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Examiners' report

FURTHER MATHEMATICS B (MEI)

H635

For first teaching in 2017

Y412/01 Summer 2023 series

Version 1

ocr.org.uk/maths

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y412/01 series overview

This is a calculator paper in which situations are modelled by discrete random variables; the suitability of models is tested using chi-squared tests. Bivariate data are investigated, with tests for correlation and association, and modelling using regression.

To do well on this paper, candidates must demonstrate their understanding of the contents of the specification. Candidates should be able to choose and apply appropriate models and be able to provide structured answers to questions involving hypothesis tests. Candidates should be aware that questions asking for 'detailed reasoning' to be given, or containing command words such as 'show that' and 'determine', indicate that final answers must be supported by relevant working.

In general, the overall performance was good this year. Candidates provided detail in solutions, in particular in hypothesis tests and in lengthier calculations. They also specified which probability distribution, with parameters, that they were using to answer the questions.

Candidates could have done better with the questions asking for a justification for using a particular probability model based on the information provided.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• showed good knowledge of probability models • stated the distribution used for probability calculations • provided all relevant detail and appropriately worded conclusions in hypothesis tests • selected appropriate probability distributions to suit the situation • gave clear, unambiguous comments when required.	• showed limited knowledge of probability models and the conditions under which they may be applied • showed limited ability to choose an appropriate probability calculation • showed limited understanding of hypothesis testing • showed limited ability to perform routine calculations such as finding E(X) and Var(X) for a given distribution, or calculating the value of Pearson's product moment correlation coefficient.

Question 1 (a)

1 Ryan has 6 one-pound coins and 4 two-pound coins. Ryan decides to select 3 of these coins at random to donate to a charity. The total value, in pounds, of these 3 coins is denoted by the random variable X.

(a) Show that $\mathrm{P}(X=3)=\frac{1}{6}$. **[2]**

Candidates were required to provide an appropriate calculation leading to the given answer. Most provided a convincing solution.

Question 1 (b)

The table below shows the probability distribution of X.

r	3	4	5	6
$\mathrm{P}(X=r)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

(b) Draw a graph to illustrate the distribution. **[2]**

Most candidates drew an appropriate line diagram. Some candidates did not fully label axes or provide a scale for the axis representing probability.

Question 1 (c)

(c) **In this question you must show detailed reasoning.**

Find each of the following.

- $\mathrm{E}(X)$
- $\mathrm{Var}(X)$ **[5]**

Most candidates found this question straightforward.

Question 1 (d)

Ryan's friend Sasha decides to give the same amount as Ryan does to the charity plus an extra three pounds. The random variable Y represents the **total** amount of money, in pounds, given by Ryan and Sasha.

(d) Determine each of the following.

- $\mathrm{E}(Y)$
- $\mathrm{Var}(Y)$ **[3]**

Having obtained correct answers to Question 1 (c), many candidates did not fully understand what was required here, despite the use of bold script to draw attention to the key word, 'total'. Many candidates treated Y as the amount given by just Sasha rather than the total amount.

The instruction 'determine' indicated that evidence of the method should be included in the response– many candidates obliged by showing sufficient detail of the calculations used.

Some of those who realised that $Y = 2X + 3$ used $\mathrm{Var}(Y) = 2 \times \mathrm{Var}(X)$ rather than $2^2 \times \mathrm{Var}(X)$.

Question 2 (a)

2 A group of friends live by the sea. Each day they look out to sea in the hope of seeing a dolphin. The probability that they see a dolphin on any day is 0.15. You should assume that this probability is not affected by whether or not they see a dolphin on any other day.

(a) Explain why you can use a geometric distribution to model the number of days that it takes for them to first see a dolphin. **[1]**

Most candidates managed to provide suitable comments though many did not pick out the feature of the geometric distribution that distinguishes it from other distributions.

Question 2 (b)

(b) Find the probability that they see a dolphin for the first time on the fifth day. **[1]**

Most candidates answered this successfully.

Question 2 (c)

(c) Find the probability that they do not see a dolphin for at least 10 days. **[1]**

Many candidates did not interpret this question correctly.

Misconception

Many candidates found the probability of taking at least ten days to see a dolphin for the first time rather than at least eleven days. The number of trials is up to (and including) the first success being counted.

Exemplar 1

2(c)	$P(x \geq 10) = 1 - P(x \leq 9)$
	$\doteq 1 - 0.76838305$
	$= 0.232$

This candidate found the probability of taking at least ten days to see a dolphin for the first time. Here, X represents the number of days until a dolphin is seen for the first time. P(X > 10) or P(X ≥ 11) was required so no mark could be given.

Question 2 (d)

(d) Determine the mean and the variance of the number of days that it takes for them to see a dolphin. **[3]**

Most candidates scored all 3 marks.

Question 3 (a)

3 At a pottery which manufactures mugs, it is known that 5% of mugs are faulty. The mugs are produced in batches of 20. Faults are modelled as occurring randomly and independently. The number of faulty mugs in a batch is denoted by the random variable X.

(a) Determine $P(X \geqslant 2)$. **[2]**

Candidates were required to use the binomial distribution to determine this probability. Most did this successfully by stating that X ~ B(20, 0.05) before writing down their probability.

Question 3 (b)

(b) Find Var(X). **[1]**

Some candidates lost this mark through using a Poisson approximation and stating the variance to be 1.

Question 3 (c)

Independently of the mugs, the pottery also manufactures cups, and it is known that 7% of cups are faulty. The cups are produced in batches of 30. Faults are modelled as occurring randomly and independently. The number of faulty cups in a batch is denoted by the random variable Y.

(c) Determine the standard deviation of $X + Y$. **[3]**

This question was not particularly well answered.

Misconception

?

A common mistake was to use $\sqrt{\mathrm{Var}(X+Y)} = \sqrt{\mathrm{Var}(X)} + \sqrt{\mathrm{Var}(Y)}$.

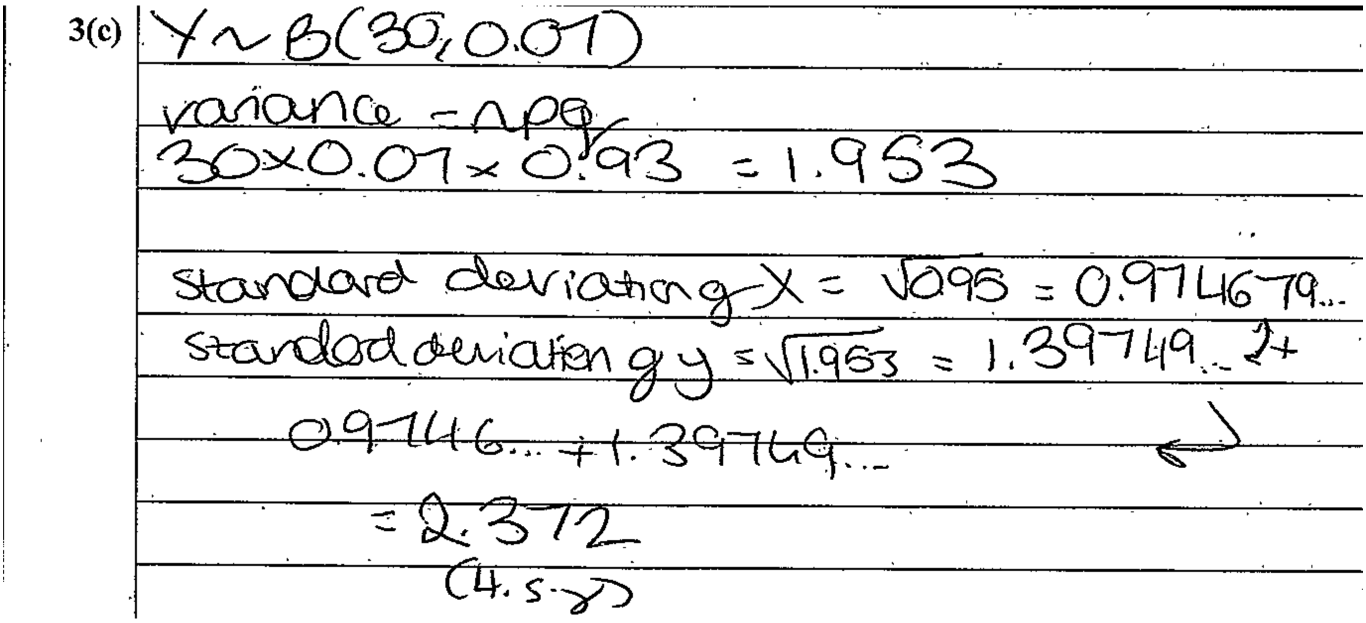
OCR support

i

For support in teaching discrete random variables, our 'Teacher Delivery Guide in Discrete Random Variables' can be downloaded from Teach Cambridge:

Delivery guide: 6.02 Further statistics: Discrete random variables

Exemplar 2


3(c) $Y \sim B(30, 0.07)$
variance $= npq$
$30 \times 0.07 \times 0.93 = 1.953$

standard deviation of X = $\sqrt{0.95} = 0.974679...$
standard deviation of y $= \sqrt{1.953} = 1.39749...$ +
$0.9746... + 1.39749...$
$= 2.372$
(4.s.f)

This candidate's working, though detailed, has used $\sqrt{\mathrm{Var}(X+Y)} = \sqrt{\mathrm{Var}(X)} + \sqrt{\mathrm{Var}(Y)}$ which is not the correct method. A method mark could be given for working out the variance of *Y* correctly though.

Question 3 (d)

When 10 batches of cups have been produced, a sample of 15 cups is tested to ensure that the handles of the cups are properly attached.

(d) Explain why it might not be sensible to select a sample of 15 cups from the same batch. **[1]**

A variety of comments were seen. Those that suggested that one batch may not be representative, usually led to the mark being given.

Question 4 (a)

4 At a parcel delivery company it is known that the probability that a parcel is delivered to the wrong address is 0.0005. On a particular day, 15 000 parcels are delivered. The number of parcels delivered to the wrong address is denoted by the random variable *X*.

(a) Explain why the binomial distribution and the Poisson distribution could both be suitable models for the distribution of *X*. **[3]**

Most candidates could recall the vocabulary that was required to provide the necessary explanation but many did not use it appropriately. Inappropriate references to the probability (of incorrect parcel delivery) being independent were seen regularly. Some candidates listed appropriate conditions without reference to the context.

Question 4 (b)

(b) Use a Poisson distribution to find each of the following.

- $P(X = 5)$
- $P(X \geqslant 8)$ **[3]**

Most candidates found this to be straightforward.

Question 4 (c) (i)

You are given that 15 000 parcels are delivered each day in a 5-day working week.

(c) **(i)** Determine the probability that at least 40 parcels are delivered to the wrong address during the week. **[2]**

In questions such as this, containing the command word, 'determine', candidates should state the distribution used. In this case, either Poisson(37.5) or B(75000, 0.0005). Most candidates picked up both marks here.

Question 4 (c) (ii)

(ii) Determine the probability that at least 8 parcels are delivered to the wrong address on each of the 5 days in the week. **[2]**

Most candidates earned both marks. Some lost the accuracy mark either through premature rounding of their value of $P(X \geq 8)$ or through not actually calculating what they had written; typically, writing 0.4754^5 as the intended method but calculating $(1 - 0.4754)^5$ to give 0.0397.

Question 5 (a)

5 Two practice GCSE examinations in mathematics are given to all of the students in a large year group. A teacher wants to check whether there is a positive relationship between the marks obtained by the students in the two examinations. She selects a random sample of 20 students. Summary data for the marks obtained in the first and second practice examinations, x and y respectively, are as follows.

$$\sum x = 565 \quad \sum y = 724 \quad \sum x^2 = 17\,103 \quad \sum y^2 = 29\,286 \quad \sum xy = 21\,635$$

The teacher decides to carry out a hypothesis test based on Pearson's product moment correlation coefficient.

(a) **In this question you must show detailed reasoning.**

Calculate the value of Pearson's product moment correlation coefficient. **[4]**

Most candidates earned all 4 marks here. Some errors in the final calculation were seen.

Question 5 (b)

(b) Carry out the test at the 5% significance level. **[5]**

Most candidates provided suitable hypotheses. In defining ρ, many candidates did not refer to population or the context of the question. Most stated the correct critical value. To accompany the comparison of the test statistic and critical value, candidates were required to state their conclusion in terms of whether to reject the null hypothesis or not to reject the null hypothesis; most did this successfully. The final conclusion needed to be in context, non-assertive and refer to the alternative hypothesis. Most candidates adopted this approach.

Question 5 (c)

(c) Given that the teacher did not draw a scatter diagram before carrying out the test, comment on the validity of the test. **[1]**

Most candidates realised that certain modelling assumptions were required and provided a suitable comment about underlying bivariate Normality. Many candidates showed a lack of understanding that the bivariate Normality applies to the population from which the data is drawn and not to the data directly.

Exemplar 3

5(c) It may be difficult to determine whether the data is bivariate normal as they cannot determine whether the shape is roughly elliptical. Therefore the test may not be valid

> This candidate did not acknowledge the need for the data to be drawn from a bivariate Normal population. This response was not given a mark.

Question 6 (a) (i)

6 An eight-sided dice has its faces numbered 1, 2, …, 8.

(a) In this part of the question you should assume that the dice is fair.

(i) State the probability that, when the dice is rolled once, the score is at least 6. **[1]**

> Most candidates picked up this mark. Several incorrectly responded $\frac{1}{8}$, perhaps having not seen 'at least' in the question.

Question 6 (a) (ii)

(ii) Show that the probability that the score is within 2 standard deviations of its mean is 1. **[4]**

> Most candidates were given the first 2 marks for stating the mean and variance. Many went on to pick up the remaining marks. Some candidates were unsure of the meaning of the requirement of the question and so did not score the last 2 marks. Some candidates provided the necessary values that were 2 standard deviations from the mean but did not explain fully enough to be given the final mark.

Question 6 (b) (i)

(b) A student thinks that the dice may be biased. To investigate this, the student decides to roll the dice 80 times and then carry out a χ^2 goodness of fit test of a uniform distribution. The spreadsheet below shows the data for the test, where some of the values have been deliberately omitted.

	A	B	C	D
1	Score	Observed frequency	Expected frequency	Chi-squared contribution
2	1	14	10	1.6
3	2	4	10	3.6
4	3	10	10	0
5	4	15	10	
6	5	6	10	1.6
7	6	11	10	0.1
8	7	7	10	0.9
9	8		10	0.9

(i) Explain why all of the expected frequencies are equal to 10. **[1]**

Most candidates managed to provide a suitable explanation to justify the expected frequencies all being 10.

Question 6 (b) (ii)

(ii) Determine the missing values in each of the following cells.

- B9
- D5 **[3]**

Few candidates lost marks here.

Question 6 (b) (iii)

(iii) **In this question you must show detailed reasoning.**

Carry out the χ^2 test at the 5% significance level. **[6]**

Most candidates applied the approach to hypothesis tests as outlined in Question 5 (b). Mistakes with regard to the number of degrees of freedom were common. In general, this question was well answered.

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