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Examiners' report

FURTHER MATHEMATICS A

H245

For first teaching in 2017

Y543/01 Summer 2023 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y543/01 series overview

This paper proved to be a good test of candidates' understanding. At this level the higher scoring candidates confidently completed questions that involved resolving velocities and forces. Standard calculations were completed with ease and formal algebraic methods were used throughout. Candidates found Questions 1, 2, 3 and 5 (a) more accessible with Questions 6 (b), 7 (b) and 8 proving more challenging for all candidates, testing their ability to model situations accurately.

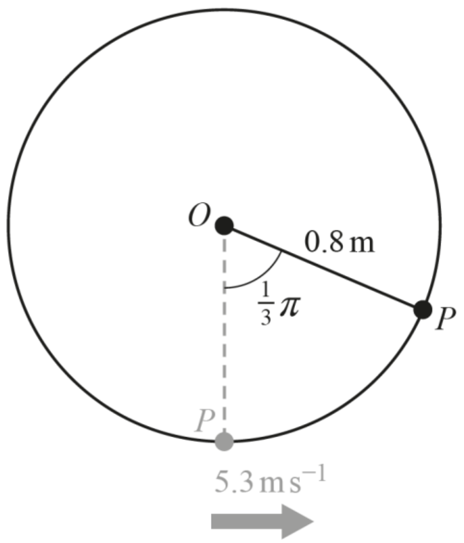
Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• included well-labelled diagrams to support their working • used clear, concise algebraic working • effectively linked calculated values to real life context.	• confused directions when resolving forces or velocities • were unable to recall key formulae • missed out steps in 'determine' and 'show that' questions.

OCR support

A poster covering key command words is available on Teach Cambridge.

Question 1 (a)

1 One end of a light inextensible string of length 0.8 m is attached to a particle P of mass m kg. The other end of the string is attached to a fixed point O. Initially P hangs in equilibrium vertically below O. It is then projected horizontally with a speed of $5.3\,\mathrm{m\,s^{-1}}$ so that it moves in a vertical circular path with centre O (see diagram).



At a certain instant, P first reaches the point where the string makes an angle of $\frac{1}{3}\pi$ radians with the downward vertical through O.

(a) Show that at this instant the speed of P is $4.5\,\mathrm{m\,s^{-1}}$. **[3]**

This question was answered effectively by most candidates. Clear working was shown and the majority of candidates gained full marks. The most common errors were incorrectly calculating the change in height using $\Delta h = 0.8\cos\left(\frac{\pi}{3}\right)$ rather than $0.8\left(1 - cos\left(\frac{\pi}{3}\right)\right)$ or not clearly showing where their value for change in height had come from.

Question 1 (b)

(b) Find the magnitude and direction of the radial acceleration of P at this instant. **[3]**

Many candidates answered this question well, although not in as great proportion as the first part of this question. Some candidates lost the final mark by attempting to describe the direction using an angle from an unspecified axis rather than stating that radial acceleration acted towards the centre. It should be noted that use of bearings is not appropriate in this situation.

Question 1 (c)

(c) Find the magnitude of the tangential acceleration of P at this instant. **[2]**

Candidates were less sure of the method needed to calculate tangential acceleration with over half of candidates gaining 0 marks in this question. Those who incorrectly expressed the magnitude as a negative number could not be credited with the final mark.

Misconception

Confusion between tangential and radial acceleration. Candidates need to make sure that they understand the difference between these and are able to recall key formulae for these.

Question 2 (a)

2 Materials have a measurable property known as the Young's Modulus, E.

If a force is applied to one face of a block of the material then the material is stretched by a distance called the extension. Young's modulus is defined as the ratio $\frac{\text{Stress}}{\text{Strain}}$ where Stress is defined as the force per unit area and Strain is the ratio of the extension of the block to the length of the block.

(a) Show that Strain is a dimensionless quantity. **[1]**

Most candidates answered this question well, showing clear consideration of the division of dimensions of length. A small number of candidates did not gain the mark, either by using units such as metres, or leaving their proof partially complete without cancelling down the dimensions to get 1 or stating that the quantity was dimensionless. Those who used a ratio rather than a division tended to be unsure how to conclude their proof, leaving L:L as their final response and therefore gaining 0 marks. Candidates need to understand that stating 'constant' or 'no units' are not equivalent to 'dimensionless'.

Question 2 (b)

(b) By considering the dimensions of both Stress and Strain determine the dimensions of E. **[2]**

Most candidates correctly calculated the dimensions of stress, gaining the first available mark. Candidates should refer carefully to the details of the question as under half of these did not show either a division by 1 or mention that strain was dimensionless gaining 1 mark out of 2 only. The 'determine' key word indicates that candidates should show clear justification for each stage of working.

Question 2 (c)

It is suggested that the speed of sound in a material, c, depends only upon the value of Young's modulus for the material, E, the volume of the material, V, and the density (or mass per unit volume) of the material, ρ.

(c) Use dimensional analysis to suggest a formula for c in terms of E, V and ρ. **[5]**

Candidates who set up a formal dimensional analysis model and constructed simultaneous equations were typically successful in gaining most marks in this part. A significant number missed out the constant *k*, leading to the loss of the final mark. Those candidates who attempted to consider combining dimensions more informally, using trial and error to divide the dimensions of one quantity by another in an effort to obtain the dimensions of *c* were much less likely to obtain a correct final formula. Examiners did credit this less formal method if it clearly led to a solution but would expect to see more formal justification in future.

Question 2 (d) (i)

(d) The speed of sound in a certain material is $500\,\mathrm{m\,s^{-1}}$.

(i) Use your formula from part **(c)** to predict the speed of sound in the material if the value of Young's modulus is doubled but all other conditions are unchanged. **[1]**

Many candidates did well in this part, showing a clear understanding of the scale factor of √2 and giving a final value for the speed of sound.

Question 2 (d) (ii)

(ii) With reference to your formula from part **(c)**, comment on the effect on the speed of sound in the material if the volume is doubled but all other conditions are unchanged. **[1]**

A minority of candidates completed this correctly. Careful reading of the question was important here as candidates needed to both comment on the lack of dependence on volume from the formula and link this to the unchanged value for speed of sound.

Question 2 (e)

(e) Suggest **one** possible limitation caused by using dimensional analysis to set up the model in part **(c)**. **[1]**

This question was completed variably by candidates with many who had not included a constant in Question 2 (c) stating that the limitation was that a constant should be included rather than stating that dimensional analysis did not allow the value of the constant to be found.

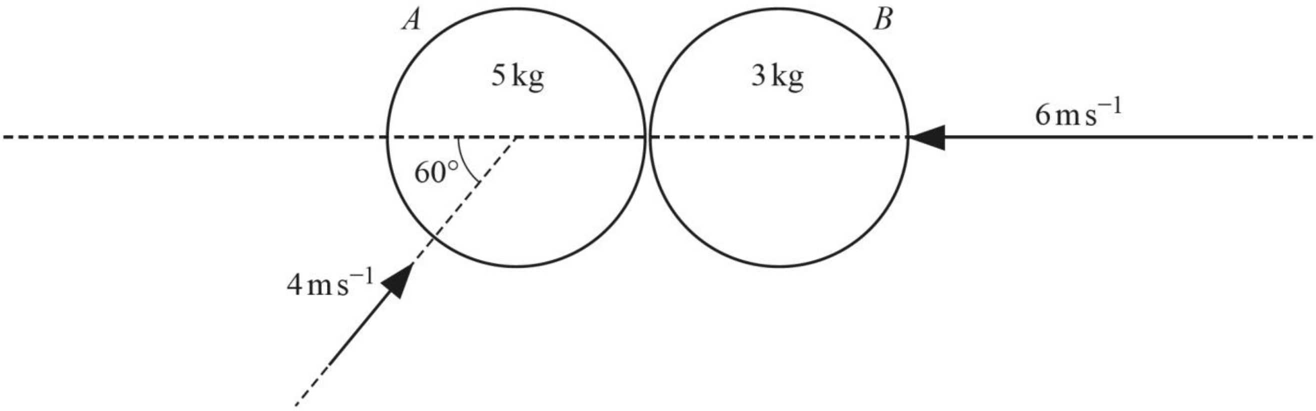
Question 3

3 Two smooth circular discs A and B are moving on a smooth horizontal plane when they collide. The mass of A is 5 kg and the mass of B is 3 kg.

At the instant before they collide,

- the velocity of A is $4\,\mathrm{m\,s^{-1}}$ at an angle of $60°$ to the line of centres,
- the velocity of B is $6\,\mathrm{m\,s^{-1}}$ along the line of centres

(see diagram).



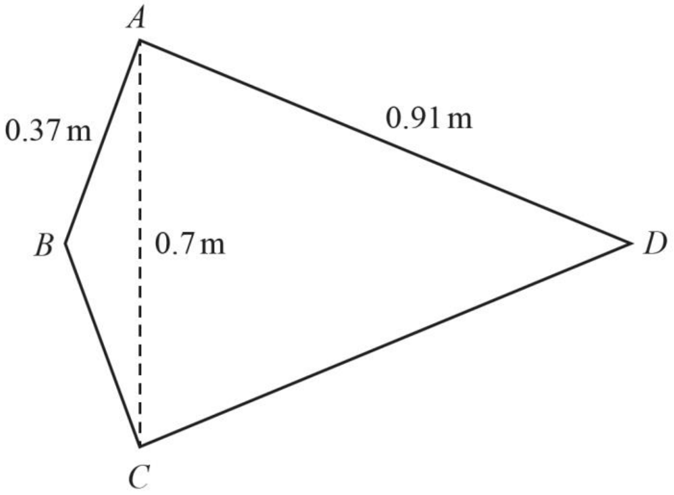
The coefficient of restitution for collisions between the two discs is $\frac{3}{4}$.

Determine the angle that the velocity of A makes with the line of centres after the collision. **[7]**

Many candidates effectively constructed conservation of momentum and restitution equations, showing a good understanding of the unchanged velocity perpendicular to the line of centres. Over half of candidates correctly found components of *A*'s velocity with most of these obtaining a correct final angle and a few quoting the complementary angle. Common errors included errors in the sign used for velocities or the use of inconsistent signs for their unknown component of *A*'s velocity. For some candidates solving the simultaneous equations introduced further errors.

Question 4 (a)

4 $ABCD$ is a uniform lamina in the shape of a kite with $BA = BC = 0.37\,\text{m}$, $DA = DC = 0.91\,\text{m}$ and $AC = 0.7\,\text{m}$ (see diagram). The centre of mass of $ABCD$ is G.



(a) Explain why G lies on BD. **[1]**

This part was completed correctly by the vast majority of candidates, demonstrating clear understanding of symmetry.

Question 4 (b)

(b) Show that the distance of G from B is $0.36\,\text{m}$. **[4]**

The lamina $ABCD$ is freely suspended from the point A.

Many candidates had a relatively strong understanding of the underlying calculation but needed to make sure that they fully showed all steps in response to the 'Show that' command included in the question. Those using a tabulated method often produced the correct answer but needed to be aware of showing key calculations and simplifying those numbers to show a final intermediary step before the given answer. Students needed to carefully consider whether the centre of mass of a triangle is measured from the base or the apex in forming a correct response.

Question 4 (c)

(c) Determine the acute angle that CD makes with the horizontal, stating which of C or D is higher. **[4]**

This question, as a 'determine' question needed to involve clear reasoning and calculation of angles. A minority of candidates managed this completely, with a small number calculating relevant angles and quoting an angle between *CD* and the vertical rather than the horizontal. Some were able to find an initial angle and then were unsure how to proceed further gaining 1 mark only. Sophisticated solutions used the idea of rotation to produce the required angle by finding the difference between angle *CAG* and angle *GDC*. A minority of students successfully used the dot product between $\overrightarrow{CD}$ and $\overrightarrow{AG}$ to find the angle.

Question 5 (a)

5 A particle P of mass $2\,\text{kg}$ moves along the x-axis.

At time $t = 0$, P passes through the origin O with speed $3\,\text{m}\,\text{s}^{-1}$.

At time t seconds the displacement of P from O is $x\,\text{m}$ and the velocity of P is $v\,\text{m}\,\text{s}^{-1}$, where $t \geqslant 0$, $x \geqslant 0$ and $v \geqslant 0$.

While P is in motion the only force acting on P is a resistive force F of magnitude $(v^2 + 1)$ N acting in the negative x-direction.

(a) Find an expression for v in terms of x. **[5]**

Candidates found this part accessible with over half gaining all marks and a large majority gaining at least 3 marks. Common errors for these candidates included missing out the negative sign for the resistive force in the original question. Many candidates were confident in the use of $v\frac{\mathrm{d}v}{\mathrm{d}x}$ for acceleration and produced clearly set out algebraic solutions from this expression with a few being unsure how to separate variables effectively. The recall of this expression for acceleration proved to be a common barrier for a small number of candidates who were unable to access this question.

Question 5 (b)

(b) Determine the distance travelled by P while its speed drops from $3\,\text{m}\,\text{s}^{-1}$ to $2\,\text{m}\,\text{s}^{-1}$. **[2]**

This was typically answered effectively by those candidates who had obtained an expression for *v* in the last part.

Question 5 (c)

Particle Q is identical to particle P. At a different time, Q is moving along the x-axis under the influence of a single constant resistive force of magnitude $1\,\text{N}$. When $t' = 0$, Q is at the origin and its speed is $3\,\text{m}\,\text{s}^{-1}$.

(c) By comparing the motion of P with the motion of Q explain why P must come to rest at some finite time when $t < 6$ with $x < 9$. **[3]**

Roughly half of candidates gained some marks on this part with many showing a good attempt at deriving the limits on *t* and *x* for particle Q whether through constant acceleration formulae, differential equations or energy considerations. Only a small number were able to compare resistive forces and relate this to comparative deceleration between the particles in order to finally state why *P* comes to rest before these.

Question 5 (d)

(d) Sketch the velocity-time graph for P. You do not need to indicate any values on your sketch. **[1]**

Over a third of candidates produced an effective sketch, indicating a progressively shallower negative gradient of the graph until the particle stopped on reaching a velocity of zero. Of those who attempted the question, common errors included using axes as asymptotes, continuing the graph into a region of negative velocity or using the constant deceleration of *Q* to sketch their graph rather than considering *P*.

Question 5 (e)

(e) Determine the maximum displacement of P from O during P's motion. **[2]**

Those candidates who had produced a usable model in part (a) were typically successful in calculating maximum displacement when velocity was zero. A minority were unable to rearrange their expression to find *x*, with some candidates forgetting to show the key steps needed for a 'determine' question, who were credited with 1 mark only for a correct answer seen.

Question 6 (a) (i)

6 A particle P of mass 3 kg is moving on a smooth horizontal surface under the influence of a variable horizontal force $\mathbf{F}$ N. At time t seconds, where $t \geqslant 0$, the velocity of P, $\mathbf{v}$ m s^{-1}, is given by

$$\mathbf{v} = (32\sinh(2t))\mathbf{i} + (32\cosh(2t) - 257)\mathbf{j}.$$

(a) **(i)** By considering kinetic energy, determine the work done by $\mathbf{F}$ over the interval $0 \leqslant t \leqslant \ln 2$. **[5]**

For this question, candidates needed to make sure that their method was clearly shown. Most commonly, successful solutions found the numerical vector for velocity at both given times before calculating the dot products to find the change in kinetic energy. Some inaccuracies in substituting into these expressions using their calculator were seen here for a significant minority of candidates. Other errors included the subtraction of kinetic energy values in the incorrect order. Candidates should be encouraged to clearly consider which is the final energy and which is the initial energy. Those candidates who found a full algebraic expression for the dot product of velocity before substituting found the question more time consuming and were less likely to arrive at the correct value. Some poor notation was seen with candidates squaring vectors to arrive at a vector expression for kinetic energy rather than the expected scalar. Methods involving calculus were generally unsuccessful with candidates attempting to integrate kinetic energy rather than the dot product of force and velocity.

Question 6 (a) (ii)

(ii) Explain the significance of the sign of the answer to part **(a)(i)**. **[1]**

Over half of students showed a good understanding on this question. Candidates needed to be careful not to confuse the force and the work done, stating clearly that the force acts in the opposite direction to motion, rather than stating that 'work is opposite to motion'. Other common errors include stating that the particle was travelling backwards or that the force is in the negative *x* direction.

Question 6 (b)

(b) Determine the rate at which **F** is working at the instant when P is moving parallel to the **i**-direction. **[6]**

A minority of candidates were able to answer this question comprehensively showing the expected level of working for a 'determine' question. Almost half of candidates made a good start to this question, gaining the first 2 marks for finding the time at which *P* is moving in the direction stated, with many of these proceeding to find the vector for velocity at this time. Some candidates were unable to link the question to the need to calculate power at this stage, using the dot product of force and velocity at that time, and incorrectly attempted to differentiate force to find a 'rate of doing work'. Scalar methods, attempting to find a value for energy and differentiate were unsuccessful and candidates should be supported to understand when to use vector based methods.

Exemplar 1

$(32\cosh(2t) - 257) = 0$

$\cosh(2t) = \frac{257}{32}$

$2t = 2.77$

$t = 1.386$

$\frac{dv}{dt} = a = 64\cosh(2t)$

$F = ma = 192\cosh 2t$

$\frac{dF}{dt} = 384\sinh(2t)$

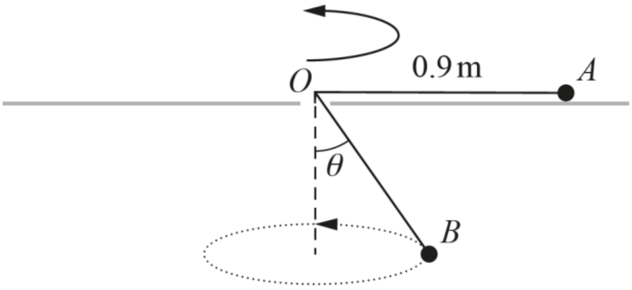
$t = 1.386$

$\frac{dF}{dt} = 720\ Ns^{-1}$

This candidate has correctly calculated the time at which *P* is moving in the **i** direction. The calculation of $P = F.v$ is not shown, and they proceeded to try and calculate the rate of change of force gaining no further marks.

Question 7 (a)

7 Two particles A and B are connected by a light inextensible string of length 1.26 m. Particle A has a mass of 1.25 kg and moves on a smooth horizontal table in a circular path of radius 0.9 m and centre O. The string passes through a small smooth hole at O. Particle B has a mass of 2 kg and moves in a horizontal circle as shown in the diagram. The angle that the portion of string below the table makes with the downwards vertical through O is θ, where $\cos\theta = \frac{4}{5}$ (see diagram).



(a) Determine the angular speed of A and the angular speed of B. **[5]**

This was an accessible question for candidates with a majority gaining at least 3 marks for this part, and a number gaining all 5 marks. Common errors seen include resolving tension as $T_B = 2g cos\theta$, calculating the radius for *B* as 0.36 rather than $0.36 \sin\theta$, or confusing $\sin\theta$ and $\cos\theta$ when resolving.

Question 7 (b)

At the start of the motion, A, O and B all lie in the same vertical plane.

(b) Find the first subsequent time when A, O and B all lie in the same vertical plane. **[2]**

This question highlighted a key misconception, with many candidates demonstrating uncertainty about how to tackle this part. Of those who correctly considered the difference in angular velocities and related this to time, many used an integer difference of 2π radians rather than understanding that a difference of π radians was sufficient for particles to lie in the same vertical plane. Those who resorted to a method involving the lowest common multiple, or ratios, often gained the 5:4 relationship for angular velocities but were unable to take this further to relate it effectively to time.

Question 8

8 One end of a light elastic string of natural length 2.1 m and modulus of elasticity 4.8 N is attached to a particle, P, of mass 1.75 kg. The other end of the string is attached to a fixed point, O, which is on a rough inclined plane. The angle between the plane and the horizontal is θ where $\sin\theta = \frac{3}{5}$. The coefficient of friction between P and the plane is 0.732.

Particle P is placed on the plane at O and then projected down a line of greatest slope of the plane with an initial speed of $2.4\,\text{m}\,\text{s}^{-1}$.

Determine the distance that P has travelled from O at the instant when it first comes to rest. You can assume that during its motion P does not reach the bottom of the inclined plane. **[8]**

Those candidates who approached this question systematically with a well-labelled diagram, showing a clear consideration of extension x and distance travelled by P as x + 2.1 were most likely to succeed. Work done against friction was often missed in the equation, and students should be encouraged to record or circle all key information as they read a question. Distances used needed to be consistent and expressed in terms of a single variable, a common error was using the same variable for distance travelled and extension. Efficient calculator use was needed for this question. Many candidates created a passable energy budget equation but lost the final 2 marks for accuracy errors. Candidates must remember to explicitly reject inappropriate final answers to be sure of gaining all the marks. Some students split the motion into two stages, calculating the velocity of the particle at zero extension as a middle step. This is not necessary, however most candidates who did so were able to proceed effectively.

Assessment for learning

Candidates should be encouraged to produce clear diagrams in the space on their printed answer booklet, annotating these with all information from the question to support them in accurately answering modelling questions.

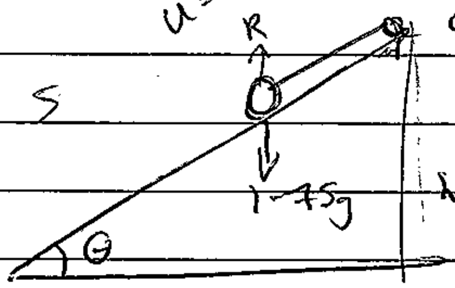
Exemplar 2

(continued)

$u=2.4$ $l_0=2.1$ $\lambda=4.8$

$\mu=0.732$

$\sin\theta = 3/5$



$F=\mu R$

$\mu=\tan\theta$

finding x

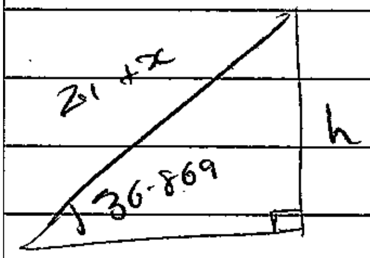
$x+2.1$

= answer

KE GPE EPE

$\frac{1}{2}\times 1.75\times 2.4^2$ $1.75\times\cancel{2}9.8\times h$

$\frac{4.8}{4.2}\times x^2$



$h : 2.1+x$

$3 : 5$

$\frac{15}{5} =$

$\times 5$ $15 : 25$

$5h : 10.5+5x$

$$\frac{h}{h+2.1+x}$$

$$\frac{3}{5} = \frac{h}{2.1+x} \qquad 1 : \frac{2.1+x}{h}$$

$$3 : 5$$

$$\boxed{h = \frac{3}{5}(2.1+x)} \qquad 3 : \frac{10.5+5x}{5h}$$

$$\frac{4.8}{4.2} \times x^2 = 5.04 + 17.15\left(\frac{3}{5}\times 2.1 + \frac{3}{5}x\right)$$

$$\frac{8}{7}x^3 = 5.04 + 21.609 + 10.29x$$

$$\frac{8}{7}x^2 - 10.29x - 26.649 = 0$$

$$x = -2.1 \text{ or } x = 11.103$$

reject as $x > 0$ for an extension

so $x = 11.103$

so total distance $= 11.103 + 2.1 = 13.2\text{ m}$

This is a typical example where the candidate has set up a diagram, and shown some understanding of the different lengths involved. Some useful scaffolding is also seen in the table to aid them in considering energy at the start and end of motion. However, this candidate has not included the work done against friction and this unfortunately means no further credit can be given after the first 3 marks.

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