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Examiners' report

FURTHER MATHEMATICS A

H235

For first teaching in 2017

Y532/01 Summer 2023 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y532/01 series overview

Candidates who did well on this paper could carry out a range of hypothesis tests with clear, accurate statements of hypotheses and conclusions, assess the validity of models and estimates on the basis of information given and apply good algebraic skills to probability formulae, including the handling of permutations and combinations.

On this paper the hypothesis test for a correlation coefficient was well done, and there has been a steady improvement in candidates' ability to handle questions involving numbers of ways of choosing or arranging.

Candidates appeared well prepared for the more routine style questions but many found questions involving more problem solving to be more challenging – for example, many are used to applying the Poisson distribution in a context that involves time intervals but struggled with contexts involving spatial intervals.

There was often confusion between binomial and Poisson assumptions and parameters.

Only a minority of candidates consistently stated hypotheses accurately. Candidates who did well appreciated the crucial fact that hypotheses refer to a population and not to a sample.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• understood the issues in questions about modelling. • read questions carefully and answer what is asked, not what has been asked in the past. • gave quantitative answers where possible, for example in assessing whether a sample mean and variance could come from a binomial distribution. • stated hypotheses in terms of the underlying population. • stated a "do not reject H_0" conclusion to a goodness-of-fit test using a double negative, e.g. "there is insufficient evidence that the model is not a good fit" • were willing to use graphs or diagrams to support algebraic answers.	• regurgitated standard phrases with little appreciation of the context of the question. • did not answer the question, for example writing only about correlation when the question asks about a diagram. • gave only qualitative answers, or use the variance when the standard deviation is more relevant. • stated hypotheses without referring to the population, or referring only to the data. • stated a "do not reject H_0" conclusion to a goodness-of-fit test by asserting that the data is a good fit • did not think of using graphs or diagrams in an algebraic context.

Question 1 (a)

1 A radar device is used to detect flaws in motorway roads before they become dangerous. The number of flaws in a 1 km stretch of motorway is denoted by X. It may be assumed that flaws occur randomly.

(a) State **two** further assumptions that are necessary for X to be well modelled by a Poisson distribution. **[2]**

Many answered this routine part of the question confidently. However, many seem not to understand the concepts involved and simply regurgitated phrases that they had learnt but which conveyed no real meaning to them. Some said "flaws have to occur at a constant rate in time", despite the fact that this is a question about distances and not time; "average constant rate" is meaningless; and any reflection on "fixed interval between flaws" should show that it is plainly wrong as it would mean that flaws were exactly predictable. Further, "the number of flaws in any 1 km is independent of the number in any other" treats each 1 km stretch as a separate object instead of the whole motorway as a continuum; like "the probability of a flaw occurring is constant", this is probably a confusion with binomial assumptions.

Some wrote that the number of flaws occurring in any 1 kilometre section of motorway was independent of the number appearing in any other km, but this is not strong enough: each flaw must occur independently of any other flaw. Again, there would seem to be confusion with a binomial distribution – this would have been relevant if the question was asking about a fixed number of separate 1 kilometre sections of motorway.

Candidates would perhaps benefit from spending some extra time considering the assumptions of a range of typical questions before tackling the calculations.

Exemplar 1

The flaws must be completely independent of one another & occur at a constant rate.

It is clear from many candidates' responses that care is needed to express the "constant average rate" assumption correctly. "Constant rate" actually means "at a regular and exactly predictable rate", and indeed some candidates actually said "the number of flaws in 1 kilometre is fixed".

Misconception

?

A detailed explanation of the second modelling assumption for a Poisson distribution might read something like this: "For any length x km of motorway, the expected number of flaws is proportional to x and is not affected by any other variables". This is what "constant average rate" means. "Constant rate" would simply mean that flaws occur at exactly predictable, regular intervals – tick, tock. It is not a good idea to express this assumption in terms of probabilities as then there is the issue that more than one flaw can occur in any length x.

Nor is this the same as the notorious "singly" condition, which simply prohibits events occurring in inseparable blocks, such as points scored in rugby football. Usually this condition is part of the scenario and candidates should not consider it, especially as it is usually covered by the independence condition anyway.

Question 1 (b)

Assume now that X can be modelled by the distribution Po(5.7).

(b) Determine the probability that in a randomly chosen stretch of motorway, of length 1 km, there are between 8 and 11 flaws, inclusive. **[2]**

This was almost always correct.

Question 1 (c)

(c) Determine the probability that in two randomly chosen, non-overlapping, stretches of motorway, each of length 5 km, there are at least 30 flaws in one stretch and fewer than 30 flaws in the other stretch. **[3]**

The substantial majority of candidates omitted the 2 factor (this is effectively B(2, 0.586), though it is not necessary to think of it like that). Some candidates misinterpreted the question and gave two separate responses.

Question 2 (a)

2 A music lover has 30 CDs arranged in a random order in a line on a shelf. Of these CDs, 7 are classed as Baroque, 10 as Classical and 13 as Romantic.

(a) Determine the probability that all 7 Baroque CDs are next to each other. **[3]**

There were many correct responses to this question. The standard of responses to questions involving arrangements and choices has risen substantially over the years.

Question 2 (b)

(b) Determine the probability that, of the 10 CDs furthest to the left on the shelf, at least 6 are Baroque. **[4]**

Although this was harder than part (a), there were still quite a few correct responses to this part.

Question 3 (a)

3 An insurance company collected data concerning the age, x years, of policy holders and the average size of claim, £y thousand. The data is summarised as follows.

$$n = 32 \quad \sum x = 1340 \quad \sum y = 612 \quad \sum x^2 = 64\,282 \quad \sum y^2 = 13\,418 \quad \sum xy = 27\,794$$

(a) Find the variance of x. **[1]**

A majority obtained the correct answer to this routine question, but a surprisingly large number did not. Some tried to use the formula for the uniform distribution.

Question 3 (b)

(b) Find the equation of the regression line of y on x. **[2]**

This was usually correct.

Question 3 (c)

(c) Hence estimate the expected size of claim from a policy holder of age 48. **[1]**

Some candidates gave their response as 20.7 rather than £20 700. The question asks for the expected claim size, not for the value of y.

Question 3 (d)

Tom is aged 48. He claims that the range of the data probably does not include people of his age because the mean age for the data is 41.875, and 48 is not close to this.

(d) Use your answer to part **(a)** to determine how likely it is that Tom's claim is correct. **[2]**

This was not well answered. Some did not understand how to approach the question, producing responses such as "the data is not reliable as it varies a lot". It is not helpful to refer merely to the variance; you need to use the standard deviation to compare the mean with 48.

Exemplar 2

Not likely as there is a very high variance of 255.3 so people of his age are likely to be included. ~~people~~

This is a typical response where more quantification should be used. The appropriate measure of the distance between 48 and the sample mean is the standard deviation, not the variance.

Exemplar 3

variance = 255.3 σx = 15.98

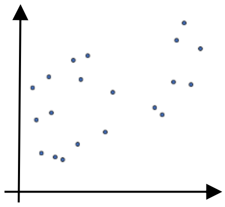
tom is less than even 1 standard deviation of x from the mean, therefore the model should be reliable for his age

By contrast, this is a fully quantified response with a conclusion that addresses the question.

Question 3 (e)

(e) Comment on the reliability of your estimate in part **(c)**. You should refer to the value of the product-moment correlation coefficient for the data, which is 0.579 correct to 3 significant figures. **[2]**

Candidates should understand that 0.579 is not strong correlation, so the estimate is not very reliable. (It is instructive to generate a scatter diagram with a PMCC of 0.579. An example is shown.)



Several candidates compared 0.579 with critical values. This is irrelevant. A hypothesis test gives evidence of whether there is any non-zero correlation, but even a very significant outcome gives no evidence about *how strong* any correlation is.

Misconception

A hypothesis test for the PMCC tests whether the correlation is zero or not. It does *not* give any indication of whether the correlation is strong. For instance, with a sample of size 60, a sample pmcc of 0.22 would give significant evidence at the 5% level that ρ is greater than zero, but it is certainly no indication that the points lie close enough to a line of best fit for estimates to be reliable. Some writers use the term "effect size" to indicate whether a correlation appears to be strong or not.

Question 4

4 A discrete random variable W has the probability distribution shown in the following table, in which a and b are constants.

w	58	59	60	61	62	63
$P(W = w)$	a	b	0.2	0.2	0.1	0.1

It is given that $E(W - 60) = 0.15$.

Determine the value of $Var(4W - 60)$. **[7]**

This question was generally well done and many correct responses were seen. Those who did not make use of (sum of probabilities) = 1 were rarely able to make any progress.

Question 5 (a)

5 A psychologist investigates the relationship between 'openness' and 'creativity' in adults. Each member of a random sample of 15 adults is given two tests, one on openness and one on creativity. Each test has a maximum score of 75. The results are given in the table.

Adult	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Openness, x	39	34	29	20	40	35	20	36	55	31	41	43	33	30	33
Creativity, y	59	34	17	29	49	46	45	54	60	38	46	35	43	56	34

$n = 15 \quad \sum x = 519 \quad \sum y = 645 \quad \sum x^2 = 19\,033 \quad \sum y^2 = 29\,751 \quad \sum xy = 23\,034$

(a) Use Pearson's product-moment correlation coefficient to test, at the 5% significance level, whether there is positive association between openness and creativity. **[6]**

Many answered this well. However, it is once again emphasised that a proper statement of hypotheses uses the value of a population parameter (usually ρ in this context), and that this parameter should be defined, for example "where ρ is the population PMCC between the results in the two tests".

Some wrong critical values were seen. Most candidates gave conclusions that were in context and not too assertive, using expressions such as "there is evidence that ...".

Question 5 (b)

(b) State what the value of Pearson's product-moment correlation coefficient shows about a scatter diagram illustrating the data. **[2]**

Many candidates did not read the question carefully enough. It is a question about a scatter diagram, so responses that referred merely to correlation, such as "moderate positive correlation", gained no credit. Some said things like "the points are quite spread out", but this does not indicate in what direction they are spread; what matters is that they do not lie very close to the line of best fit.

A number of candidates said "the points must form an approximate ellipse", but this is not a *consequence* of the PMCC value; it is a *necessary condition* for the test to be valid and so is irrelevant to the question.

Exemplar 4

because the number 0.4869 only suggests a weak positive correlation, this data illustrated on a scatter diagram would likely be in a shallow pattern, with a shallow ~~[illegible]~~ line of best fit.

This candidate appears to have confused the gradient of the regression line with the strength of the correlation.

Question 5 (c)

(c) A student suggests that there is a way to obtain a more accurate measure of the correlation. Before carrying out the test it would be better to standardise the test scores so that they have the same mean and variance.

Explain whether you agree with this suggestion. **[1]**

Only a minority of candidates realised that standardising the test scores was a linear transformation and so would have no effect on the PMCC.

Question 6 (a)

6 A machine is used to toss a coin repeatedly. Rosa believes that the outcome of each toss made by the machine is not independent of the previous toss. Rosa gets the machine to toss a coin 6 times and record the number of heads, X, obtained. After recording the number of heads obtained, Rosa resets the machine and gets it to toss the coin 6 more times. Rosa again records the number of heads obtained and she repeats this procedure until she has recorded 88 independent values of X.

(a) The sample mean and sample variance of X are 3.35 and 3.392 respectively.

Explain what these results suggest about the validity of a binomial model $B(6, p)$ for the data. **[3]**

This question proved challenging for the majority of candidates. The wording of the question showed that it was necessary to use both the mean and the variance, but not to consider modelling assumptions. Further, the question refers to "a binomial model B(6, p)", in other words to *any* binomial distribution with $n = 6$, and not to whether the coin is fair, so it was wrong to take $p = 0.5$.

A substantial minority of candidates said "mean ≈ variance, so a binomial distribution is valid", but that of course is a confusion with the Poisson distribution. Some tried to remember something that they hoped was relevant but was not, such as "the value of *p* is not constant".

Successful approaches included the following:

find p from $6p = 3.35$ and substitute into $6p(1 - p)$ to get an answer very different from 3.392,

show that the equation $6p(1 - p) = 3.392$ has no real solutions,

argue that $npq > np$ so q seems to be bigger than 1 which is impossible.

Question 6 (b)

Rosa uses a computer spreadsheet to work out the probabilities for a more sophisticated model in which the outcome of each toss is dependent on the outcome of the previous toss. Her model suggests that the probabilities $P(X = x)$, for $x = 0, 1, 2, 3, 4, 5, 6$, are approximately in the ratio $5 : 6 : 7 : 8 : 7 : 6 : 5$. She carries out a χ^2 test to investigate whether this model is a good fit for the data.

The following table shows the full results of the experiments, together with some of the calculations needed for the test.

x	0	1	2	3	4	5	6	Total
Observed frequency	7	10	16	15	15	11	14	88
Expected frequency								
Contribution to χ^2 statistic	0.9	0.3333	0.2857	0.0625	0.0714			

(b) In the Printed Answer Booklet, complete the table. **[3]**

This caused few problems apart from those who wrote 0 in the final cell, no doubt calculating $(O - E)^2/E$ from the totals. Such candidates could gain full marks provided they used the correct total in part (c). A few wrote 0.0833… as 0.833…

Question 6 (c)

(c) Carry out the test, using a 10% significance level. **[4]**

Candidates seemed less familiar with chi-squared hypothesis tests. In particular, hypotheses were often wrongly stated, or the wrong way round. It is not correct to say "H_0: the results are in the specified ratio"; the *results* are definitely in the ratio 7:10:16:15:15:11:14. The issue is whether the *underlying population* is in the suggested ratio. Nor is it correct to say "H_0: there is evidence that …"; the words "evidence that" belong in the conclusion, but hypothesis statements are about what is actually true, rather than what the experiment shows. The importance of the difference between population and sample should not be under-estimated. Some said "H_0: the coin tosses are independent", etc., which is not the right approach.

The conclusion was often not well stated. It is wrong to say "there is significant evidence that the model fits the data"; hypotheses tests never give positive evidence that H_0 is true. It is necessary to use a double negative: "there is insufficient evidence that the model does not fit the data", or words to that effect.

Question 6 (d)

(d) Rosa says that the results definitely show that one of the two proposed models is correct.

Comment on this statement. **[2]**

Candidates who performed well realised that the use of the word "definite" is inappropriate as the use of sample statistics or hypotheses tests can never produce definite conclusions about a population. A different sample might produce a wholly different set of results.

Question 7 (a)

7 A town council is planning to introduce a new set of parking regulations. An interviewer contacts randomly chosen people in the town and asks them whether they are in favour of the proposal. The first person who is not in favour of the regulation is the Rth person interviewed. It can be assumed that the probability that any randomly chosen person is not in favour of the proposal is a constant p, and that p does not equal 0 or 1.

Assume first that $E(R) = 10$.

(a) Determine $P(R \geqslant 14)$. **[3]**

This was generally well done, although the command word "Determine" was overlooked by a few candidates who showed insufficient working.

Question 7 (b)

Now, without the assumption that $\mathrm{E}(R) = 10$, consider a general value of p.

It is given that $\mathrm{P}(R = 3) - 0.4 \times \mathrm{P}(R = 2) - a \times \mathrm{P}(R = 1) = 0$, where a is a positive constant.

(b) Determine the range of possible values of a. **[7]**

This was certainly the hardest question on the paper, although most candidates were able to get an appropriate quadratic equation such as $p^2 - 1.6p + 0.6 - a = 0$. (Actually it is much easier to use q instead of p.) However, many struggled to make further progress. It is inappropriate to use '$b^2 > 4ac$', as the issue is not "do solutions for p exist?" but "are there solutions in the range $0 < p < 1$?" The issue is complicated by the fact that the quadratic function is not increasing, or decreasing, throughout the range, so simply substituting the end points is not valid. Most candidates who made any progress attempted to find an explicit formula for p and use the inequality $0 < p < 1$. However, one or two candidates found a much easier way: simply write $a = p^2 - 1.6p + 0.6$ and sketch a graph of a against p. This makes it clear at once that *in the relevant domain* the graph is decreasing, so it is easy to write down the end points.

It is perhaps worth reflecting on the fact that so few exam candidates seem to be willing or able to use diagrams to help their arguments; they seem irrevocably wedded to algebra.

Several candidates correctly attempted to find an explicit formula for p in terms of a but then wrote $\sqrt{a + 0.04} = \sqrt{a} + 0.2$, which is unexpected at this level.

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