OCR
Oxford Cambridge and RSA

Friday 16 June 2023 – Afternoon

A Level Further Mathematics A

Y543/01 Mechanics

Time allowed: 1 hour 30 minutes

9976884382

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

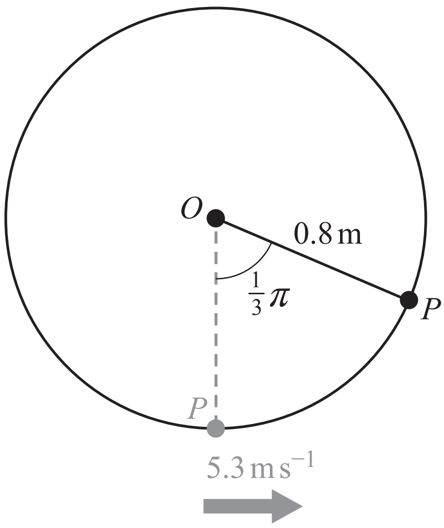
QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 One end of a light inextensible string of length 0.8 m is attached to a particle P of mass m kg. The other end of the string is attached to a fixed point O. Initially P hangs in equilibrium vertically below O. It is then projected horizontally with a speed of $5.3\,\mathrm{m\,s^{-1}}$ so that it moves in a vertical circular path with centre O (see diagram).



At a certain instant, P first reaches the point where the string makes an angle of $\frac{1}{3}\pi$ radians with the downward vertical through O.

(a) Show that at this instant the speed of P is $4.5\,\mathrm{m\,s^{-1}}$. **[3]**

(b) Find the magnitude and direction of the radial acceleration of P at this instant. **[3]**

(c) Find the magnitude of the tangential acceleration of P at this instant. **[2]**

2 Materials have a measurable property known as the Young's Modulus, E.

If a force is applied to one face of a block of the material then the material is stretched by a distance called the extension. Young's modulus is defined as the ratio $\frac{\text{Stress}}{\text{Strain}}$ where Stress is defined as the force per unit area and Strain is the ratio of the extension of the block to the length of the block.

(a) Show that Strain is a dimensionless quantity. **[1]**

(b) By considering the dimensions of both Stress and Strain determine the dimensions of E. **[2]**

It is suggested that the speed of sound in a material, c, depends only upon the value of Young's modulus for the material, E, the volume of the material, V, and the density (or mass per unit volume) of the material, ρ.

(c) Use dimensional analysis to suggest a formula for c in terms of E, V and ρ. **[5]**

(d) The speed of sound in a certain material is $500\,\text{m}\,\text{s}^{-1}$.

(i) Use your formula from part **(c)** to predict the speed of sound in the material if the value of Young's modulus is doubled but all other conditions are unchanged. **[1]**

(ii) With reference to your formula from part **(c)**, comment on the effect on the speed of sound in the material if the volume is doubled but all other conditions are unchanged. **[1]**

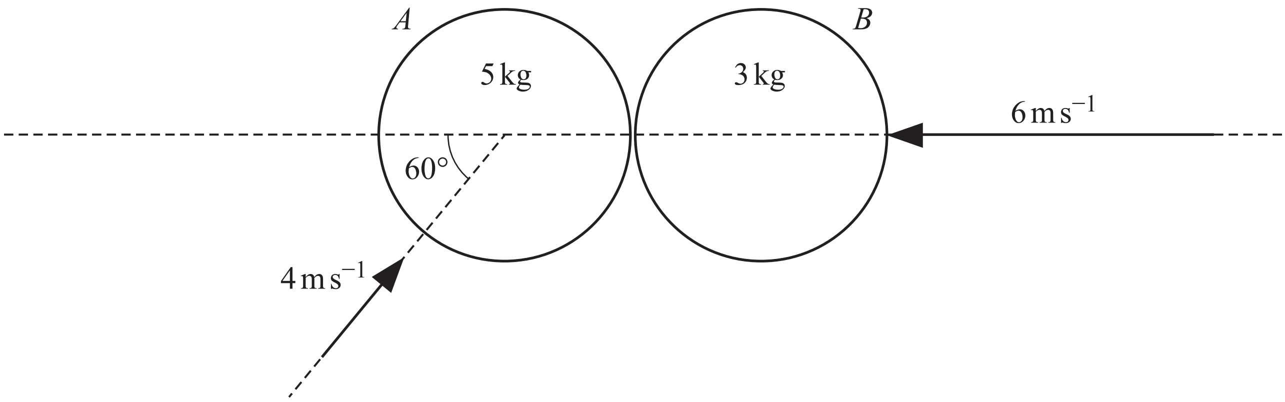
(e) Suggest **one** possible limitation caused by using dimensional analysis to set up the model in part **(c)**. **[1]**

3 Two smooth circular discs A and B are moving on a smooth horizontal plane when they collide. The mass of A is $5\,\text{kg}$ and the mass of B is $3\,\text{kg}$.

At the instant before they collide,

- the velocity of A is $4\,\text{m}\,\text{s}^{-1}$ at an angle of $60°$ to the line of centres,
- the velocity of B is $6\,\text{m}\,\text{s}^{-1}$ along the line of centres

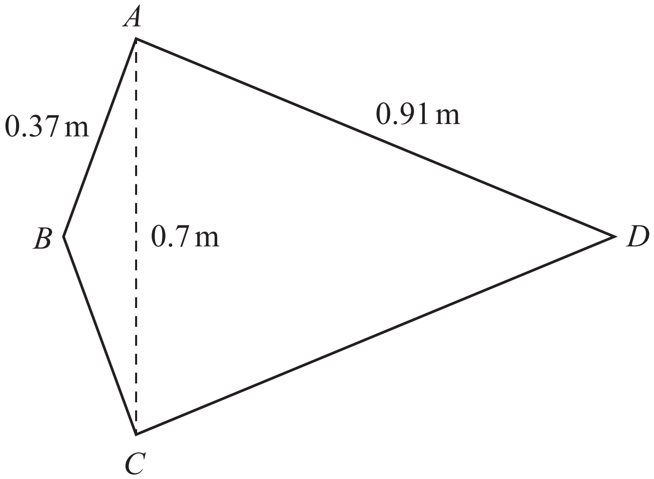
(see diagram).



The coefficient of restitution for collisions between the two discs is $\frac{3}{4}$.

Determine the angle that the velocity of A makes with the line of centres after the collision. **[7]**

4 $ABCD$ is a uniform lamina in the shape of a kite with $BA = BC = 0.37\,\text{m}$, $DA = DC = 0.91\,\text{m}$ and $AC = 0.7\,\text{m}$ (see diagram). The centre of mass of $ABCD$ is G.



(a) Explain why G lies on BD. **[1]**

(b) Show that the distance of G from B is $0.36\,\text{m}$. **[4]**

The lamina $ABCD$ is freely suspended from the point A.

(c) Determine the acute angle that CD makes with the horizontal, stating which of C or D is higher. **[4]**

5 A particle P of mass $2\,\text{kg}$ moves along the x-axis.

At time $t = 0$, P passes through the origin O with speed $3\,\text{m}\,\text{s}^{-1}$.

At time t seconds the displacement of P from O is $x\,\text{m}$ and the velocity of P is $v\,\text{m}\,\text{s}^{-1}$, where $t \geqslant 0$, $x \geqslant 0$ and $v \geqslant 0$.

While P is in motion the only force acting on P is a resistive force F of magnitude $(v^2 + 1)$ N acting in the negative x-direction.

(a) Find an expression for v in terms of x. **[5]**

(b) Determine the distance travelled by P while its speed drops from $3\,\text{m}\,\text{s}^{-1}$ to $2\,\text{m}\,\text{s}^{-1}$. **[2]**

Particle Q is identical to particle P. At a different time, Q is moving along the x-axis under the influence of a single constant resistive force of magnitude $1\,\text{N}$. When $t' = 0$, Q is at the origin and its speed is $3\,\text{m}\,\text{s}^{-1}$.

(c) By comparing the motion of P with the motion of Q explain why P must come to rest at some finite time when $t < 6$ with $x < 9$. **[3]**

(d) Sketch the velocity-time graph for P. You do not need to indicate any values on your sketch. **[1]**

(e) Determine the maximum displacement of P from O during P's motion. **[2]**

6 A particle P of mass $3\,\text{kg}$ is moving on a smooth horizontal surface under the influence of a variable horizontal force $\mathbf{F}\,\text{N}$. At time t seconds, where $t \geqslant 0$, the velocity of P, $\mathbf{v}\,\text{m}\,\text{s}^{-1}$, is given by

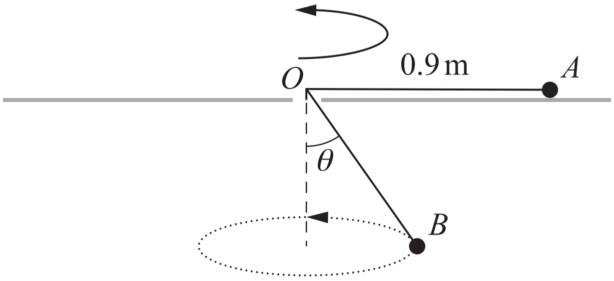
$$\mathbf{v} = (32\sinh(2t))\mathbf{i} + (32\cosh(2t) - 257)\mathbf{j}.$$

(a) **(i)** By considering kinetic energy, determine the work done by $\mathbf{F}$ over the interval $0 \leqslant t \leqslant \ln 2$. **[5]**

(ii) Explain the significance of the sign of the answer to part **(a)(i)**. **[1]**

(b) Determine the rate at which $\mathbf{F}$ is working at the instant when P is moving parallel to the $\mathbf{i}$-direction. **[6]**

7 Two particles A and B are connected by a light inextensible string of length 1.26 m. Particle A has a mass of 1.25 kg and moves on a smooth horizontal table in a circular path of radius 0.9 m and centre O. The string passes through a small smooth hole at O. Particle B has a mass of 2 kg and moves in a horizontal circle as shown in the diagram. The angle that the portion of string below the table makes with the downwards vertical through O is θ, where $\cos\theta = \frac{4}{5}$ (see diagram).



(a) Determine the angular speed of A and the angular speed of B. **[5]**

At the start of the motion, A, O and B all lie in the same vertical plane.

(b) Find the first subsequent time when A, O and B all lie in the same vertical plane. **[2]**

8 One end of a light elastic string of natural length 2.1 m and modulus of elasticity 4.8 N is attached to a particle, P, of mass 1.75 kg. The other end of the string is attached to a fixed point, O, which is on a rough inclined plane. The angle between the plane and the horizontal is θ where $\sin\theta = \frac{3}{5}$. The coefficient of friction between P and the plane is 0.732.

Particle P is placed on the plane at O and then projected down a line of greatest slope of the plane with an initial speed of $2.4\,\text{m s}^{-1}$.

Determine the distance that P has travelled from O at the instant when it first comes to rest. You can assume that during its motion P does not reach the bottom of the inclined plane. **[8]**

END OF QUESTION PAPER

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