OCR
Oxford Cambridge and RSA

Monday 5 June 2023 – Afternoon

A Level Further Mathematics A

Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 **(a)** The matrix **P** is given by $\mathbf{P} = \begin{pmatrix} 1 & 0 & -2 & 2 \\ 4 & 2 & -2 & 3 \end{pmatrix}$.

(i) Write down the dimensions of **P**. **[1]**

(ii) Write down the transpose of **P**. **[1]**

(b) The matrices **Q**, **R** and **S** are given by $\mathbf{Q} = \begin{pmatrix} 1 & 2 \end{pmatrix}$, $\mathbf{R} = \begin{pmatrix} 3 & -4 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{S} = \begin{pmatrix} 3 & -2 \end{pmatrix}$.

Write down the sum of the two of these matrices which are conformable for addition. **[1]**

(c) The dimensions of matrix **A** are 4 by 5. The matrices **A** and **B** are conformable for multiplication so that the matrix **C** = **BA** can be formed. The matrix **C** has 6 rows.

(i) Write down the number of columns that **C** has. **[1]**

(ii) Write down the dimensions of **B**. **[1]**

(iii) Explain whether the matrix **AB** can be formed. **[1]**

(d) Find the value of c for which $\begin{pmatrix} -2 & 3 \\ 6 & 10 \end{pmatrix}\begin{pmatrix} c & 5 \\ 10 & 13 \end{pmatrix} = \begin{pmatrix} c & 5 \\ 10 & 13 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 6 & 10 \end{pmatrix}$. **[2]**

2 **In this question you must show detailed reasoning.**

(a) Write the complex number $-24 + 7\mathrm{i}$ in modulus-argument form. **[3]**

(b) Solve the simultaneous equations given below, giving your answers in cartesian form.

$$\begin{aligned} \mathrm{i}z + 3w &= -7\mathrm{i} \\ -6z + 5\mathrm{i}w &= 3 + 13\mathrm{i} \end{aligned}$$

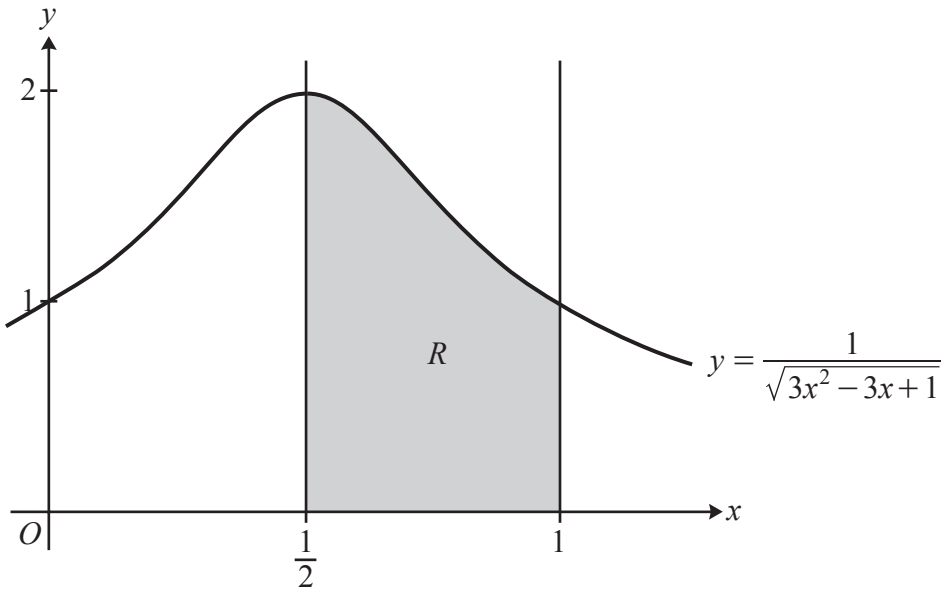
[4]

3 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}u}(\sinh^{-1} u) = \frac{1}{\sqrt{u^2 + 1}}$. **[2]**

(b) Find the equation of the normal to the graph of $y = \sinh^{-1} 2x$ at the point where $x = \sqrt{6}$. Give your answer in the form $y = mx + c$ where m and c are given in exact, non-hyperbolic form. **[4]**

4 In this question you must show detailed reasoning.

The region R is bounded by the curve with equation $y = \dfrac{1}{\sqrt{3x^2 - 3x + 1}}$, the x-axis and the lines with equations $x = \dfrac{1}{2}$ and $x = 1$ (see diagram). The units of the axes are cm.



A pendant is to be made out of a precious metal. The shape of the pendant is modelled as the shape formed when R is rotated by 2π radians about the x-axis.

Find the exact value of the volume of precious metal required to make the pendant, according to the model. **[4]**

5 In this question you must show detailed reasoning.

(a) Using the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, show that $\sinh 2x \equiv 2 \sinh x \cosh x$. **[2]**

(b) Solve the equation $15 \sinh x + 16 \cosh x - 6 \sinh 2x = 20$, giving all your answers in logarithmic form. **[5]**

6 The equation of the plane Π is $\mathbf{r} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$.

(a) Find the acute angle between Π and the plane with equation $\mathbf{r}.\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} = 4$. **[4]**

The point A has coordinates $(9, -7, 20)$.

The point F is the point of intersection between Π and the perpendicular from A to Π.

(b) Determine the coordinates of F. **[4]**

7 **In this question you must show detailed reasoning.**

(a) Show that

$$\sum_{r=1}^{n} \frac{5r+6}{r^3+r^2} = \frac{a}{n+1} + b + c\sum_{r=1}^{n} \frac{1}{r^2}$$

where a, b and c are integers whose values are to be determined. **[6]**

You are given that $\sum_{r=1}^{\infty} \frac{1}{r^2}$ exists and is equal to $\frac{1}{6}\pi^2$.

(b) Show that $\sum_{r=1}^{\infty} \frac{5r+6}{r^3+r^2}$ exists and is equal to $(\pi-1)(\pi+1)$. **[2]**

8 A surge in the current, I units, through an electrical component at a time, t seconds, is to be modelled. The surge starts when $t = 0$ and there is initially no current through the component. When the current has surged for 1 second it is measured as being 5 units. While the surge is occurring, I is modelled by the following differential equation.

$$(2t - t^2)\frac{\mathrm{d}I}{\mathrm{d}t} = (2t - t^2)^{\frac{3}{2}} - 2(t-1)I$$

(a) By using an integrating factor show that, according to the model, while the surge is occurring, I is given by $I = (2t - t^2)(\sin^{-1}(t-1) + 5)$. **[6]**

The surge lasts until there is again no current through the component.

(b) Determine the length of time that the surge lasts according to the model. **[2]**

(c) Determine, according to the model, the rate of increase of the current at the start of the surge. Give your answer in an exact form. **[3]**

9 A function is defined by $y = \mathrm{f}(t)$ where $\mathrm{f}(t) = \ln(1 + at)$ and a is a constant.

(a) By considering $\frac{\mathrm{d}y}{\mathrm{d}t}$, $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$, $\frac{\mathrm{d}^3y}{\mathrm{d}t^3}$ and $\frac{\mathrm{d}^4y}{\mathrm{d}t^4}$, make a conjecture for a general formula for $\frac{\mathrm{d}^ny}{\mathrm{d}t^n}$ in terms of n and a for any integer $n \geqslant 1$. **[3]**

(b) Use induction to prove the formula conjectured in part **(a)**. **[4]**

(c) In the case where $\mathrm{f}(t) = \ln(1 + 2t)$, find the rate at which the 6^{th} derivative of $\mathrm{f}(t)$ is varying when $t = \frac{3}{2}$. **[2]**

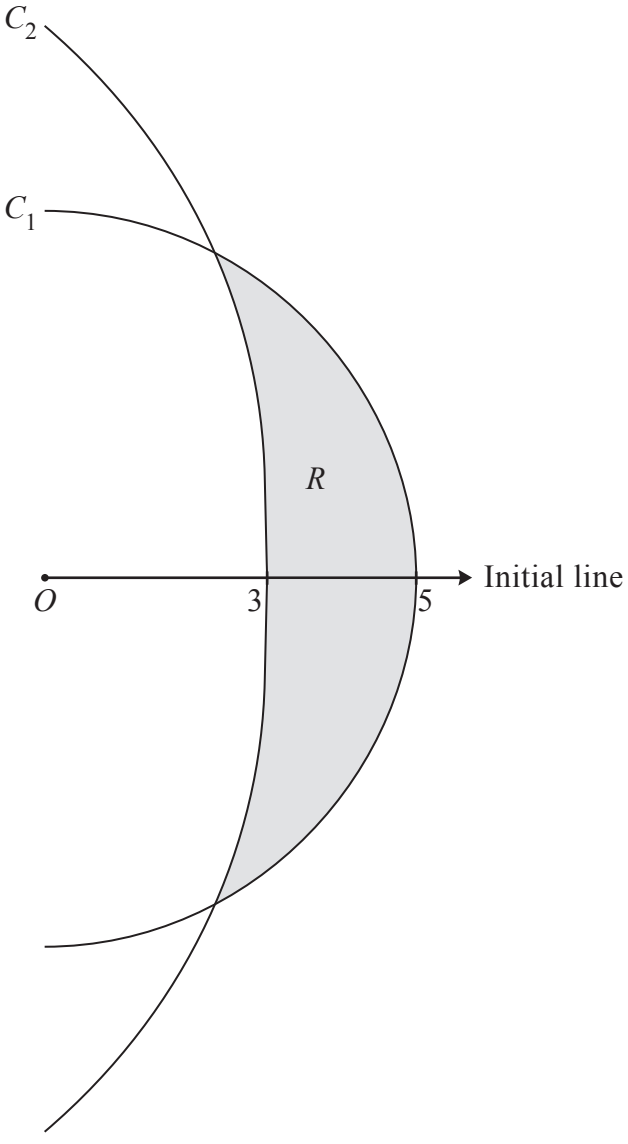
10 In this question you must show detailed reasoning.

A region, R, of the floor of an art gallery is to be painted for the purposes of an art installation. A suitable polar coordinate system is set up on the floor of the gallery with units in metres and radians. R is modelled as being the region enclosed by two curves, C_1 and C_2. The polar equations of C_1 and C_2 are

$C_1 : r = 5, \qquad -\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$

$C_2 : r = 3\cosh\theta, \qquad -\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$

Both curves are shown in the diagram, with R indicated.



The gallery must buy tins of paint to paint R. Each tin of paint can cover an area of $0.5\,\text{m}^2$.

Determine the smallest number of tins of paint that the gallery must buy in order to be able to paint R completely. **[7]**

END OF QUESTION PAPER

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