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Examiners' report

FURTHER MATHEMATICS A

H245

For first teaching in 2017

Y540/01 Summer 2023 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y540/01 series overview

This paper, along with Y541, assesses the compulsory core content of the A Level Further Mathematics qualification. Questions in each paper can assess any part of the core specification. This paper had a number of straightforward questions, and most candidates had a good attempt at these questions. However, there were some unfamiliar problem solving aspects which looked daunting, and some candidates struggled to process the issues that the questions posed in order to approach their response in a productive way.

Most candidates appeared to be able to complete the paper in the time available.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• showed a secure grasp of all standard techniques • communicated well using mathematical language correctly • applied the breadth of their mathematical knowledge to find ways to tackle problems unfamiliar to them • made minimal arithmetic and algebraic errors • organised their time well so that they had time to check (and possibly correct) their work.	• had clear gaps in their knowledge of standard techniques • produced unclear or incomplete mathematical arguments, often resulting in unfortunate algebraic and arithmetic errors • could not find routes into problems unfamiliar to them.

Candidates must give a full analytical solution where the whole question, or specific parts of a question, are preceded by the bold statement request '**In this question you must show detailed reasoning'**. Answers cannot just be given without any justification and any values obtained by a calculator must be supported with sufficient evidence that the mathematical technique is understood. Questions using 'Show that' and 'Determine' also require clear working to be shown.

OCR support

A guide to the command words used in OCR A Level Maths exams can be found on Teach Cambridge.

Question 1

1 **In this question you must show detailed reasoning.**

Determine the value of $\sum_{r=1}^{50} r^2(16-r)$. **[3]**

This question was, in general, tackled well. However, a number of candidates did not appreciate what they had to do to comply with the 'detailed reasoning' demand. Consequently there were steps missing in the working.

Assessment for learning

Most of the questions in this paper asked candidates to show detailed reasoning, were 'show that' questions, or used the command word 'determine'. In these questions, candidates are required to demonstrate understanding of the topic being assessed and this requires them to show all the steps in their working. Candidates would benefit from having an understanding of these commands.

Exemplar 1

$\sum_{r=1}^{50} r^2(16-r)$

$= \sum_{r=1}^{50} 16r^2 - r^3$

$= 16\sum_{r=1}^{50} r^2 - \sum r^3$

$= 16\left(\frac{1}{6}n(n+1)(2n+1)\right) - \frac{1}{4}n^2(n+1)^2$

$n=50 \quad = 5181175$

This candidate has correctly separated the terms for both formulae for the first method mark. Had the candidate shown their substitution, required by the '**detailed reasoning**' request, they may have avoided the arithmetical error and achieved the correct final result for full credit.

Exemplar 2

$$\sum_{r=1}^{50} r^2(16-r) = \sum_{r=1}^{50} 16r^2 - r^3 = 16\sum_{r=1}^{50} r^2 - \sum_{r=1}^{50} r^3$$

$$= \frac{16}{6}\, 50(50+1)(100+1) - \frac{1}{4}\, 50^2(51)^2$$

$$= 686800 - 1625625$$

$$= -938825$$

This candidate has shown all the steps of the working, including the substitution of *n*, and so could be given full marks.

Question 2 (a)

2 In this question you must show detailed reasoning.

The equation $z^4 + 4z^3 + 9z^2 + 10z + 6 = 0$ has roots α, β, γ and δ.

(a) Show that a quartic equation whose roots are $\alpha + 1$, $\beta + 1$, $\gamma + 1$ and $\delta + 1$ is $w^4 + 3w^2 + 2 = 0$. **[3]**

The substitution $z = w - 1$ was the expected method for this question. As this is also a '**detailed reasoning'** question, it was expected that the terms in the complete expansion would be collected and clearly produced the transformed equation. Given that the equation was given, it was important to see all the steps to demonstrate that the answer had not been just copied.

The alternative method was to deal with symmetric roots and this required a great deal of algebraic manipulation. Most candidates doing it this way were able to obtain the coefficient of w^3 to be 0, a few managed the obtain the coefficient of w^2 but very few were able to complete the process to obtain all 4 coefficients.

Question 2 (b)

(b) Hence determine the exact roots of the equation $z^4 + 4z^3 + 9z^2 + 10z + 6 = 0$. **[3]**

A small number of candidates did not understand the command word 'hence' and started with attempting to solve the quartic equation. Solving the transformed equation gave two values for w^2 and thus the 4 roots. A large number of candidates were successful in finding these roots.

Question 3 (a)

3 **(a)** Show that $\frac{-3+\sqrt{3}\,\mathrm{i}}{2} = \sqrt{3}\,\mathrm{e}^{\frac{5}{6}\pi\mathrm{i}}$. **[2]**

It was acceptable to tackle this question from either direction – i.e. showing that the left hand side was equal to the right hand side or the right hand side to be equal to the left hand. In either direction, this was a 'show that' question which meant that all steps needed to be shown, and this was not always the case.

Question 3 (b)

(b) Hence determine the exact roots of the equation $z^5 = \frac{9(-3+\sqrt{3}\,\mathrm{i})}{2}$, giving the roots in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $r > 0$ and $0 \leqslant \theta < 2\pi$. **[3]**

Many candidates obtained all the roots of the equation. Because the fifth roots were to be found, the fifth root of the modulus had to be given in its simplest form which many candidates chose not to do. The argument required consideration of values by adding 2π 4 times before dividing by 5 to obtain all the roots in the required range.

Question 4 (a)

4 The transformations $\mathrm{T_A}$ and $\mathrm{T_B}$ are represented by the matrices **A** and **B** respectively, where

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

(a) Describe geometrically the **single** transformation consisting of $\mathrm{T_A}$ followed by $\mathrm{T_B}$. **[2]**

The easiest way to answer this question was to multiply the matrices in the correct order and to consider what the product represented. A number of candidates defined the transformation that each matrix represented and considered the result of carrying out both transformations. For both methods most candidates obtained the correct answer.

Question 4 (b)

(b) By considering the transformation $\mathrm{T_A}$, determine the matrix $\mathbf{A}^{423}$. **[3]**

'By considering the transformation T_A' required candidates to define the transformation and many did not do so. To note that the transformation represented an anticlockwise rotation of 90º meant that $A^4 = I$. Many did not immediately realise this and found successive powers of A and only then noting that $A^4 = I$ and that it was possible to deduce that $A^{423} = A^3$.

Question 4 (c) (i)

The transformation T_C is represented by the matrix **C**, where

$$\mathbf{C} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}.$$

The region R is defined by the set of points (x, y) satisfying the inequality $x^2 + y^2 \leqslant 36$.

The region R' is defined as the image of R under T_C.

(c) **(i)** Find the exact area of the region R'. **[2]**

The fact that the determinant of C transformed the area within the circle to the area of the transformed circle was well known, but it was surprising how many struggled to write down the area of the circle.

Question 4 (c) (ii)

(ii) Sketch the region R', specifying all the points where the boundary of R' intersects the coordinate axes. **[4]**

A graph of the shape of *R*' was usually correct but many candidates did not shade the required area, thus losing a mark.

Question 5 (a)

5 **(a)** Find the general solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$. **[2]**

This question was well done. A few candidates lost a mark by not presenting their general solution in the form $y = f(x)$.

Question 5 (b)

(b) Hence find the general solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = x(4 - 5x)$. **[4]**

The form of the particular integral was chosen appropriately by most of the candidates. A few algebraic errors aside, most were able to obtain the correct general solution to this differential equation,

Question 6

6 In this question you must show detailed reasoning.

The power output, p watts, of a machine at time t hours after it is switched on can be modelled by the equation $p = 20 - 20\tanh(1.44t)$ for $t \geqslant 0$.

Determine, according to the model, the **mean** power output of the machine over the first half hour after it is switched on. Give your answer correct to 2 decimal places. **[4]**

The integral for the mean power output was usually correct. Those who knew that the integral for tanh(1.44t) was $\frac{1}{1.44}\ln|\cosh 1.44t|$ usually obtained the correct answer. There were other forms for this integral including exponentials but generally responses using these alternate forms were less successful.

Exemplar 3

mean power: $\frac{1}{0.5-0}\int_0^{0.5} 20 - 20\tanh(1.44t)\,dt$

$= 2 \times 20 \int_0^{0.5} 1 - \tanh(1.44t)\,dt$

$= 40\left[t - \frac{1}{1.44}\ln(\cosh(1.44t))\right]_0^{0.5}$

$= 40\left(0.5 - \frac{1}{1.44}\ln(1.2706) + \frac{0}{1.44}\ln 1\right)$

$= 40(0.5 - 0.1663)$

$= 13.35W$ to 2d.p.

All the steps are shown in this exemplar but this is not always the case. Calculators will give the answer to this definite integral but if that is done, the candidate has not demonstrated knowledge of the integral of k tanh1.44t.

Question 7 (a) (i)

7 An engineer is modelling the motion of a particle P of mass 0.5 kg in a wind tunnel.

P is modelled as travelling in a straight line. The point O is a fixed point within the wind tunnel. The displacement of P from O at time t seconds is x metres, for $t \geqslant 0$.

You are given that $x \geqslant 0$ for all $t \geqslant 0$ and that P does not reach the end of the wind tunnel.

If $t \geqslant 0$, then P is subject to three forces which are modelled in the following way.

- The first force has a magnitude of $5(t+1)\cosh t$ N and acts in the positive x-direction.
- The second force has a magnitude of $0.5x$ N and acts towards O.
- The third force has a magnitude of $\left|\frac{dx}{dt}\right|$ N and acts in the direction of motion of the particle.

(a) The engineer applies the equation "$F = ma$" to the model of the motion of P and derives the following differential equation.

$$5(t+1)\cosh t - 0.5x + \frac{dx}{dt} = 0.5\frac{d^2x}{dt^2}$$

(i) Explain the sign of the $\frac{dx}{dt}$ term in the engineer's differential equation. **[1]**

There was a variety of responses to this question; the most common error was to describe the term as the velocity rather than a force.

Question 7 (a) (ii)

When $t = 0$ the displacement of P is 6 m, and it is travelling towards O with a speed of $5\,\text{m s}^{-1}$.

(ii) Without attempting to solve the differential equation, find the acceleration of P when $t = 0$. **[2]**

The substitutions were usually correct, though the most frequent error was to ignore the statement that the particle was travelling towards O.

Question 7 (b) (i)

Let the particular solution to the differential equation in part **(a)** be a function f such that $x = f(t)$ for $t \geqslant 0$.

The particular solution to the differential equation can be expressed as a Maclaurin series.

(b) **(i)** Show that the Maclaurin series for $f(t)$ up to and including the term in t is $6 - 5t$. **[1]**

In order to obtain the first two terms as a Maclaurin's series, it was necessary to give the general form of the first two terms and often this was not done.

Question 7 (b) (ii)

(ii) Use your answer to part **(a)(ii)** to show that the term in t^2 in the Maclaurin series for $\mathrm{f}(t)$ is $-3t^2$. **[1]**

Likewise, it was necessary to state the general form of the third term so that the correct conclusion could be reached.

Question 7 (b) (iii)

(iii) By differentiating the differential equation in part **(a)** with respect to t, show that the term in t^3 in the Maclaurin series for $\mathrm{f}(t)$ is $0.5t^3$. **[4]**

The 4th term in this series required differentiation of the equation for the model using the product rule. A number did not remember that the series was a function of *t* and differentiated the term kx to give k rather than $k\frac{\mathrm{d}x}{\mathrm{d}t}$.

Question 7 (c) (i)

You are given that the complete Maclaurin series for the function f is valid for all values of $t \geqslant 0$.

After 0.25 seconds P has travelled 1.43 m towards the origin.

(c) (i) By using the Maclaurin series for $\mathrm{f}(t)$ up to and including the term in t^3, evaluate the suitability of the model for determining the displacement of P from O when $t = 0.25$. **[1]**

The information given in the stem of the last two parts of the question was often ignored.

Question 7 (c) (ii)

(ii) Explain why it might not be sensible to use the Maclaurin series for $\mathrm{f}(t)$ up to and including the term in t^3 to evaluate the suitability of the model for determining the displacement of P from O when $t = 10$. **[1]**

The information given in the stem of the question was often ignored. Comments such as 'Maclaurin's series is not valid for $t \geq 1$' contradicted the information that the complete series expansion was valid for all positive *t*. Responses that referred to the problem being because the series so far only had the first 4 terms and so was not complete were more successful.

Question 8 (a)

8 The points P, Q and R have coordinates (0, 2, 3), (2, 0, 1) and (1, 3, 0) respectively.

The acute angle between the line segments PQ and PR is θ.

(a) Show that $\sin\theta = \frac{2}{11}\sqrt{22}$. **[3]**

Use of the scalar product usually found $\cos\theta$ successfully. However, this was a 'show that' question and the calculation of $\sin\theta$ was often not convincing. Use of the cross product usually found $\sin\theta$ directly.

Question 8 (b)

The triangle PQR lies in the plane Π.

(b) Determine an equation for Π, giving your answer in the form $ax + by + cz = d$, where a, b, c and d are integers. **[3]**

Arithmetic errors aside, the equation for the plane was found by most candidates.

Question 8 (c)

The point S has coordinates $(5, 3, -1)$.

(c) By finding the shortest distance between S and the plane Π, show that the volume of the tetrahedron $PQRS$ is $\frac{14}{3}$.

[The volume of a tetrahedron is $\frac{1}{3} \times$ area of base $\times$ perpendicular height] **[4]**

A significant majority of candidates laid out their response to this part well. The formula for the volume of the tetrahedron was given as the product of the area of the base triangle and the height divided by 3, so the response stating the value of the area of the triangle and the height led easily to the correct answer. In some responses it was not clear what the candidate was calculating.

Question 8 (d)

The tetrahedron $PQRS$ is transformed to the tetrahedron $P'Q'R'S'$ by a rotation about the y-axis.

The x-coordinate of S' is $2\sqrt{2}$.

(d) By using the matrix for a rotation by angle θ about the y-axis, as given in the Formulae Booklet, determine in exact form the possible coordinates of R'. **[5]**

The first step, to find an equation in $\cos\theta$ and $\sin\theta$ was well done. However, the (correct) observation that this equation was satisfied by $\theta = \frac{\pi}{4}$ missed the fact that there were in fact two values for θ.

Candidates who found a quadratic equation in either $\sin\theta$ or $\cos\theta$ and solved it to find the corresponding value of the other trigonometrical function were able to make better progress.

Question 9 (a)

9 In this question you must show detailed reasoning.

(a) Use de Moivre's theorem to determine constants A, B and C such that $\sin^4\theta \equiv A\cos 4\theta + B\cos 2\theta + C$. **[5]**

Candidates who used the fact that $z = (\cos\theta + \mathrm{i}\sin\theta)$ leads to $2\mathrm{i}\sin\theta = \left(z - \frac{1}{z}\right)$ then used De Moivre's theorem to find the required identity quite easily.

Others started with $(\cos\theta + \mathrm{i}\sin\theta)^4$, equating De Moivre's theorem to the binomial expansion and equating imaginary parts. These candidates had rather more work to do to obtain the required result, giving rather more opportunities for making algebraic or arithmetic errors.

Question 9 (b)

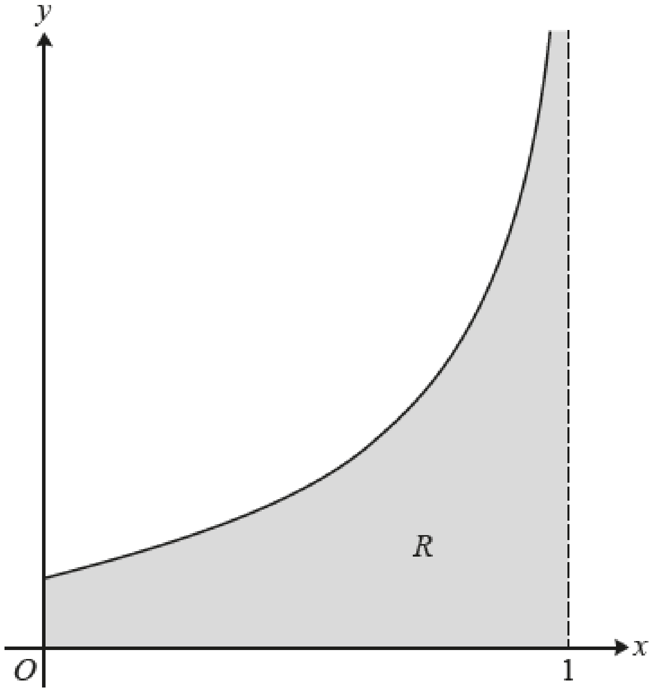
The function f is defined by

$$\mathrm{f}(x) = \sin\left(4\sin^{-1}\left(x^{\frac{1}{5}}\right)\right) - 8\sin\left(2\sin^{-1}\left(x^{\frac{1}{5}}\right)\right) + 12\sin^{-1}\left(x^{\frac{1}{5}}\right), \qquad x \in \mathbb{R},\ 0 \leqslant x < 1.$$

(b) Show that $\mathrm{f}'(x) = \dfrac{32}{5\sqrt{1 - x^{\frac{2}{5}}}}$. **[6]**

The candidates who realised that part (a) was an identity to be used usually made good progress. Others struggled with this rather daunting function without any clear understanding of how to obtain the required result and so usually did not get there.

Question 9 (c)



The diagram shows the curve with equation $y = \dfrac{1}{\sqrt{1-x^{\frac{2}{3}}}}$ for $0 \leqslant x < 1$ and the asymptote $x = 1$. The region R is the unbounded region between the curve, the x-axis, the line $x = 0$ and the line $x = 1$.

You are given that the area of R is finite.

(c) Determine the exact area of R. **[3]**

The clue to the necessary approach to this part, that the area *R* was unbounded, was often missed and so did not use a limiting process. Many also did not realise that part (b) was to be used.

Assessment for learning

It is sometimes the case that in finding the answer to a problem, a key fact is required. Candidates may therefore be helped with this by giving them the key fact. Usually this is done by asking candidates to 'Show that' a key fact is true. Then, should the candidate not be able to show it, the fact is there to be used.

This was the case in Question 9. The identity required to solve part (b) was $\sin^4\theta = \frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 2\theta + \frac{3}{8}$ and the proof of this was asked in part (a). Candidates made good progress in part (b) when they realised that part (a) was relevant.

Likewise, part (c) required the answer to part (b) to be used.

Candidates would benefit from asking themselves the question: Are the results of earlier parts relevant to this part?

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