# OCR
Oxford Cambridge and RSA

# Thursday 22 June 2023 – Afternoon

## A Level Further Mathematics B (MEI)

### Y434/01 Numerical Methods

**Time allowed: 1 hour 15 minutes**

**You must have:**
• the Printed Answer Booklet
• the Formulae Booklet for Further Mathematics B (MEI)
• a scientific or graphical calculator

**QP**

## INSTRUCTIONS
• Use black ink. You can use an HB pencil, but only for graphs and diagrams.
• Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
• Fill in the boxes on the front of the Printed Answer Booklet.
• Answer **all** the questions.
• Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
• Give your final answers to a degree of accuracy that is appropriate to the context.
• Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

## INFORMATION
• The total mark for this paper is **60**.
• The marks for each question are shown in brackets **[ ]**.
• This document has **12** pages.

## ADVICE
• Read each question carefully before you start your answer.

**1** You are given that $(x_1, y_1) = (0.9, 2.3)$ and $(x_2, y_2) = (1.1, 2.7)$.

The values of $x_1$ and $x_2$ have been **rounded** to **1** decimal place.

**(a)** Determine the range of possible values of $x_2 - x_1$. **[2]**

The values of $y_1$ and $y_2$ have been **chopped** to **1** decimal place.

**(b)** Determine the range of possible values of $y_2 - y_1$. **[2]**

You are given that $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

**(c)** Determine the range of possible values of $m$. **[2]**

**(d)** Explain why your answer to part **(c)** is much larger than your answer to part **(a)** and your answer to part **(b)**. **[1]**

**2** A car tyre has a slow puncture. Initially the tyre is inflated to a pressure of 34.5 psi. The pressure is checked after 3 days and then again after 5 days. The time $t$ in days and the pressure, $P$ psi, are shown in the table below. You are given that the pressure in a car tyre is measured in pounds per square inch (psi).

| $t$ | 0 | 3 | 5 |
|---|---|---|---|
| $P$ | 34.5 | 29.4 | 27.0 |

The owner of the car believes the relationship between $P$ and $t$ may be modelled by a polynomial.

**(a)** Explain why it is not possible to use Newton's forward difference interpolation method for these data. **[1]**

**(b)** Use Lagrange's form of the interpolating polynomial to find an interpolating polynomial of degree 2 for these data. **[4]**
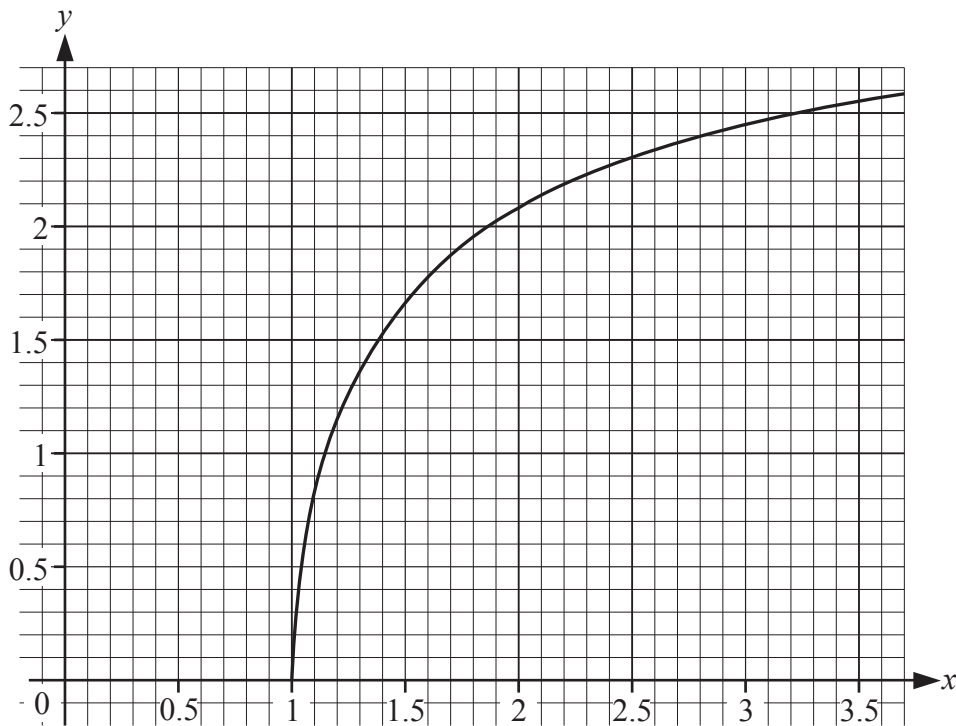
The car owner uses the polynomial found in part **(b)** to model the relationship between $P$ and $t$.

Subsequently it is found that when $t = 6$, $P = 26.0$ and when $t = 10$, $P = 24.4$.

**(c)** Determine whether the owner's model is a good fit for these data. **[2]**

**(d)** Explain why the model would not be suitable in the long term. **[1]**

**3**    The diagram shows the graph of $y = f(x)$ for values of $x$ from 1 to 3.5.



The table shows some values of $x$ and the associated values of $y$.

| $x$ | 1.5 | 2 | 2.5 |
|---|---|---|---|
| $y$ | 1.682137 | 2.094395 | 2.318559 |

**(a)** Use the forward difference method to calculate an approximation to $\dfrac{dy}{dx}$ at $x = 2$.    **[2]**

**(b)** Use the central difference method to calculate an approximation to $\dfrac{dy}{dx}$ at $x = 2$.    **[2]**

**(c)** On the copy of the diagram in the Printed Answer Booklet, show how the central difference method gives the approximation to $\dfrac{dy}{dx}$ at $x = 2$ which was found in part **(b)**.    **[1]**

**(d)** Explain whether your answer to part **(a)** or your answer to part **(b)** is likely to give a better approximation to $\dfrac{dy}{dx}$ at $x = 2$.    **[1]**

**4**   A spreadsheet is used to approximate $\int_a^b f(x)\,dx$ using the midpoint rule with 1 strip.

The output is shown in the table below.

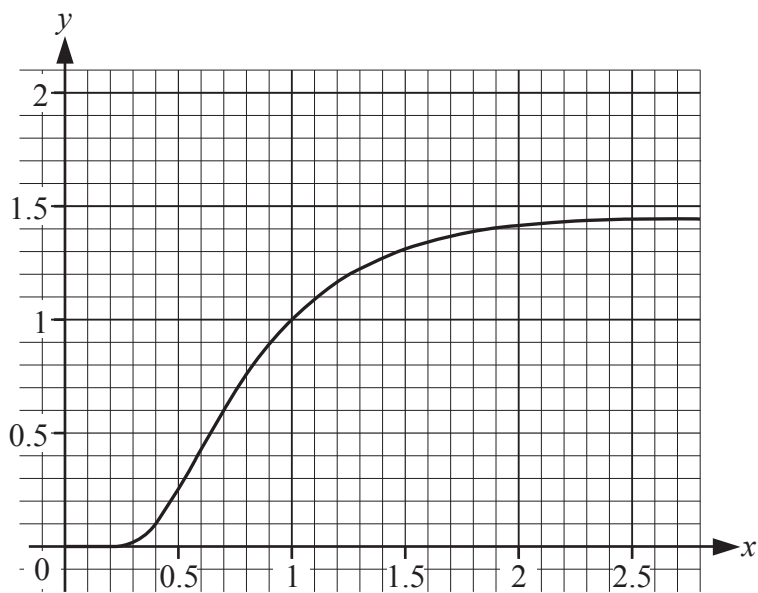|   | B | C | D |
|---|---|---|---|
| 3 | $x$ | $f(x)$ | $M_1$ |
| 4 | 1.5 | 1.3103707 | 0.65518535 |

The formula in cell C4 is  $=B4\wedge(1/B4)$  .

The formula in cell D4 is  $=0.5*C4$  .

**(a)**  Write the integral in standard mathematical notation.    **[3]**

A graph of $y = f(x)$ is included in the diagram below.
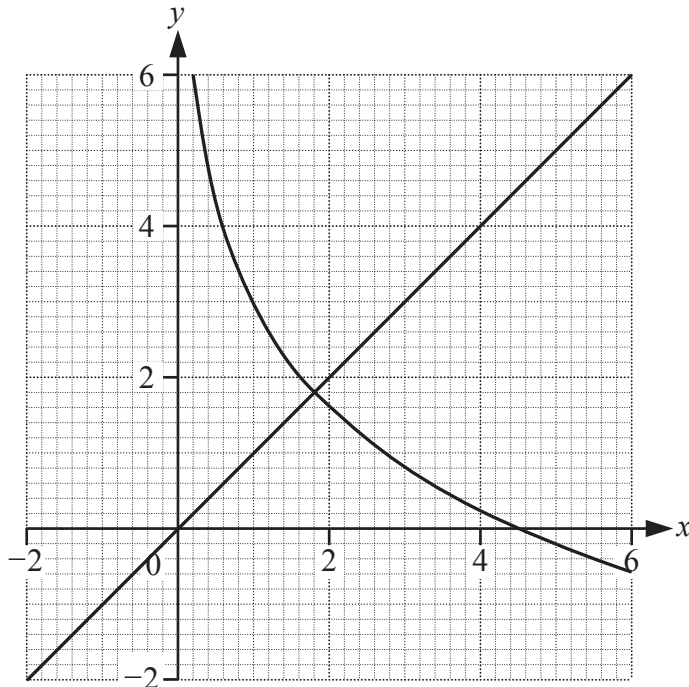


**(b)**  Explain whether 0.65518535 is an over-estimate or an under-estimate of $\int_a^b f(x)\,dx$.    **[1]**

**5** The equation $3 - 2\ln x - x = 0$ has a root near $x = 1.8$.

A student proposes to use the iterative formula $x_{n+1} = g(x_n) = 3 - 2\ln x_n$ to find this root.

The diagram shows the graphs of $y = x$ and $y = g(x)$ for values of $x$ from $-2$ to 6.



**(a)** With reference to the graph, explain why it might not be possible to use the student's iterative formula to find the root near $x = 1.8$. **[1]**

**(b)** Use the relaxed iteration $x_{n+1} = \lambda g(x_n) + (1-\lambda)x_n$, with $\lambda = 0.475$ and $x_0 = 2$, to determine the root correct to **6** decimal places. **[3]**

A student uses the same relaxed iteration with the same starting value. Some analysis of the iterates is carried out using a spreadsheet, which is shown in the table below.

| $r$ | difference | ratio |
|---|---|---|
| 0 | | |
| 1 | $-0.1834898$ | |
| 2 | $-0.0049137$ | 0.02678 |
| 3 | $-6.44\text{E-}06$ | 0.00131 |
| 4 | $-3.862\text{E-}09$ | 0.0006 |
| 5 | $-2.313\text{E-}12$ | 0.0006 |

**(c)** Explain what the analysis tells you about the order of convergence of this sequence of approximations. **[2]**

**6** **(a)** **(i)** Calculate the relative error when $\pi$ is **chopped** to **2** decimal places in approximating $\pi^2 + 2$. **[2]**

    **(ii)** **Without** doing any calculation, explain whether the relative error would be the same when $\pi$ is **chopped** to **2** decimal places when approximating $(\pi + 2)^2$. **[1]**

The table shows some spreadsheet output. The values of $x$ in column A are exact.

| | A | B | C |
|---|---|---|---|
| 1 | $x$ | $10^x$ | $\log_{10} 10^x$ |
| 2 | 1E-12 | 1 | 1.00001E-12 |
| 3 | 1E-11 | 1 | 9.99998E-12 |

The formula in cell B2 is     $=10\text{^}A2$     .

This has been copied down to cell B3.

The formula in cell C2 is     $=\text{LOG}(B2)$     .

This formula has been copied down to cell C3.

**(b)** **(i)** Write the value displayed in cell C2 in standard mathematical notation. **[1]**

    **(ii)** Explain why the values in cells C2 and C3 are neither zero nor the same as the values in cells A2 and A3 respectively. **[2]**

**7** The value of a function, $y = f(x)$, and its gradient function, $\dfrac{dy}{dx}$, when $x = 2$, is given in **Table 7.1**.

**Table 7.1**

| $x$ | $f(x)$ | $\dfrac{dy}{dx}$ |
|---|---|---|
| 2 | 6 | $-2.8$ |

**(a)** Determine the approximate value of the error when f(2) is used to estimate f(2.03). **[2]**

The Newton-Raphson method is used to find a sequence of approximations to a root, $\alpha$, of the equation $f(x) = 0$. The spreadsheet output showing the iterates, together with some further analysis, is shown in **Table 7.2**.

**Table 7.2**

| | A | B | C | D |
|---|---|---|---|---|
| 1 | $r$ | $x_r$ | difference | ratio |
| 2 | 0 | 12 | | |
| 3 | 1 | $-13.1165572$ | $-25.1165572$ | |
| 4 | 2 | 1.76283279 | 14.87939004 | $-0.5924136$ |
| 5 | 3 | 2.18052157 | 0.41768878 | 0.02807163 |
| 6 | 4 | 2.182419024 | 0.001897454 | 0.00454275 |
| 7 | 5 | 2.182419066 | 4.13985E-08 | 2.1818E-05 |

**(b) (i)** Explain what the values in column D tell you about the order of convergence of this sequence of approximations. **[2]**

**(ii)** **Without** doing any further calculation, state the value of $\alpha$ as accurately as you can, justifying the precision quoted. **[2]**

**8** The graph of $y = 0.2\cosh x - 0.4x$ for values of $x$ from 0 to 3.32 is shown on the graph below.
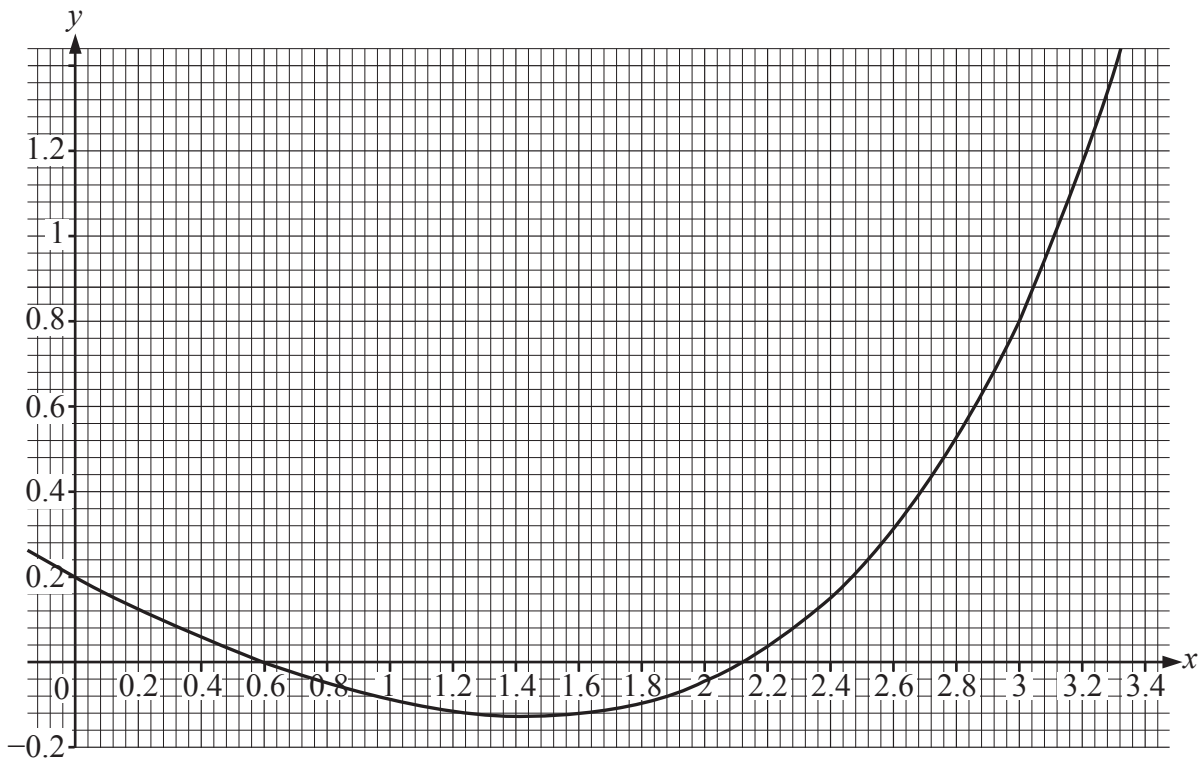


The equation $0.2\cosh x - 0.4x = 0$ has two roots, $\alpha$ and $\beta$ where $\alpha < \beta$, in the interval $0 < x < 3$. The secant method with $x_0 = 1$ and $x_1 = 2$ is to be used to find $\beta$.

**(a)** On the copy of the graph in the Printed Answer Booklet, show how the secant method works with these two values of $x$ to obtain an improved approximation to $\beta$. **[1]**

The spreadsheet output in the table below shows the result of applying the secant method with $x_0 = 1$ and $x_1 = 2$.

| | I | J | K | L | M |
|---|---|---|---|---|---|
| 2 | $r$ | $x_r$ | $f(x_r)$ | $x_{r+1}$ | $f(x_{r+1})$ |
| 3 | 0 | 1 | −0.0914 | 2 | −0.0476 |
| 4 | 1 | 2 | −0.0476 | 3.08529 | 0.95784 |
| 5 | 2 | 3.08529 | 0.95784 | 2.05134 | −0.0298 |
| 6 | 3 | 2.05134 | −0.0298 | 2.08259 | −0.0181 |
| 7 | 4 | 2.08259 | −0.0181 | 2.13042 | 0.00155 |
| 8 | 5 | 2.13042 | 0.00155 | 2.12664 | −7E-05 |

**(b)** Write down a suitable cell formula for cell J4. **[1]**

**(c)** Write down a suitable cell formula for cell L4. **[2]**

**(d)** Write down the most accurate approximation to $\beta$ which is displayed in the table. **[1]**

**(e)** Determine whether your answer to part **(d)** is correct to **5** decimal places. You should **not** calculate any more iterates. **[2]**

**(f)** It is decided to use the secant method with starting values $x_0 = 1$ and $x_1 = a$, where $a > 1$, to find $\alpha$. State a suitable value for $a$. **[1]**

**9** The trapezium rule is used to calculate 3 approximations to $\int_0^1 \sqrt[3]{\sinh(x)}\,dx$ with 1, 2 and 4 strips respectively. The results are shown in **Table 9.1**.

**Table 9.1**

| $n$ | $\mathrm{T}_n$ |
|-----|----------------|
| 1 | 0.52764369 |
| 2 | 0.66617652 |
| 4 | 0.72534275 |

**(a)** **Use these results** to determine **two** approximations to $\int_0^1 \sqrt[3]{\sinh(x)}\,dx$ using Simpson's rule. **[2]**

**(b)** Use your answers to part **(a)** to state the value of $\int_0^1 \sqrt[3]{\sinh(x)}\,dx$ as accurately as you can, justifying the precision quoted. **[1]**

**Table 9.2** shows some further approximations found using the trapezium rule, together with some analysis of these approximations.

**Table 9.2**

| $n$ | $\mathrm{T}_n$ | difference | ratio |
|-----|----------------|------------|-------|
| 1 | 0.5276437 | | |
| 2 | 0.6661765 | 0.138533 | |
| 4 | 0.7253427 | 0.059166 | 0.42709 |
| 8 | 0.7498821 | 0.024539 | 0.41475 |
| 16 | 0.7598858 | 0.010004 | 0.40766 |
| 32 | 0.7639221 | 0.004036 | 0.40348 |
| 64 | 0.7655404 | 0.001618 | 0.40095 |

**(c)** Explain what can be deduced about the order of the method in this case. **[2]**

**(d)** Use extrapolation to obtain the value of $\int_0^1 \sqrt[3]{\sinh(x)}\,dx$ as accurately as you can, justifying the precision quoted. **[4]**

**END OF QUESTION PAPER**

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