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OCR
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AS LEVEL

Examiners' report

FURTHER MATHEMATICS A

H235

For first teaching in 2017

Y531/01 Summer 2023 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y531/01 series overview

Y531 is the mandatory paper for AS Further Mathematics A. It is taken alongside at least two other papers which can be freely chosen from a choice of four. It tests knowledge of proof, complex numbers, matrices, vectors and further algebra as well as testing the understanding of the overarching themes of mathematical argument, problem solving and modelling. To do well on this paper, candidates need a thorough understanding of the techniques covered and they need to support their answers with detailed working. They also need to have good algebraic and numerical manipulation skills.

Candidates seemed to be well prepared for the paper, although some candidates did not follow the instructions in the question and used a method other than the one requested.

Some candidates had presentation or layout which was hard to follow, and in some cases candidates could not read their own writing. It was not unusual to see candidates introduce wrong signs or coefficients as they progressed through Question 1 or Question 7, where sustained algebraic manipulation was needed, due to a lack of clear layout.

Questions 2, 4, 5 and 6 were particularly well done by candidates, and the vast majority of candidates appeared well prepared for the requirements of these questions.

The "If and only if" proofs in Question 8 were found to be difficult and only a minority of candidates seemed to be aware of the need to show that the implication works in both directions.

Candidates must take careful note of any "command words" given in the question. In particular, if a question asks for "**In this question you must show detailed reasoning**", candidates must show all of their working. Calculators can be used as a check for each step, but the solutions must be fully detailed in order to gain full credit. This was a particular issue in Question 7, but also affected Question 3 in places.

There was evidence to suggest that some candidates struggled with time management and either rushed or omitted some of the parts on Questions 8 and Question 9.

OCR support

A guide to the command words used in OCR A Level Maths exams can be found on Teach Cambridge.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• set out their working clearly and logically which helped avoid algebraic errors • applied routine methods accurately • read the questions carefully to make sure that they used the requested method and that they had satisfied all requirements • used calculators appropriately and efficiently.	• did not show sufficient working to justify their solutions to Detailed reasoning questions • made arithmetical or algebraical mistakes when applying routine methods, often involving negative numbers.

Question 1

1 The roots of the equation $4x^4 - 2x^3 - 3x + 2 = 0$ are α, β, γ and δ. By using a suitable substitution, find a quartic equation whose roots are $\alpha + 2$, $\beta + 2$, $\gamma + 2$ and $\delta + 2$ giving your answer in the form $at^4 + bt^3 + ct^2 + dt + e = 0$, where a, b, c, d, and e are integers. **[5]**

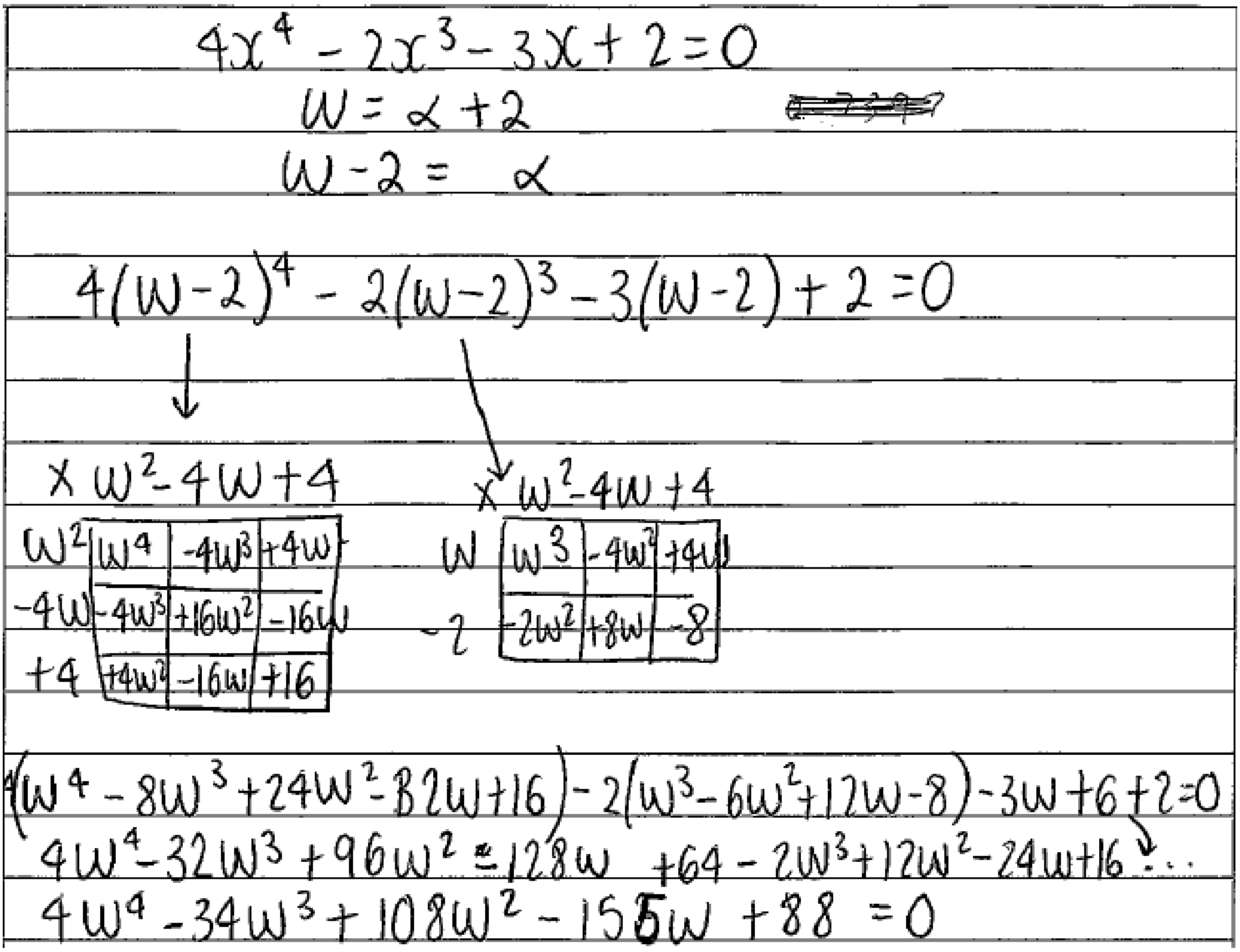
In this question, the method to be used was specified, and those candidates who employed a different method, such as using Vieta's formulas (sum and product of coefficients formulae), scored no credit.

Of those who used the required method, the most common errors were making a mistake when expanding (usually made by those who considered expanding the brackets manually rather than using binomial expansions) and failing to write their final result as an equation, i.e. omitting the "= 0" part. A few candidates used an incorrect substitution; these candidates usually gained the 3 method marks available for the question.

Assessment for learning

Candidates should be reminded of the importance of reading the question carefully and recognise when the method they should use is dictated by the question.

Exemplar 1



$4x^4 - 2x^3 - 3x + 2 = 0$

$W = \alpha + 2$

$W - 2 = \alpha$

$4(W-2)^4 - 2(W-2)^3 - 3(W-2) + 2 = 0$

×	W^2	$-4W$	$+4$
W^2	W^4	$-4W^3$	$+4W^2$
$-4W$	$-4W^3$	$+16W^2$	$-16W$
$+4$	$+4W^2$	$-16W$	$+16$

×	W^2	$-4W$	$+4$
W	W^3	$-4W^2$	$+4W$
-2	$-2W^2$	$+8W$	-8

$4(W^4 - 8W^3 + 24W^2 - 32W + 16) - 2(W^3 - 6W^2 + 12W - 8) - 3W + 6 + 2 = 0$

$4W^4 - 32W^3 + 96W^2 - 128W + 64 - 2W^3 + 12W^2 - 24W + 16 \ldots$

$4W^4 - 34W^3 + 108W^2 - 155W + 88 = 0$

This candidate chose to expand the brackets rather than using the binomial expansion formula. Usually candidates made mistakes when using this method, but this candidate used a clear method to show their working which helped make sure that they did this accurately.

Question 2 (a)

2 The lines L_1 and L_2 have the following equations.

$$L_1 : \mathbf{r} = \begin{pmatrix} -5 \\ 6 \\ 15 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -2 \\ -2 \end{pmatrix}$$

$$L_2 : \mathbf{r} = \begin{pmatrix} 24 \\ 1 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix}$$

(a) Show that L_1 and L_2 intersect, giving the position vector of the point of intersection. **[5]**

This question was answered well, with a large majority gaining full marks, and most of the rest gaining 4 out of the 5 marks available.

The most common reason for not gaining full credit was not clearly showing that their values of λ and μ satisfied all three equations, with some other candidates making sign errors or not writing down the point of intersection of the lines.

Candidates who used their calculators to solve the simultaneous equations had to make it clear to the examiners which two equations they had used, otherwise it was not possible to give them credit for showing that their values worked in the third equation.

Use of calculators

Candidates who use their calculators to do algebraic manipulation (such as solving simultaneous equations as in this question) should make it clear to examiners that they understand the mathematics underpinning the results generated by the calculator. This is especially important in "Show that" questions.

Question 2 (b)

(b) Find the equation of the line which intersects L_1 and L_2 and is perpendicular to both. Give your answer in cartesian form. **[3]**

This question was generally done well, with the most common mistakes being sign errors when finding the cross product; not using the intersection point from the previous part; and not turning their vector equation into cartesian form. Follow through applied in this question so candidates could gain credit for each step even if they had made a mistake in the previous part.

Question 3 (a)

3 In this question you must show detailed reasoning.

In this question the principal argument of a complex number lies in the interval $[0, 2\pi)$.

Complex numbers z_1 and z_2 are defined by $z_1 = 3 + 4\text{i}$ and $z_2 = -5 + 12\text{i}$.

(a) Determine $z_1 z_2$, giving your answer in the form $a + b\text{i}$. **[2]**

There are two indications here that working was required, firstly this is a "Detailed reasoning" question, and secondly the request in the question was "Determine". Candidates had to show at least one line of working before they arrived at the final answer in order to gain credit for this question.

Detailed reasoning

If a question requests "**In this question you must show detailed reasoning**" then candidates must make sure that they show full justification for their answers. Calculators can be used, but the steps of the working must still be shown.

Question 3 (b)

(b) Express z_2 in modulus-argument form. **[3]**

Again, justification was required to gain full credit. Candidates were allowed to "spot" a Pythagorean triple for the module, but to find the argument it was necessary to show use of trigonometry.

Some candidates used trigonometry correctly but ended up with an incorrect value. In many of these cases a quick sketch to show where the point lies in the Argand plane would have been helpful.

A few candidates did not take notice of the instruction to give the argument as a value in the interval $[0, 2\pi)$. Candidates who worked in degrees but then converted to radians could gain full credit as long as their working was accurate enough to make sure the final answer was correct to 3 significant figures.

Question 3 (c)

(c) Verify, by direct calculation, that $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$. **[3]**

Some of the same mistakes made in part (b) were repeated in this question, such as working in degrees or not showing justification of the argument for z_1z_2. Some candidates who had rounded previous results to 3 significant figures found it difficult to verify the result as their calculations did not agree.

Accuracy of intermediate results

If it is required that the final calculation is to be correct to 3 significant figures then it is advisable to record intervening results to a greater degree of accuracy.

Question 4

4 The vector **p**, all of whose components are positive, is given by $\mathbf{p} = \begin{pmatrix} a^2 \\ a-5 \\ 26 \end{pmatrix}$ where a is a constant.

You are given that **p** is perpendicular to the vector $\begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$.

Determine the value of a. **[4]**

This question was answered well with the majority of candidates gaining full credit, and most of the rest gained 3 out of 4 marks, having not rejected the a = -9 case, or not rejected it with a fully correct reason.

Candidates who stated that "a had to be positive" did not gain the last mark as the key point was that the components of **p** that had to be positive. However "$a > 5$" was sufficient as this implies that the components must be positive.

Question 5

5 **In this question you must show detailed reasoning.**

The roots of the equation $5x^2 - 3x + 12 = 0$ are α and β.

By considering the symmetric functions of the roots, $\alpha + \beta$ and $\alpha\beta$, determine the exact value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. **[4]**

This question was answered well, and very few candidates attempted the question using a method other than the one specified (contrasting with Question 1 where a much larger proportion tried to use an alternative method).

Question 6

6 Prove by induction that $4 \times 8^n + 66$ is divisible by 14 for all integers $n \geqslant 0$. **[6]**

This question was answered well, with logical working shown clearly by a majority of candidates. A few different methods were seen, including considering the difference between P_{k+1} and P_k. The most common reasons for losing a mark were considering *n* = 1 as the base case, or for not wording the conclusion precisely enough.

For the conclusion candidates needed to clearly state both that if the proposition was true when *n* = *k* then it must be true when *n* = *k* + 1, and also that since the proposition is true when *n* = 0 it must therefore be true for all non-negative integers. Note that "positive integers" was not sufficient as this excludes 0.

Exemplar 2

So if true for $n=0$, $n=k$ and $n=k+1$
then true for all $n \geq 0$.

To gain the last mark, candidates needed to make a clear and fully correct conclusion to the proof. In this case, the candidate has not clearly obtained the "When the proposition is true for *n* = *k* it is also true for *n* = *k* + 1, and since it is true when *n* = 0…" format so could not be given the last mark.

Question 7

7 **In this question you must show detailed reasoning.**

Matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & -6 & a-3 \\ a+9 & a & 4 \\ 0 & -13 & a-1 \end{pmatrix}$ where a is a constant.

Find all possible values of a for which det **A** has the same value as it has when $a = 2$. **[6]**

The most common marks given for this question were 2 and 4. The candidates who gained 2 marks tended to make a mistake when evaluating the determinant in terms of *a* or when *a* = 2. Candidates who gained 4 marks usually found the correct cubic equation but then wrote down the roots with no further working. Since this was an "**In this question you must show detailed reasoning**" question, to gain full credit candidates needed to show the factorisation of the cubic into a linear and quadratic factor, and then some evidence of how they solved the quadratic.

Question 8 (a)

8 **(a)** Solve the equation $\omega + 2 + 7\text{i} = 3\omega^* - \text{i}$. **[4]**

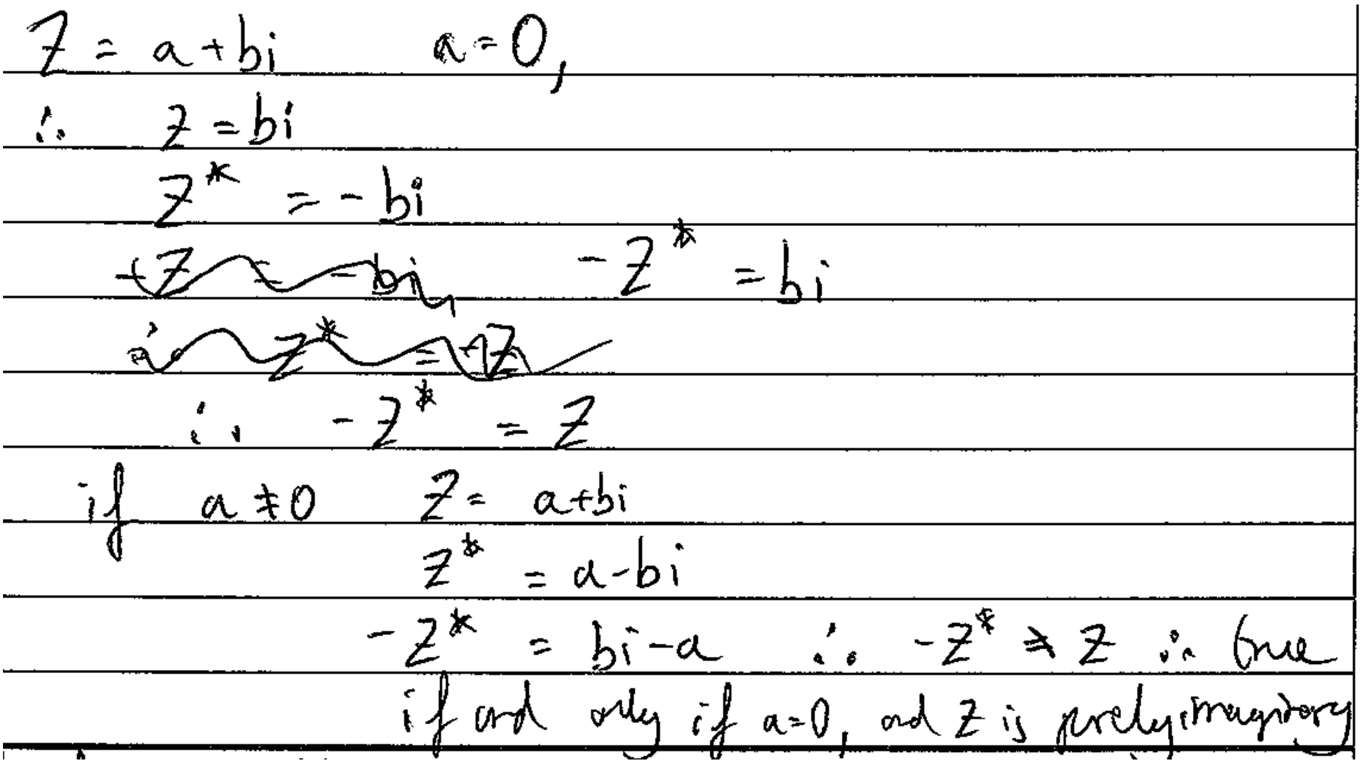
This question was generally answered well, with over half of candidates gaining full credit. Of those who didn't, common mistakes included not writing down the value of ω at the end of the question, or making an algebraic slip when working out the values of the real and imaginary parts. A few candidates did not know how to start this question.

Question 8 (b)

(b) Prove algebraically that, for non-zero z, $z = -z^*$ if and only if z is purely imaginary. **[2]**

Candidates found the concept of "If and only if" difficult, and most only showed one direction of the implication. Those candidates who were successful in gaining full credit tended to consider the cases when *z* was purely imaginary and when *z* had a non-zero real part rather than following the method in the mark scheme.

Exemplar 3



$z = a + bi \quad a = 0,$

$\therefore \; z = bi$

$z^* = -bi$

$-z^* = bi$

$\therefore \; -z^* = z$

if $a \neq 0 \quad z = a + bi$

$z^* = a - bi$

$-z^* = bi - a \quad \therefore \; -z^* \neq z \quad \therefore$ true

if and only if $a = 0$, and z is purely imaginary

In this case the candidate has taken a different approach to that shown in the mark scheme. This is an alternative method to show the "if and only if", which turned out to be the more popular approach.

Assessment for learning

The concept of "If and only if" is covered in section 1.01 of the single A Level Mathematics specification, but it is important to make sure that Further Mathematics candidates are familiar with the concept.

There are some examples of tasks that can be used in the section 1.01 delivery guide on Teach Cambridge.

Question 8 (c) (i)

(c) The complex numbers z and z^* are represented on an Argand diagram by the points A and B respectively.

(i) State, for any z, the single transformation which transforms A to B. **[1]**

Since the transformation was in the Argand plane, candidates needed to state that it was a reflection in the real axis (rather than the *x*-axis).

Question 8 (c) (ii)

(ii) Use a geometric argument to prove that $z = z^*$ if and only if z is purely real. **[2]**

This was found to be the hardest question on the paper, and a large number of candidates omitted it. As in Question 8 (b), those candidates who attempted the question almost always did not consider both directions of implication. Many candidates missed the request to give a geometrical argument, instead using an algebraic one of a similar nature to that used in part (b).

Question 9 (a)

9 Matrix $\mathbf{R}$ is given by $\mathbf{R} = \begin{pmatrix} a & 0 & -b \\ 0 & 1 & 0 \\ b & 0 & a \end{pmatrix}$ where a and b are constants.

(a) Find $\mathbf{R}^2$ in terms of a and b. **[2]**

This was almost always answered correctly, with just a few candidates who made a sign error or who did not simplify *ab* + *ab*. A very small number wrote down the central element as 0.

Question 9 (b)

The constants a and b are given by $a = \frac{\sqrt{2}}{4}(\sqrt{3}+1)$ and $b = \frac{\sqrt{2}}{4}(\sqrt{3}-1)$.

(b) By determining exact expressions for ab and $a^2 - b^2$ and using the result from part **(a)**,

show that $\mathbf{R}^2 = k\begin{pmatrix} \sqrt{3} & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & \sqrt{3} \end{pmatrix}$ where k is a real number whose value is to be determined.

[2]

This was a good question for using a calculator to assist with the algebraic manipulation. The majority of candidates gained 2 marks, but some did not pull out a factor of ½ convincingly (since the question is a "show that" full justification for the value of *k* needs to be given).

Question 9 (c)

(c) Find $\mathbf{R}^6$, $\mathbf{R}^{12}$ and $\mathbf{R}^{24}$. **[3]**

This question had quite a high omit rate. It was a good example of a question where the judicious use of a calculator would be beneficial to candidates.

The most common mistakes were using the matrix found at the end of part (b) as their **R** (and so actually calculating $\mathbf{R}^{12}$, $\mathbf{R}^{24}$ and $\mathbf{R}^{48}$), and from having a 1 at the centre matrix from part (b).

Question 9 (d)

(d) Describe fully the transformation represented by **R**. **[3]**

This question also had a high omit rate, with evidence on some scripts to suggest a few candidates were short of time at the end of the paper. Most candidates gained at least 1 mark for recognising that the transformation was a rotation. Some candidates described the transformation represented by $\mathbf{R}^2$, and so getting an erroneous angle of 30°.

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