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Examiners' report

FURTHER MATHEMATICS B (MEI)

H635

For first teaching in 2017

Y411/01 Summer 2023 series

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y411/01 series overview

Candidates found this to be a more accessible paper than those from past years, with many producing good responses to most parts of the first four questions. They were able to demonstrate a sound understanding of a range of topics. Some parts, such as Questions 4 (d), 5 (d) and 6 (c) caused difficulties for almost all the candidates.

Assessment for learning

Candidates should be encouraged to draw diagrams, set out their work clearly, and explain what they are doing (for example, 'Resolving vertically', 'Taking moments about A', 'By the conservation of momentum/energy', and so on).

When the answer is wrong, or is given on the question paper, marks can only be given when the working is clear.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• attempted all parts of all the questions • set out their work clearly and in a logical order • explained where their equations came from • drew diagrams • produced substantially correct responses to Questions 1 to 4, and to some parts of Questions 5 and 6.	• left many parts unanswered, particularly towards the end of the paper • presented disorganised working which was hard to follow • did not explain what they were doing.

Question 1 (a)

1 **Throughout all parts of this question**, the resistance to the motion of a car has magnitude kv^2 N, where $v\,\text{m s}^{-1}$ is the speed of the car and k is a constant.

At first, the car travels along a straight horizontal road with constant speed $20\,\text{m s}^{-1}$. The power developed by the car at this speed is 5000 W.

(a) Show that $k = \frac{5}{8}$. **[3]**

Almost all candidates knew the relationship between power, speed and driving force, and were able to use this to obtain the given answer.

Question 1 (b)

(b) Find the power the car must develop in order to maintain a constant speed of $28\,\text{m s}^{-1}$ when travelling along the same horizontal road. **[1]**

This part was also very well answered. The only common error was taking the resistance (and hence the driving force) to be the same as in part (a).

Question 1 (c)

The car climbs a hill which is inclined at an angle of 2° to the horizontal. The power developed by the car is 13 000 W, and the car has a constant speed of $20\,\text{m s}^{-1}$.

(c) Determine the mass of the car. **[3]**

Most candidates answered this correctly, by resolving the weight and considering forces parallel to the slope. Some used the mass instead of the weight, and many omitted the resistance from their calculation.

Question 2 (a)

2 A ball P of mass m kg is held at a height of 12.8 m above a horizontal floor. P is released from rest and rebounds from the floor. After the first bounce, P reaches a maximum height of 5 m above the floor.

Two models, A and B, are suggested for the motion of P.

Model A assumes that air resistance may be neglected.

(a) Determine, according to model A, the coefficient of restitution between P and the floor. **[3]**

Most candidates found the speed just before hitting the ground and the speed just afterwards, and hence obtained the coefficient of restitution. Others quoted a formula such as $h_2 = e^2h_1$ which is acceptable as the answer is not given.

Question 2 (b)

Model B assumes that the collision between P and the floor is perfectly elastic, but that work is done against air resistance at a constant rate of E joules per metre.

(b) Show that, according to model B, $E = \frac{39}{89}mg$. **[3]**

This was a less familiar situation, and some candidates omitted it altogether. There were many efficient solutions, equating the loss of potential energy (12.8 – 5)mg to the work done against air resistance (12.8 + 5)E, as in the following exemplar.

Exemplar 1

KE ~~At~~ Work Done = 17.8E

$17.8E = \sout{\tfrac{1}{2}m(v^2 - u^2)}$

GPE loss = $mg \times (12.8 - 5)$

$\frac{39}{5} mg = 17.8E$

$E = \frac{39}{89} mg$.

The candidate has made it clear where the terms $17.8E$ and $\frac{39}{5}mg$ come from, but did not refer to the collision at all. This method only works because the collision is given to be perfectly elastic so that no energy is lost during the collision. Candidates were not expected to mention this when considering the complete bounce, and very few actually did so.

Other candidates applied the work-energy principle to the two stages (falling and rising); they needed to equate the speeds just before and just after the collision (because it is perfectly elastic).

The speeds just before and just after the collision calculated in part (a) quite often appeared in work-energy equations intended for model B, thus mixing the two models.

Question 2 (c)

(c) Show that both models predict that P will attain the same maximum height after the second bounce. **[4]**

Most candidates found the height predicted by model A correctly. Those who had successfully answered part (b) were usually able to adapt their argument to find the height predicted by model B.

Question 3 (a)

3 The time period T of a satellite in circular orbit around a planet satisfies the equation

$$GMT^2 = 4\pi^2 R^3,$$

where

- G is the universal gravitational constant,
- M is the mass of the planet,
- R is the radius of the orbital circle.

(a) Find the dimensions of G. **[2]**

Most candidates used the given equation to find the dimensions of *G* correctly. Some assumed that *G* was an acceleration.

Question 3 (b)

A student suggests the following formula to model the approach speed between two orbiting bodies.

$$v = kG^{\alpha}c^{\beta}r^{\gamma}m_1 m_2(m_1 + m_2),$$

where

- v is the approach speed of the two bodies,
- k is a dimensionless constant,
- c is the speed of light,
- r is the distance between the two bodies,
- m_1 and m_2 are the masses of the bodies.

(b) Use dimensional analysis to determine the values of α, β and γ. **[5]**

The method of using dimensions to find unknown indices was very well understood, although there were often errors in setting up the dimensions equation, and in solving the resulting equations for α, β and γ. The expression $m_1m_2(m_1 + m_2)$ was sometimes ignored, and often given dimensions M^n with n different from the correct value 3.

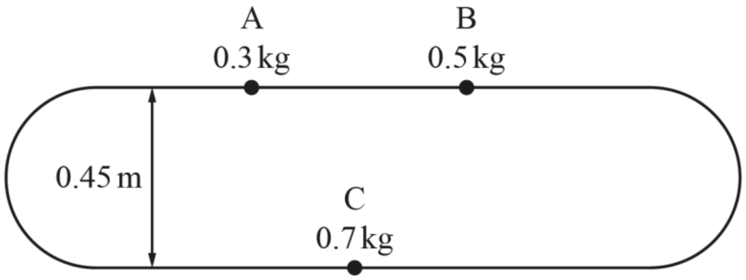
Question 3 (c)

(c) Calculate, according to the student's model, how many times greater the approach speed is between a pair of stars which are 6.13 light-years apart and the same pair of stars if they were 8.64 light-years apart. (A light-year is a unit of distance.) **[2]**

Candidates who realised that it was only necessary to consider the ratio of the distances quite often answered this correctly. However, very many candidates wrote nothing of merit, or omitted this part.

Question 4 (a)

4 The diagram shows three beads, A, B and C, of masses 0.3 kg, 0.5 kg and 0.7 kg respectively, threaded onto a smooth wire circuit consisting of two straight and two semi-circular sections. The circuit occupies a **vertical** plane, with the two straight sections horizontal and the upper section 0.45 m directly above the lower section.



Initially, the beads are at rest. A and B are each given an impulse so that they move towards each other, A with a speed of $8\,\mathrm{m\,s^{-1}}$ and B with a speed of $1.6\,\mathrm{m\,s^{-1}}$. In the subsequent collision between A and B, A is brought to rest.

(a) Show that the coefficient of restitution between A and B is $\frac{1}{3}$. **[3]**

Most candidates answered this correctly, by first using the conservation of momentum to find the final speed of B. A common error was writing the initial momentum as $0.3 \times 8 + 0.5 \times 1.6$ instead of $0.3 \times 8 - 0.5 \times 1.6$ (the beads A and B are moving in opposite directions). Also, when finding the coefficient of restitution, the relative speed of impact was often given as 6.4 instead of 9.6.

Question 4 (b)

Bead B next collides with C.

(b) Show that the speed of B before this collision is $4.37\,\mathrm{m\,s^{-1}}$, correct to **3** significant figures. **[2]**

This needed an application of the conservation of energy, starting with the speed found in part (a), and most candidates obtained the given answer convincingly. Some used formulae for constant acceleration, which is not valid here as the acceleration is not constant.

Question 4 (c)

In this collision between B and C, B is brought to rest.

(c) Determine whether C next collides with A or with B. **[3]**

Most candidates used the conservation of momentum to find the speed of C correctly. Some did not appreciate that further calculation was required, simply remarking that C was moving round the wire towards A and so would collide with A next. Others used a variety of methods to show that C does in fact reach the upper section. Most considered the kinetic energy of C and compared it with the potential energy needed, and some calculated the maximum height that C could reach, comparing it to 0.45 m.

Question 4 (d)

(d) Explain why, if B has a greater mass than C, B could **not** be brought to rest in their collision. **[2]**

To earn both marks, candidates were expected to explain that, if B were brought to rest, conservation of momentum (requiring the speed of C to be greater than the initial speed of B) would be incompatible with the coefficient of restitution $e \leq 1$ (requiring the speed of C not to exceed the initial speed of B). Few candidates achieved this, although many scored one mark for a partly correct explanation (such as saying that the speed of C would be greater than the initial speed of B). There were a lot of imprecise statements such as 'B would have too much momentum to stop' which did not earn any marks.

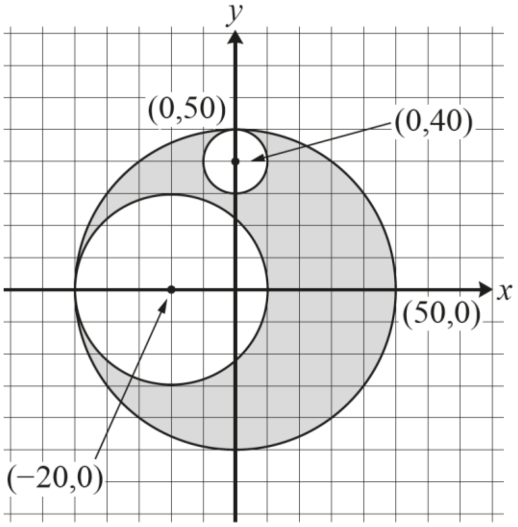
Question 5 (a)

5 **Fig. 5.1** shows the uniform cross-section of a solid S which is formed from a cylinder by boring two cylindrical tunnels the entire way through the cylinder. The radius of S is 50 cm, and the two tunnels have radii 10 cm and 30 cm.

The material making up S has uniform density.

Coordinates refer to the axes shown in **Fig. 5.1** and the units are centimetres.

Fig. 5.1



The centre of mass of S is $(\bar{x}, \bar{y})$.

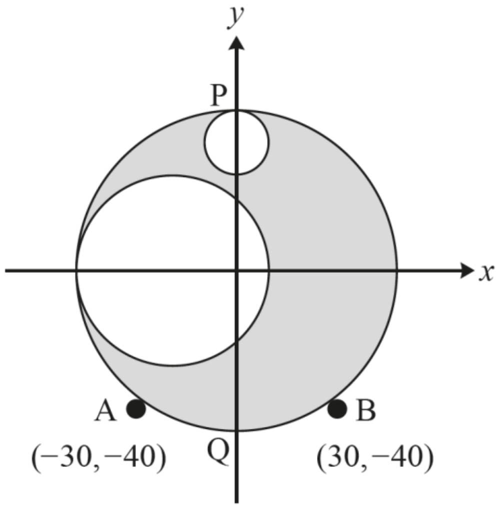
(a) Show that $\bar{x} = 12$ and find the value of $\bar{y}$. **[4]**

Most candidates understood how to find the centre of mass of the remaining solid S, and set out their working clearly. Some made errors with the signs, often obtaining a positive value for $\bar{y}$.

Question 5 (b)

Solid S is placed onto two rails, A and B, whose point of contacts with S are at $(-30, -40)$ and $(30, -40)$ as shown in **Fig. 5.2**. Two points, $P(0, 50)$ and $Q(0, -50)$, are marked on **Fig. 5.2**.

Fig. 5.2



At first, you should assume that the contact between S and the two rails is **smooth**.

(b) Determine the angle PQ makes with the vertical, after S settles into equilibrium. **[2]**

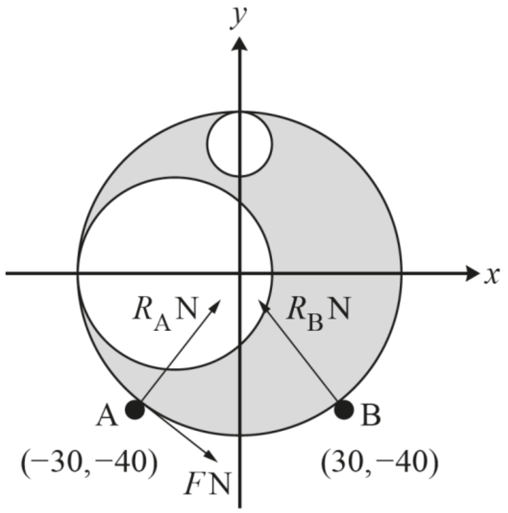
Common misconception

A large number of candidates assumed that the centre of mass would be vertically below P, instead of vertically below the origin.

Question 5 (c)

For the remainder of the question, you should assume that the contact between S and A is **rough**, that the contact between S and B is **smooth**, and that S does not move when placed on the rails. **Fig. 5.3** shows **only** the forces exerted on S by the rails. The normal contact forces exerted by A and B on S have magnitude R_A N and R_B N respectively. The frictional force exerted by A on S has magnitude F N.

Fig. 5.3



The weight of S is W N.

(c) By taking moments about the origin, express F in the form λW, where λ is a constant to be determined. **[2]**

This part was quite often omitted. Those who realised that the only forces which had non-zero moments about the origin were the weight and the friction, usually wrote down the moments equation correctly. Some did not have the correct distance of the line of action of F from the origin.

Question 5 (d)

(d) Given that S is in limiting equilibrium, find the coefficient of friction between A and S. **[5]**

This was the most challenging item on the paper, with many candidates omitting it altogether. The friction F was expressed in terms of W in part (c), so to find the coefficient of friction it was necessary to find the normal reaction R_A. Candidates usually attempted to do this by resolving horizontally and vertically, obtaining simultaneous equations for R_A and R_B. However, F was very often omitted from these equations, and few candidates resolved all the forces correctly. Some candidates took moments (about A or B) which could lead to a more efficient solution.

The following exemplar is one of the best responses to this part.

Exemplar 2

$F = \mu R_A$

~~$\mu R_A = 0.24W$~~ ~~$\mu = \frac{0.24W}{R_A}$~~

$W = R_A \sin(53.13) + R_B \sin(53.13) - F \sin(90 - 53.13)$

$W = 0.8R_A + 0.8R_B - 0.6F$

$R_A \cos(53.13) + F\cos(90-53.13) = R_B \cos(53.13)$

$0.6R_A + 0.8F = 0.6R_B$

$0.6W - 0.48R_A + 0.36F = 0.48R_A + 0.64F$

$0.6W = 0.96R_A + 0.28F$

$F = 0.24W$

$0.6W = 0.96R_A + 0.0672W$

$W = \frac{0.96R_A}{0.6672}$

$F = 0.24W$

$\mu R_A = 0.24W \qquad \mu = \frac{0.24W}{R_A}$

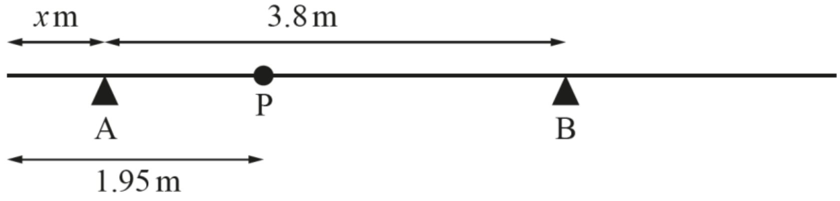
$\mu = \frac{0.24 \times \frac{200}{139} R_A}{R_A} = 0.35 = \mu$

There was no diagram drawn in the answer space (the candidate may have used the diagram on the question paper), and there was no explanation. However, it is clear that the candidate has resolved vertically and horizontally, then used these equations, together with the value of *F* found in part (c), to obtain the coefficient of friction. This was only spoilt by a numerical error near the end (where the 0.6672 should be 0.5328).

Question 6 (a)

6 A uniform beam of length 6m and mass 10kg rests horizontally on two supports A and B, which are 3.8m apart. A particle P of mass 4kg is attached 1.95m from one end of the beam (see **Fig. 6.1**).

Fig. 6.1



When A is x m from the end of the beam, the supports exert forces of equal magnitude on the beam.

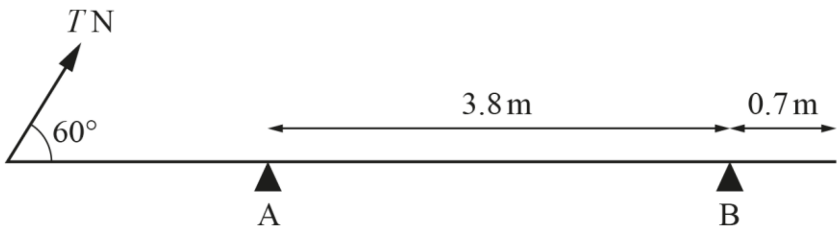
(a) Determine the value of x. **[4]**

Candidates who realised that the reactions at A and B were both 7*g* usually obtained the value of *x* correctly by taking moments about any suitable point, such as the left-hand end of the beam. Some used 5*g* instead of 7*g*. Many candidates took moments about two points (usually A and B) and used $R_A = R_B$ without needing to know the common value. This method was considerably more complicated and was very prone to errors, particularly in giving the distances in terms of *x*. Another approach, used by just a few candidates, was to find the centre of mass of the beam with P attached, which must be at the midpoint of AB when the reactions are equal.

Question 6 (b)

P is now removed. The same beam is placed on the supports so that B is 0.7 m from the end of the beam. The supports remain 3.8 m apart (see **Fig. 6.2**).

Fig. 6.2



The contact between A and the beam is smooth. The contact between B and the beam is rough, with coefficient of friction 0.4.

A small force of magnitude T N is applied to one end of the beam. The force acts in the same vertical plane as the beam and the angle the force makes with the beam is 60°.

As T is increased, forces T_L and T_S are defined in the following way.

- T_L is the value of T at which the beam would start lifting, assuming that is not already sliding.
- T_S is the value of T at which the beam would start sliding, assuming that it has not already lifted.

(b) Show that $T_L = 49.1$, correct to **3** significant figures. **[2]**

Many candidates answered this convincingly, usually by taking moments about B. Some candidates did not realise that $R_A = 0$ when the beam is about to lift.

Question 6 (c)

(c) Determine whether the beam will first slide or lift. **[5]**

Very few candidates completed this successfully. The question is leading candidates towards finding T_S and comparing it with T_L, and most attempts followed this course. There were several steps: using the laws of friction, resolving horizontally and vertically, taking moments, and finally solving the equations. Many candidates did not draw diagrams or explain what they were trying to do, which made awarding part marks difficult. For example, there was a mark for writing $F = 0.4R_B$. Candidates who wrote $F = 0.4R$ did not earn this mark unless it was clear that their R was the reaction at B.

The question does not require T_S to be found. A few candidates continued with the scenario in part (b) with the beam about to lift, calculated the friction and the reaction at B, and showed that this required a coefficient of friction greater than 0.4, therefore concluding that the beam would have slid first.

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