-3)$$

$$= 5(8p) + 5(3^k)$$

so true for  $n=k+1$

as  $8p$  and  $3^k$  are both multiples of 5, where  $p$  is any positive integer

so shown true for  $n=1$ .

and when assumed true for  $n=k$ , it can be proven true that for  $n=k+1$  is also divisible by 5, so together it is true that  $8^n - 3^n$  is divisible by 5 for all positive integers  $n$ .

Here is an example of a proof which does not explicitly show that the expression is divisible by 5 when  $n = k + 1$ , meaning that the first accuracy mark could not be given. Since all previous marks were required for the final accuracy mark to be given for a correct conclusion, this response scored 3 marks.

## Question 9

- 9 In an electrical circuit, the alternating current  $I$  amps is given by  $I = a \sin nt$ , where  $t$  is the time in seconds and  $a$  and  $n$  are positive constants. The RMS value of the current, in amps, is defined to be the square root of the mean value of  $I^2$  over one complete period of  $\frac{2\pi}{n}$  seconds.

Show that the RMS value of the current is  $\frac{a}{\sqrt{2}}$  amps. [6]

Approximately half of candidates were given full marks for this question. Common errors were to lose the  $n$  from the argument of the sine function when using the double angle formula or when integrating and to make a sign error in the integration.

## Question 10 (a)

- 10 The equation  $x^3 - 4x^2 + 7x + c = 0$ , where  $c$  is a constant, has roots  $\alpha$ ,  $\beta$  and  $\alpha + \beta$ .

(a) Determine the roots of the equation. [6]

The majority of candidates were given full marks on this question. Marks were most commonly lost following a slip in the substitution step when candidates did not provide a method for solving their quadratic equation. As the question asks for candidates to 'determine' the roots, these candidates could not receive any further credit for their answers. Examiners noted that sign errors when setting up the initial equations in  $\alpha$  and  $\beta$  were less frequent than in previous series.

### Assessment for learning



When a question asks candidates to determine a result they should give justification for any results found, including working where appropriate.

## Question 10 (b)

(b) Find  $c$ .

[1]

This question was also well answered. Sign errors were more common in this part of the question and 6 was the most common incorrect answer.

## Question 11

11 Solve the differential equation  $\cosh x \frac{dy}{dx} - 2y \sinh x = \cosh x$ , given that  $y = 1$  when  $x = 0$ . [7]

Most candidates were able to achieve at least partial credit for their answers. Most were able to divide through by  $\cosh x$  and many correctly negotiated the negative sign in the integral for their integrating factor. Completing the required integration proved challenging for most candidates, however, with commonly seen approaches for doing so including writing  $\tanh x$  in exponential form and using integration by parts, typically with no subsequent progress made. The integration of  $\operatorname{sech}^2 x$  proved similarly challenging. Many sign errors and arithmetical errors, particularly with indices, were seen in this question. Candidates should check their working carefully to make sure they have the required level of accuracy in their answers.

## Question 12

12 Show that  $\sin^5 \theta = a \sin 5\theta + b \sin 3\theta + c \sin \theta$ , where  $a$ ,  $b$  and  $c$  are constants to be determined. [7]

Those candidates who knew to start by considering  $(z - z^{-1})^5$  generally performed well on this question. Many started off by considering  $(\cos \theta + i \sin \theta)^5$  instead, attempting to equate imaginary components. Typically candidates who used this method were unclear on the difference between  $\sin 5\theta$  and  $\sin^5 \theta$  and made little to no creditworthy progress. Of those who did make significant progress through the question a significant minority missed the  $i$  in their expressions throughout and so they did not present a complete method. Some candidates used  $c + i$  notation in their responses and in doing so often did not distinguish between  $\cos \theta$ ,  $\cos^5 \theta$  and  $\cos 5\theta$ .

### Assessment for learning



The command words 'Show that' indicate that a given result must be achieved from the starting information. As the result has been given, the explanation has to be sufficiently detailed to cover every step of the working.



## Question 13 (a) (i)

13 (a) On separate Argand diagrams, show the set of points representing each of the following inequalities.

(i)  $|z| \leq \sqrt{5}$  [3]

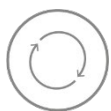
This question was very well answered. A small minority of candidates misidentified the radius of the circle as 5.

## Question 13 (a) (ii)

(ii)  $|z+2-4i| \geq |z-2-6i|$  [3]

This was less well answered than part (i). The vast majority of candidates identified the correct points but some drew circles around them. Others drew the perpendicular bisector incorrectly. It was common for candidates to shade in the wrong side of the perpendicular bisector.

## Assessment for learning



When asked to show the set of points representing a single inequality, candidates should shade the set of points which satisfy it. Shading the set of points which is not required is acceptable but they should clearly indicate that this is what they are doing.

## Question 13 (b)

(b) Show that there is a unique value of  $z$ , which should be determined, for which both  $|z| \leq \sqrt{5}$  and  $|z+2-4i| \geq |z-2-6i|$ . [8]

This question was answered well with over half of candidates given full marks. A majority of successful responses identified the equations of the perpendicular bisector and circle from correct diagrams in part (a) and solved these simultaneously. A relatively common error was to give the point (2,1) as the answer rather than the value of  $z$  which corresponded to this. Candidates who followed the alternative method were typically less successful. Candidates who assumed that there was a unique value of  $z$  at the beginning of their response received no credit for their answers as the question asked for this to be shown.

## Question 14 (a)

14 Three planes have equations

$$\begin{aligned} kx - z &= 2, \\ -x + ky + 2z &= 1, \\ 2kx + 2y + 3z &= 0, \end{aligned}$$

where  $k$  is a constant.

- (a) By considering a suitable determinant, show that the three planes meet at a point for all values of  $k$ .

[5]

The majority of candidates were able to find the correct determinant, although arithmetic slips were common. Many candidates' further work demonstrated only that the determinant could not equal 0 without also making the distinct point that the planes would always meet at a point. Responses where candidates gave an explanatory sentence on what their algebraic work showed were the most successful.

### Misconception



Some candidates' work showed no distinction between planes meeting and always meeting at point. It is possible for three planes to form a sheaf so that they do not meet at a unique point.

## Question 14 (b)

- (b) Using a matrix method, find, in terms of  $k$ , the coordinates of the point of intersection of the planes.

[8]

Most candidates understood the procedure for finding an inverse matrix and multiplied the correct vector with it but a lack of accuracy meant that less than one third of responses received full marks. Candidates typically made at least one arithmetic or algebraic mistake in finding the elements of the inverse matrix and often several. In their final answer many candidates gave the position vector of the point of intersection, rather than its coordinates, for which they could not receive full credit.

### Assessment for learning



Many topics covered in Y420, but particularly work with matrices, require significant amounts of numerical and algebraic manipulation. Students should be encouraged to check their workings carefully for sign slips and other numerical errors as these are more common under exam conditions than when similar questions are attempted in class.

## Question 15

**15** In this question you must show detailed reasoning.

Evaluate  $\int_1^2 \frac{1}{\sqrt{1+2x-x^2}} dx$ , giving your answer in terms of  $\pi$ . [5]

This question was well answered with the majority of candidates achieving full marks. Less successful responses saw candidates having difficulty with completing the square, typically resulting from the negative  $x^2$  term, not integrating correctly or making sign errors when using the substitution  $u = 1 - x$ .

### Assessment for learning



Candidates should be aware of which formulae are given to them in the formula booklet and which are not.

## Question 16

**16** The point P (4, 1, 0) is equidistant from the plane  $2x + y + 2z = 0$  and the line  $\frac{x-3}{2} = \frac{y-1}{b} = \frac{z+5}{3}$ , where  $b > 0$ .

Determine the value of  $b$ . [10]

Given the length and complexity of this question, it was answered well overall with almost half of candidates being given full marks. Most successful responses considered a vector product and this was the most efficient method. Many candidates used alternative methods to achieve a fully correct answer but these methods required much more algebraic manipulation, making slips and errors much more common. Less successful responses saw candidates typically not setting out their work clearly, providing responses with little structure.

## Question 17 (a) (i)

17 Two similar species, X and Y, of a small mammal compete for food and habitat. A model of this competition assumes, in a particular area, the following.

- In the absence of the other species, each species would increase at a rate proportional to the number present with the same constant of proportionality in each case.
- The competition reduces the rate of increase of each species by an amount proportional to the number of the other species present.

So if the numbers of species X and Y present at time  $t$  years are  $x$  and  $y$  respectively, the model gives the differential equations

$$\frac{dx}{dt} = kx - ay \quad \text{and} \quad \frac{dy}{dt} = ky - bx,$$

where  $k$ ,  $a$  and  $b$  are positive constants.

- (a) (i) Show that the general solution for  $x$  is  $x = Ae^{(k+n)t} + Be^{(k-n)t}$ , where  $n = \sqrt{ab}$  and  $A$  and  $B$  are arbitrary constants. [6]

Most candidates showed an awareness of what was required to solve simultaneous differential equations although sign errors were common, both in deriving the correct differential equation and in solving the auxiliary equation. The most successful responses saw candidates note that the question asked them to 'show that' and so explicitly gave the correct differential and auxiliary equations, a method for solving the auxiliary equation and its roots before reaching the given answer. Less successful responses saw candidates omit one or more of these steps.

## Question 17 (a) (ii)

- (ii) Hence find the general solution for  $y$  in terms of  $A$ ,  $B$ ,  $k$ ,  $n$ ,  $a$  and  $t$ . [2]

A majority of candidates differentiated  $x$  and substituted their result for an expression for  $y$  but many did not simplify their answers. Some answers contained  $b$  although this was not allowed by the question.

### Assessment for learning



It is stated in section 2b of the H645 specification that candidates are expected to simplify algebraic and numerical expressions when giving their final answers, even if the examination question does not explicitly ask them to do so.

## Question 17 (b) (i)

Observations suggest that suitable values for the model are  $k = 0.015$ ,  $a = 0.04$  and  $b = 0.01$ . You should use these values in the rest of this question.

(b) When  $t = 0$ , the numbers present of species X and Y in this area are  $x_0$  and  $y_0$  respectively.

(i) Show that  $x = \frac{1}{2}(x_0 - 2y_0)e^{0.035t} + \frac{1}{2}(x_0 + 2y_0)e^{-0.005t}$ . [3]

Many candidates were reluctant to show numerical work establishing the values of 0.035 and -0.005. A significant number, although fewer, did not establish the values of A and B from  $x_0$  and  $y_0$ .

## Question 17 (b) (ii)

(ii) Hence show that  $y = \frac{1}{4}(x_0 + 2y_0)e^{-0.005t} - \frac{1}{4}(x_0 - 2y_0)e^{0.035t}$ . [1]

Nearly half of candidates did not attempt this question, the most of any question on this paper. Of those who did, most gave successful responses.

## Question 17 (c) (i)

(c) Use initial values  $x_0 = 500$  and  $y_0 = 300$  with the results in part (b) to determine what the model predicts for each of the following questions.

(i) What numbers of each species will be present after 25 years? [2]

Around half of candidates answered this question correctly. Those candidates who did not most typically gave non-integer solutions.

## Question 17 (c) (ii)

(ii) **In this question you must show detailed reasoning.**

When will the numbers of the two species be equal? [4]

This was well answered by those candidates who attempted the question with sufficient working typically given for a question which asked for detailed reasoning. The most common error was to make a sign error in substituting the initial values, typically leading to a response of  $t = 59.9$ . Even those candidates that struggled with some of the earlier questions on the paper were still able to get at least partial credit this late on in the paper, showing the importance of attempting every question.

## Question 17 (c) (iii)

(iii) Does either species ever disappear from the area? Justify your answer.

[3]

Fewer than half of candidates supported their answers with sufficient justification to score any marks on this question. Examiners did not credit responses which stated what each population would tend to as  $t \rightarrow \infty$  without considering what would happen to the individual terms in the formulae for  $x$  and  $y$ .

## Exemplar 3

17(c)(iii)	yes $x$ will disappear
	$x$ tends to <del>100</del> $-\infty$ as $t$ increases
	$y$ tends to $\infty$
	$x$ will go below 0
	$y$ will not

Although both conclusions are correct in this response, they are not supported by sufficient justification so it scored 0 marks.

## Question 17 (d) (i)

(d) Different initial values will apply in other areas where the two species compete, but previous studies indicate that one species or the other will eventually dominate in any given area.

(i) Identify a relationship between  $x_0$  and  $y_0$  where the model does **not** predict this outcome.

[1]

Of those candidates who provided an answer to part (i), approximately half identified the relationship correctly. A number of candidates did not answer either part of Question 17 (d) but there is no clear evidence to suggest that this was specifically due to running out of time at the end of a long paper or whether they had difficulty accessing these particular question parts.

**Question 17 (d) (ii)**

- (ii)** Explain what the model predicts in the long term for this exceptional case. **[2]**

As in Question 17 (c) (iii), few candidates provided a sufficient explanation. Most responses described a long term trend without any justification for their answer.

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