

**AS LEVEL**

**Examiners' report**

# **FURTHER MATHEMATICS A**

**H235**

For first teaching in 2017

**Y533/01 Summer 2023 series**

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## Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

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## Paper Y533/01 series overview

This is one of the optional papers in AS Further Maths, alongside the compulsory Pure Core, and the other options in Statistics, Discrete Mathematics and Additional Pure Mathematics. This paper complements the mechanics content in AS Mathematics and requires candidates to solve practical problems using a variety of techniques including energy, power, momentum, connected particles, horizontal and vertical circles, and dimensional analysis.

To do well on this paper, candidates needed to be able to work systematically and accurately with the standard principles of mechanics. They often needed to make sense of more complex situations, for example, correctly combining several forces and/or types of energy in a variety of situations, such as two or more sources of gravitational potential and/or kinetic energy and resistance. They also need to have some understanding of the purpose of mathematical modelling.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
<ul style="list-style-type: none"> <li>• made good use of diagrams to help them understand a problem before solving it</li> <li>• recognised the different forces that were acting and when they needed resolving</li> <li>• understood the different quantities in mechanics, such as work, kinetic and potential energy, momentum, impulse, power, angular velocity, and the correct formula required to work with each of them, together with the associated dimensions</li> <li>• were able to use Newton's Second Law (<math>F = ma</math>) confidently together with the relationship between force and energy</li> <li>• were able to apply their knowledge to complex and unfamiliar problems such as horizontal circles with unknown angles</li> <li>• were efficient and effective in applying their knowledge, always choosing the simplest possible route to the solution</li> <li>• understood the difference between a vector and a scalar.</li> </ul>	<ul style="list-style-type: none"> <li>• did not consider the different directions that objects might be travelling, especially in collisions and momentum questions</li> <li>• did not appreciate the connection between force and acceleration, or the need to resolve a force when calculating quantities such as acceleration and work done</li> <li>• did not always distinguish between the different quantities that can be found or used to solve problems or select the correct quantity for the given problem</li> <li>• made avoidable errors in calculations.</li> </ul>

## Question 1 (a)

- 1 Two particles  $A$ , of mass  $m$  kg, and  $B$ , of mass  $3m$  kg, are connected by a light inextensible string and placed together at rest on a smooth horizontal surface with the string slack.  $A$  is projected along the surface, directly away from  $B$ , with a speed of  $2.4 \text{ ms}^{-1}$ .

(a) Find the speed of  $B$  immediately after the string becomes taut.

[2]

Most candidates scored full marks on this question on moments. A small number scored only 1 mark, mostly due to avoidable errors such as getting the masses confused. However, a significant minority scored no marks. Common reasons included misunderstanding the nature of the question, e.g., that the presence of a connecting string, when taut, meant that the particles acted as if they had coalesced rather than having different velocities. Some candidates transferred all the momentum to one or other of the objects, usually  $B$ , to get  $0.8 \text{ m}$ . Another error that came up was to assume that the initial velocities of the two particles were both zero, meaning that no collision could take place.

## Exemplar 1

1(a)

$$0 \times m + 0 \times 3m = -2.4 \times m + v \times 3m$$

$$0 = -2.4m + 3vm$$

$$3v - 2.4 = 0$$

$$3v = 2.4$$

$$v = 0.8 \text{ ms}^{-1}$$

This is a good illustration of a candidate who has misinterpreted the question, thinking that since both particles are said to be initially at rest, this must be the state immediately before the 'collision'. The candidate then has no option but to assume that the given velocity of  $A$ ,  $2.4 \text{ m/s}$ , refers to the velocity after the 'collision' rather than beforehand. Conservation of momentum then leads to  $B$  having equal but opposite momentum to  $A$  and therefore a speed of  $0.8 \text{ m/s}$  rather than  $0.6 \text{ m/s}$ , scoring no marks.

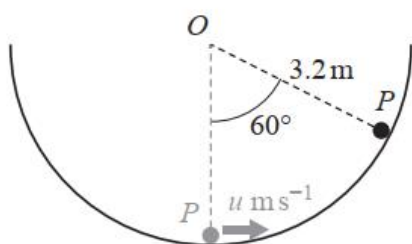
## Question 1 (b)

- (b) Find, in terms of  $m$ , the magnitude of the impulse exerted on  $B$  as a result of the string becoming taut. [2]

Nearly all candidates who scored full marks in part (a) also scored full marks in part (b). However, some did not find the magnitude and therefore lost a mark. Others used the combined mass of  $A$  and  $B$  to get an answer of 2.4 m or used the mass of  $A$  instead of  $B$  to give 0.6 m, while there were also some cases where candidates confused the change in kinetic energy with the impulse.

## Question 2 (a)

2



A small body  $P$  of mass  $3\text{ kg}$  is at rest at the lowest point of the inside of a smooth hemispherical shell of radius  $3.2\text{ m}$  and centre  $O$ .

$P$  is projected horizontally with a speed of  $u\text{ m s}^{-1}$ . When  $P$  first comes to instantaneous rest  $OP$  makes an angle of  $60^\circ$  with the downward vertical through  $O$ .

- (a) Find the value of  $u$ .

[4]

This question was done very well, with most candidates gaining full marks. Most marks were lost to careless slips like numerical errors, using sine instead of cosine, using  $\frac{1}{2}mv$  instead of  $\frac{1}{2}mv^2$  or missing out  $g$ . There were some who confused horizontal and vertical circles, or tried to use suvat, but these were not seen often.

## Question 2 (b)

- (b) State **one** assumption made in modelling the motion of  $P$ .

[1]

As is often the case with questions on modelling, this one was not done well, and most candidates did not score this mark. The key issue here was that the assumption needed to be distinct from anything mentioned in the question. In particular, the surface is stated as being smooth, so any reference to friction was excluded, and anything that was too vague, such as 'no resistance to motion' or 'no other forces' was also excluded. Acceptable responses usually mentioned air resistance or the particle nature of the model, both of which were not given in the question.

### Assessment for learning

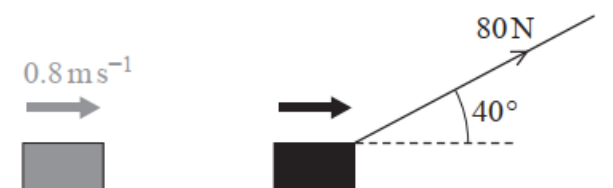


Collect several modelling questions with different responses and get students to discuss in small groups which ones are worthy of credit, which ones are not, and which ones they are unsure of and why. Include questions in a variety of contexts and with different amounts of information given in the question.

### Question 3 (a)

- 3 A crate of mass 45 kg is sliding with a speed of  $0.8 \text{ ms}^{-1}$  in a straight line across a smooth horizontal floor. One end of a light inextensible rope is attached to the crate. At a certain instant a builder takes the other end of the rope and starts to pull, applying a constant force of 80 N for 5 seconds.

While the builder is pulling the crate, the rope makes a constant angle of  $40^\circ$  above the horizontal. Both the rope and the velocity of the crate lie in the same vertical plane (see diagram).



It may be assumed that there is no resistance to the motion of the crate.

- (a) Determine the work done by the builder in pulling the crate.

[4]

This question was not generally done well, with 1 and 4 being by far the most common marks. Many candidates did not resolve the force, with some even appearing to assume that the crate was moving in the direction of the force, or that the vertical component was also doing work.

The most common error was not to realise that, as there was no resistance to motion, the crate must be accelerating. Therefore, the distance travelled required the use of suvat, which many candidates did not use, so were only able to score M1 for multiplying the horizontal component of the force by their distance. Others resolved the force to find the acceleration, but then multiplied the distance by the unresolved force, or vice versa. A small number confused the impulse with the work done, while a few made careless errors, such as using  $t$  instead of  $t^2$  when attempting to find the distance travelled.

A small number of candidates used either the acceleration or the impulse generated by the force to find the new velocity and then calculated the work done by comparing the initial and final KE. This is a perfectly acceptable less conventional alternative, but not generally recommended as it requires more work to get the marks.

Exemplar 2

3(a)

$0.8$   
 $0.5$   
 $80\text{N}$   
 $40^\circ$   
 $45$   
 Power of builder =  $Dv$   
 $= 0.8 + 80\cos 40$   
 $= 49.01$   
 $P = \frac{WD}{t}$   
 $= f \times v$   
 $= 80\cos 40$   
 $WD = \frac{Dv}{t} = \frac{0.8 + 80\cos 40 + v}{t}$   
 $WD = \text{Power} \times t$   
 $= 49.01 \times 5$   
 $= 245.13$   
 $WD + KE = KE + \frac{1}{2} \times 45 \times v^2$   
 $= \text{TOTAL KE}$   
 $0.8 \times 45 \times 0.5 + 80\cos 40 + 0.8 = \frac{1}{2} KE$

In this response, the candidate has made the very common error of not realising that the crate will be subject to a resultant force, and therefore accelerating. We can see a variety of different attempts to get a value for the work done, including  $\frac{\text{power}}{\text{time}}$ ,  $\frac{Dv}{t}$  and kinetic energy, before finally settling on power  $\times$  time, which gets M1 only as the acceleration and distance travelled (or average speed) have not been calculated. The candidate could instead have found the final velocity and kinetic energy and subtracted the initial energy.



### Question 3 (b) (i) and (ii)

- (b) (i)** Find the kinetic energy of the crate at the instant when the builder stops pulling the crate. [2]
- (ii)** Explain why the answers to part (a) and part (b)(i) are **not** equal. [1]

Most candidates who did well in part (a) tackled (b) (i) successfully by first attempting to find the final velocity, although a simpler method is to add the work done from part (a) (assuming that is correct) to the initial KE. Most candidates gained at least 1 mark for this, although we excluded those who did not consider the acceleration, as the answer would then have to be the same as the initial KE. Again, some candidates misinterpreted the question, finding things like the change in KE (= work done as in part (a)), or thought they needed to find just the initial KE, or even used the work done to find the new velocity.

Part (b) (ii) was done much less successfully, with many candidates not realising that the presence of the initial KE meant that the final KE would not be the same as the work done. Others realised this but did not explain it adequately, merely saying that the KE was the total energy for example, without mentioning the initial KE. Some attributed the difference to work done by the vertical component or by gravity, or both. Others mentioned the tension in the rope, presumably alluding to elastic potential energy even though it is described as inextensible, while others thought that it was because only the horizontal component was pulling the crate, or that friction had caused energy to be lost. Finally, one or two attributed it to not being able to measure values exactly, which was not relevant to the question.

### Question 3 (c) and (d)

- (c)** Find the average power developed by the builder in pulling the crate. [1]
- (d)** Calculate the total impulse exerted on the crate by the builder. [2]

For part (c), most candidates used their result from part (a) and divided this by the time taken, as would be expected. Some calculated the average speed and multiplied by the force, which is an acceptable but more laborious alternative, except in the cases where the candidate forgot to resolve the force. We excluded responses based on an acceleration of 0 where this was used in part (a), as this should have given zero power output. Some candidates confused instantaneous power (= force  $\times$  velocity) with average power and worked out the power at the end of the 5 seconds of motion instead. Others confused the force with the work done or divided the force by the time for example.

In part (d), most candidates calculated the change in momentum, although a simpler and better method that many used is to calculate force  $\times$  time, which has the added advantage that the data from the question can be used directly, rather than relying on possibly incorrect prior worked values and getting the wrong final answer.

### Question 4 (a)

- 4 A rower is rowing a boat in a straight line across a lake. The combined mass of the rower, boat and oars is 240 kg. The maximum power that the rower can generate is 450 W.

In a model of the motion of the boat it is assumed that the total resistance to the motion of the boat is 150 N at any instant when the boat is in motion.

- (a) Find the maximum possible acceleration of the boat, according to the model, at an instant when its speed is  $0.5 \text{ ms}^{-1}$ . [2]

This question required candidates to find the force from the given power and velocity and use Newton's Second Law with the given resistance to find the acceleration. This was generally tackled very well, with most candidates getting full marks. Some candidates omitted the resistance or the driving force, while some made errors such as working out the power associated with the friction and then counting that as a force. Others incorrectly multiplied the power by the velocity instead of dividing.

### Question 4 (b)

At one stage in its motion the boat is travelling at a constant speed and the rower is generating power at an average rate of 210 W, which is assumed to be constant. The boat passes a pole and then, after travelling 350 m, a second pole.

- (b) Determine how long it takes, according to the model, for the boat to travel between the two poles. [4]

This question was handled very well by almost all candidates, with just a few getting less than full marks.

Most candidates started by working out the speed from the given power, then equated the driving force with the resistance and used this to find the time taken to travel 350 m, which worked very well. A small number realised that they could express the power as force  $\times$  distance  $\div$  time and thus find the time more directly.

There were some careless slips and conceptual errors, for example, equating the work done to distance  $\times$  time, or confusing different types of quantity to a greater or lesser degree.

## Question 4 (c)

- (c) State a reason why the assumption that the rower's generated power is constant may be unrealistic. [1]

Most candidates responded very well to this question, with most alluding to the fact that humans do not produce a constant power output as they may get tired over time, for example. Some candidates realised that the periodic motion of the oars dipping in and out of the water meant that there would be regular interruptions in the power output throughout the journey. Where candidates did not directly mention power, e.g., changes in resistance due to wind or waves, or changes in speed, this needed to be linked explicitly to a change in power output to gain this mark, which did not always happen.

## Question 5 (a) (i), (ii) and (iii)

- 5 Two identical spheres,  $A$  and  $B$ , each of mass  $4\text{ kg}$ , are moving directly towards each other along the same straight line on a smooth horizontal surface until they collide. Before they collide, the speeds of  $A$  and  $B$  are  $5\text{ ms}^{-1}$  and  $3\text{ ms}^{-1}$  respectively. Immediately after they collide, the speed of  $A$  is  $2\text{ ms}^{-1}$  and its direction of motion has been reversed.
- (a) (i) Determine the velocity of  $B$  immediately after  $A$  and  $B$  collide. [3]
- (ii) Show that the coefficient of restitution between  $A$  and  $B$  is  $\frac{3}{4}$ . [2]
- (iii) Calculate the total loss of kinetic energy due to this collision. [2]

Question 5 (a) (i) was a simple conservation of motion problem, which most candidates scored full marks on. However, quite a few did not appreciate that velocity means speed and direction, and where a direction was not clearly indicated, either by stating the direction of motion of  $B$  or seeing it indicated on their diagram, this mark was not given. Candidates should be encouraged to include the requisite arrows for their calculated velocities. A few candidates scored only 1 mark, usually by making a slip such as having  $B$ 's initial velocity in the same direction as  $A$ 's velocity as well as not indicating the direction of the final velocity.

Part (ii) was, likewise, done very well and those who scored at least 2 marks in part (i) were able to use the standard formula to find the coefficient of restitution. A few made errors such as calculating approach  $\div$  separation speed, getting the velocity signs wrong (especially for sphere  $B$ ), adding instead of subtracting velocities, or even trying to calculate  $V_B$  afresh from the given value of  $e$ . As it is a given answer, sufficient working must be shown to gain credit, which did not happen in a very small number of cases.

Part (iii) was also done very well, with a large majority of candidates gaining both marks. A small number made simple numerical slips or only calculated the change for one of the objects rather than both. Occasionally a candidate calculated the difference in KE between  $A$  and  $B$  rather than the sum, or used the wrong formula for KE, or even combined the velocities of  $A$  and  $B$  before squaring.

## Question 5 (b)

Sphere  $B$  goes on to strike a fixed wall directly. As a result of this collision  $B$  moves along the same straight line with a speed of  $4 \text{ ms}^{-1}$ .

- (b) Find the coefficient of restitution between  $B$  and the wall, stating whether the collision between  $B$  and the wall is perfectly elastic. [2]

This question was completed successfully by nearly all candidates. The main issues were with a small number of candidates who found a negative value of  $e$ , forgot to interpret the value, or used an incorrect description such as 'completely elastic'. One or two candidates also calculated a value greater than one, either by getting the wrong value in part (a) which was less than 4, or by dividing approach by separation when the answer to part (b) was greater than 4.

## Question 5 (c)

- (c) Determine the magnitude of the impulse that  $B$  exerts on  $A$  the next time that they collide. [5]

This part required candidates to set up simultaneous equations using conservation of momentum and Newton's law of elasticity. This proved to be rather more challenging than the previous parts of the question, and most of the previous questions too, apart from Question 3 (a). Only about half of candidates scored full marks, but most of the rest made a reasonable attempt at it nonetheless or at least attempted either the momentum or the restitution equation, or both.

Most issues were caused by numerical errors, and slips such as sign errors in the restitution equation, e.g. using  $-e$  instead of  $e$ , or vice versa, leading to the wrong values for  $V_A$  and/or  $V_B$ . Some candidates did not find the magnitude of the impulse as instructed and left it as a negative value, or even calculated the change in KE instead. A very small number had  $B$  still moving away from  $A$  before the second collision, meaning that a collision was impossible, and could therefore gain no marks, while some assumed that  $A$  and  $B$  had the same post-collision velocities like in Question 1.

## Question 6 (a)

- 6 The physical quantity pressure, denoted by  $P$ , can be calculated using the formula  $P = \frac{F}{A}$  where  $F$  is a force and  $A$  is an area.

- (a) Find the dimensions of  $P$ . [1]

This question started off a series of questions related to dimensional analysis, which was again generally done well, with nearly all candidates gaining at least 5 out of the 10 marks available.

Nearly all candidates answered part (a) correctly, with a small number making careless slips, such as dividing  $T^{-2}$  by  $L^2$  to get  $T^{-4}$ , for example, or dividing by  $M^2$  instead of  $L^2$  (probably confusing  $M$  with metres), or not simplifying the expression. Very few candidates used incorrect notation, but one or two gave responses such as  $\text{N/m}^2$ , using SI units instead of dimensions, or a combination of both.

## Question 6 (b) and (c)

An object of mass  $m$  is moving on a smooth horizontal surface subject to a system of forces which begin to act at time  $t = 0$ . The initial velocity of the object is  $u$  and its velocity and acceleration at time  $t$  are denoted by  $v$  and  $a$  respectively. The object exerts a pressure  $P$  on the surface. The total work done by the forces is denoted by  $W$ .

A Mathematics class suggests three formulae to model the quantity  $W$ .

The first suggested formula is  $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 + mP$ .

**(b)** Use dimensional analysis to show that this formula cannot be correct. [2]

The second suggested formula is  $W = ku^\alpha v^\beta t^\gamma$  where  $k$  is a dimensionless constant.

**(c)** Use dimensional analysis to show that this formula cannot be correct for any values of  $\alpha$ ,  $\beta$  and  $\gamma$ . [2]

These questions required candidates to identify issues where there was a mismatch between the dimensions of a quantity and the dimensions suggested by the proposed formula. Again, these were answered very well by most candidates, with good use being made of dimensional equations, which now seems to be well bedded into the teaching of the topic.

Part (c) was generally done slightly better than part (b), where some candidates did not explain convincingly that the dimensions were not compatible between at least two of the terms, while others made errors calculating the correct dimensions of one or both sides of the equation, e.g.  $[W] = \text{MLT}^{-2}$ , thereby losing the accuracy mark.

This was less of an issue in part (c) as use of their incorrect value of  $[W]$ , provided it included an element of M, was allowed. However, some candidates struggled to explain adequately the problem of M only appearing on one side of the equation, with some saying that there was no M or no mass without specifying that there was still an element of mass on the left side of the equation, for example.

## Question 6 (d) (i), (ii) and (iii)

The third suggested formula is  $W = ku^\alpha a^\beta m^\gamma t^\delta$  where  $k$  is a dimensionless constant.

- (d) (i) Explain why it is **not** possible to use dimensional analysis to determine the values of  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  for the third suggested formula. [1]
- (ii) Given that  $\alpha = 3$ , use dimensional analysis to determine the values of  $\beta$ ,  $\gamma$  and  $\delta$  for the third suggested formula. [3]
- (iii) By considering what the formula predicts for large values of  $t$ , explain why the formula derived in part (d)(ii) is likely to be incorrect. [1]

These questions required candidates to recognise and explain the problem of having too many unknowns for the number of available dimensions, use a given value to find the remaining values using dimensional analysis, and interpret the result by comparing it to real life.

In part (i), many candidates struggled to explain the key issue adequately, namely that there are four unknowns, but dimensional analysis can only give us three equations, so a unique solution is not possible. Simply stating that there were 'too many unknowns' is not an adequate explanation, although we did give credit to those candidates who demonstrated all three equations in four unknowns and followed up by saying that they could not be solved, because of the number of unknowns. Some candidates ignored the equation for  $M$ , saying that there were 3 unknowns and only 2 equations, which was not deemed adequate.

Part (ii) was generally done well by the great majority of candidates, who used the dimensional equation (sometimes found in part (i)) correctly to form equations for  $\beta$ ,  $\gamma$  and  $\delta$  using the given value of  $\alpha$ . Aside from arithmetic slips, the main issues were errors such as having the wrong dimensions for  $[W]$ , using the wrong dimensions for  $a$ , or just transcribing the equations slightly wrongly from the dimensional equation. One or two candidates regarded the dimensional equation as optional, which also lost them a mark as well as making them more prone to errors in the individual logarithmic equations for each power.

As with many of the explanatory questions, part (iii) was done much less successfully than other parts, with only about half of candidates scoring this mark. The most successful responses saw candidates contrast the long-term value of  $[W]$ , predicted by the equation to tend to zero, with the fact that work done would be expected to increase over time, which is why the formula would be unlikely to be correct. As a result, responses based on a value of  $\delta$  that was greater than zero were excluded, as that would then also predict an increase in work done. Those that clearly contrasted the trend (as opposed to the long-term values) suggested by the formula with real-life expectations were also given the mark.

## Question 7 (a)

- 7 Two identical light, inextensible strings  $S_1$  and  $S_2$  are each of length 5 m. Two identical particles  $P$  and  $Q$  are each of mass 1.5 kg.

One end of  $S_1$  is attached to  $P$ . The other end of  $S_1$  is attached to a fixed point  $A$  on a smooth horizontal plane.  $P$  moves with constant speed in a horizontal circular path with  $A$  as its centre (see Fig. 1).

One end of  $S_2$  is attached to  $Q$ . The other end of  $S_2$  is attached to a fixed point  $B$ .  $Q$  moves with constant speed in a horizontal circular path around a point  $O$  which is vertically below  $B$ . At any instant,  $BQ$  makes an angle of  $\theta$  with the downward vertical through  $B$  (see Fig. 2).

Fig. 1

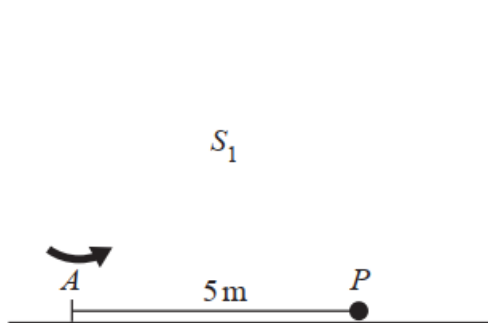
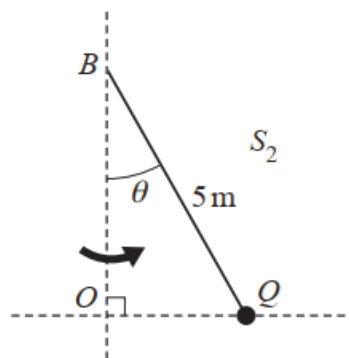


Fig. 2



- (a) Given that the angular speed of  $P$  is the same as the angular speed of  $Q$ , show that the tensions in  $S_1$  and  $S_2$  have the same magnitude. [3]

This question required candidates to apply the standard technique of resolving horizontally (and/or vertically) to solve a conical pendulum problem. In this case, only the horizontal component with  $F = ma = mr\omega^2$  was required to find the tension in each string as the angular speed was already given and the angle in  $B$  cancels out.

Most candidates managed to find a suitable expression for the tension in at least one of the particles  $P$  and  $Q$ , usually  $P$ , although a substantial minority did not, so centres may wish to pay particular attention to this in the future.

To get the tension in the string attached to  $Q$ , candidates needed to resolve both the tension and the length of the string horizontally by multiplying each by  $\sin\theta$ . Those that managed to do this successfully also simplified their equation by cancelling  $\sin\theta$  on each side to give the same value as for particle  $A$  and generally gained all 3 marks, with only a few candidates gaining 2 marks.

There were a small number of candidates that confused the horizontal circular motion with a vertical circle and ended up using incorrect methods. Some tried to use the speed instead of the angular speed even though the speeds are different for each object due to the different radii, while the angular speed is given in the question as being the same for each.

Other errors noted include applying the element of  $\sin\theta$  on only one or neither side of the equation of motion of Q, omitting the element of mass and/or length of the string, resolving Q vertically instead of horizontally, or trying to find the value of  $\theta$ .

## Exemplar 3

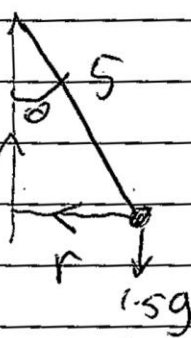
7(a)

$S_1$   $F=ma$   
 $F=1.5 \times \frac{V^2}{5}$   $5 \sin\theta = r$

$T_1 = 0.3V^2$

$S_2$   $\uparrow$

$T_2 \cos\theta = 1.5g$   
 $T_2 = \frac{1.5g}{\cos\theta}$   
 $\rightarrow T_2 \sin\theta = \frac{1.5g \sin\theta}{\cos\theta} = 1.5 \times \frac{V^2}{5 \sin\theta}$



$T_2 = \frac{1.5V^2}{5 \sin\theta} \times \frac{\sin\theta}{\cos\theta} = T_2 \sin\theta = \frac{1.5V^2}{5 \sin\theta}$

$T_2 = \frac{1.5V^2}{5} = 0.3V^2 = T_1$

Here the candidate has attempted every possible approach including resolving vertically and horizontally and recognised that  $r = 5\sin\theta$  for particle Q. They have also improperly cancelled out the  $\sin\theta$  term to seemingly equalise the tension acting on particle Q with the tension on particle P. However, as it is done in terms of  $V$  instead of  $\omega$ , which is different for the two particles, it gets no marks.



## Question 7 (b)

- (b) You are given instead that the kinetic energy of  $P$  is 39.2 J and that this is the same as the kinetic energy of  $Q$ .

Determine the difference between the times taken by  $P$  and  $Q$  to complete one revolution.  
Give your answer in an exact form. [7]

This question required candidates to find the speed using particle  $P$  and hence its time period. Candidates then had to resolve the forces on  $Q$  both horizontally and vertically, combine them and use the common speed from  $P$  plus a suitable trigonometric identity to find the angle of deflection of  $Q$ . They then have to find the time period for  $Q$  by using the speed or the angular speed, and finally compare this with the time found for  $P$ .

Most candidates that attempted the question were able to find the speed of  $P$  and often its time period from the given kinetic energy and the radius of 5 m.

Almost all candidates struggled to make much progress with particle  $Q$ . Some realised that they needed to resolve in one or both directions, particularly the horizontal direction, but did not make further progress as they assumed that they needed to find the acceleration in terms of  $\omega$  rather than  $v$  which was given as being the same as for particle  $P$ . Some also attempted to resolve vertically but most did not try to combine the two equations and did not know how to progress or deal with  $\omega$ . A small number did manage to make a credible attempt to combine the two equations, usually using the known value of  $v$  rather than  $\omega$ , although this could subsequently be eliminated using the equation  $v = r\omega$  if needed. A very small number of candidates made it all the way to the end without making any errors.

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