OCR
Oxford Cambridge and RSA

Friday 26 May 2023 – Afternoon

AS Level Further Mathematics B (MEI)

Y413/01 Modelling with Algorithms

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

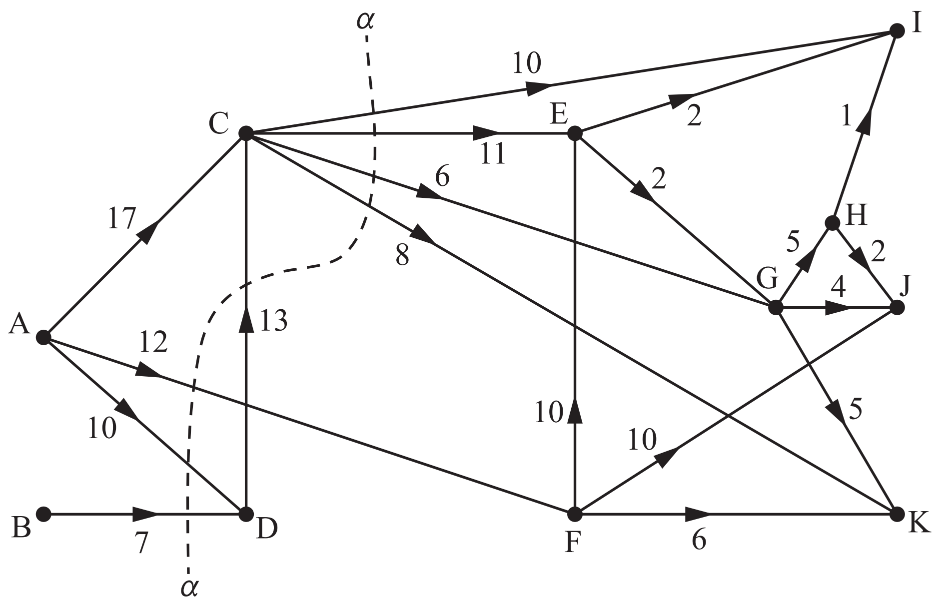
QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

1 **Fig. 1.1**



The diagram in **Fig. 1.1** represents a system of pipes through which a fluid can flow from two sources to three sinks. It also shows a cut α.

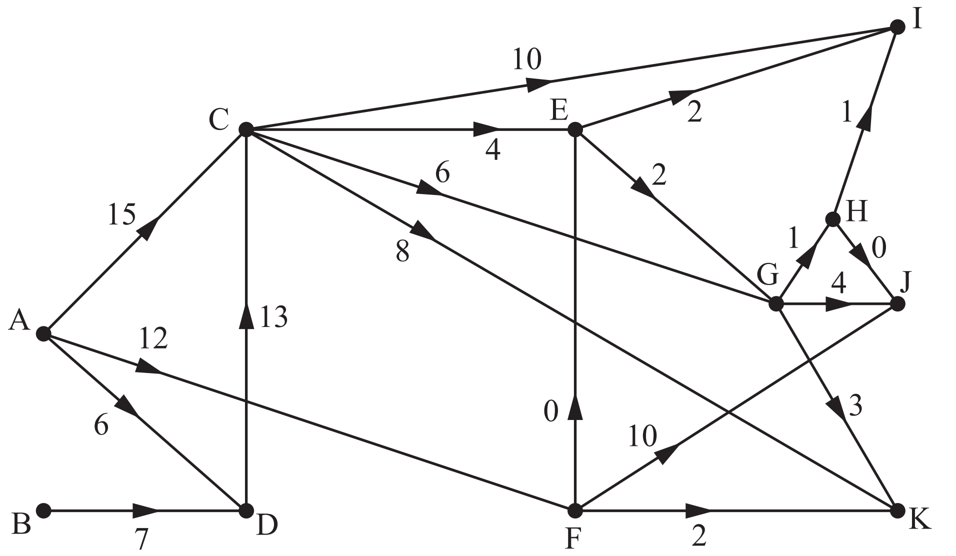
The weights on the arcs show the capacities of the pipes in gallons per minute.

(a) Add a supersource S and a supersink T to the network in the Printed Answer Booklet, giving appropriate weightings and directions to the connecting arcs. **[2]**

(b) Calculate the capacity of the cut α. **[1]**

The diagram in **Fig. 1.2** shows a feasible flow through the network.

Fig. 1.2



(c) Write down the amount of fluid, in gallons per minute, that **Fig. 1.2** shows flowing from the sources to the sinks. **[1]**

(d) Prove that the flow shown in **Fig. 1.2** is the maximum possible flow through the network. **[2]**

2 The journey times (in minutes) between eight towns, A, B, …, H, for which there is a direct route are given in the table in **Fig. 2**.

Fig. 2

	A	B	C	D	E	F	G	H
A	–	26	11	43	–	–	–	–
B	26	–	12	–	34	17	6	–
C	11	12	–	–	–	–	–	–
D	43	–	–	–	14	7	15	–
E	–	34	–	14	–	–	–	9
F	–	17	–	7	–	–	5	38
G	–	6	–	15	–	5	–	45
H	–	–	–	–	9	38	45	–

(a) Using the vertices given in the Printed Answer Booklet, draw a network to represent the information shown in **Fig. 2**. **[2]**

(b) **(i)** Apply Dijkstra's algorithm to the network drawn in part **(a)**, to find the length of the quickest route from A to H. **[5]**

(ii) Write down the quickest route from A to H. **[1]**

(c) Using your answers to part **(b)**, determine the total length of the arcs in a minimum spanning tree for the network given in **Fig. 2**.

You should not apply an algorithm to find this minimum spanning tree. **[2]**

3 The list below shows the sizes of 10 items.

17	15	18	9	23	20	14	12	25	11

(a) Use the quick sort algorithm to sort the list of numbers above into **descending** order. You should use the first value as the pivot for each sublist. **[3]**

The first fit decreasing algorithm is used to pack items with the sizes listed above into bins that have a capacity of m, where m is a positive integer.

The following packing is achieved.

Bin 1:	25	20	
Bin 2:	23	18	
Bin 3:	17	15	14
Bin 4:	12	11	9

(b) **(i)** By considering the placement of the three largest items, determine the possible values of m. **[1]**

(ii) By considering the total in each bin, further refine the possible values of m. **[1]**

(c) Using the original list, show the result of applying the first fit algorithm to pack items with the sizes listed above into bins that have a capacity of m, where m takes any of the possible values found in part **(b)(ii)**. **[2]**

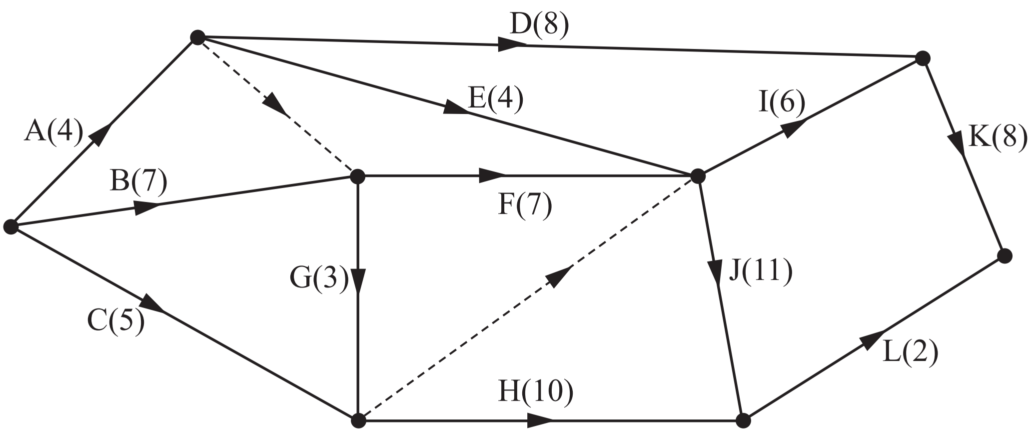
A list of n numbers is to be sorted into descending order using the quick sort algorithm. The number of comparisons made is used as a measure of complexity of the quick sort algorithm.

(d) **(i)** By considering a worst case, determine an expression in terms of n, for the total number of comparisons required to sort the list using the quick sort algorithm in the worst case. **[2]**

(ii) Hence state, in the worst case, the complexity of the quick sort algorithm. **[1]**

4 **Fig. 4.1** shows an activity network for a project. The arc weights show activity durations in days.

Fig. 4.1



(a) Complete the table in the Printed Answer Booklet to show the immediate predecessors for each activity. **[2]**

(b) Carry out a forward pass and a backward pass through the entire network to find

- The minimum completion time for the project
- The critical activities **[5]**

(c) Determine the total float for activity H. **[2]**

The table in **Fig. 4.2** shows the number of workers required for each activity.

It is given that when an activity is started it must be completed without interruption.

Fig. 4.2

Activity	Number of workers
A	1
B	1
C	2
D	1
E	2
F	1
G	1
H	1
I	2
J	1
K	1
L	3

(d) **(i)** Draw a resource histogram to show the number of workers required each day when each activity begins at its earliest possible start time. **[2]**

(ii) Hence state the minimum number of workers required to complete the project (in the minimum time), according to the histogram drawn in part **(d)(i)**. **[1]**

Turn over for question 5

5 Four students, Jamal (J), Kai (K), Layla (L), and Mia (M) are planning to participate in a 200 m swimming relay race.

Each of the four students must swim exactly one 50 m length. Furthermore, the students must each swim a different type of swimming stroke. The four types of swimming stroke are backstroke (A), front crawl (C), breaststroke (R) and butterfly (U).

The table in **Fig. 5.1** shows the average time, in seconds, each of the four students took to complete each of the four types of swimming stroke during practice.

Fig. 5.1

	A	C	R	U
J	39	32	37	41
K	39	34	38	42
L	40	36	41	40
M	42	33	42	43

The four students need to know, based on the times in **Fig. 5.1**, who should be allocated to swim each stroke to give them the best chance of winning the relay race.

The constraints for an LP formulation for this problem are as follows.

$$JA + JC + JR + JU = 1$$
$$KA + KC + KR + KU = 1$$
$$LA + LC + LR + LU = 1$$
$$MA + MC + MR + MU = 1$$
$$JA + KA + LA + MA = 1$$
$$JC + KC + LC + MC = 1$$
$$JR + KR + LR + MR = 1$$
$$JU + KU + LU + MU = 1$$

(a) Explain the purpose of the line JA + JC + JR + JU = 1 in the LP formulation. **[1]**

(b) Write down the objective function of the LP formulation. **[2]**

The complete LP was run in an LP solver and the output is shown in the table in **Fig. 5.2**.

Fig. 5.2

Variable	Value
JA	0.000000
JC	0.000000
JR	1.000000
JU	0.000000
KA	1.000000
KC	0.000000
KR	0.000000
KU	0.000000
LA	0.000000
LC	0.000000
LR	0.000000
LU	1.000000
MA	0.000000
MC	1.000000
MR	0.000000
MU	0.000000

(c) State the predicted total time for the team to complete the relay race, according to the output given in **Fig. 5.2**. **[1]**

A fifth student, Nina (N), is now available to swim one of the four 50 m lengths. Her times, on average, in seconds, to complete each of the four types of swimming stroke are shown in the table in **Fig. 5.3**.

Fig. 5.3

	A	C	R	U
N	38	43	37	42

(d) Explain how to adapt the LP formulation so that the LP solver can determine which of the five students should be allocated to swim each of the four 50 m lengths. **[3]**

When the adapted LP formulation is run through the LP solver it determines that Jamal should be allocated the front crawl (C) and Kai should be allocated the breaststroke (R).

(e) Determine the difference in the predicted times for the team to complete the relay race when Nina is in the team compared with when she is not in the team. **[2]**

6 **Fig. 6.1**

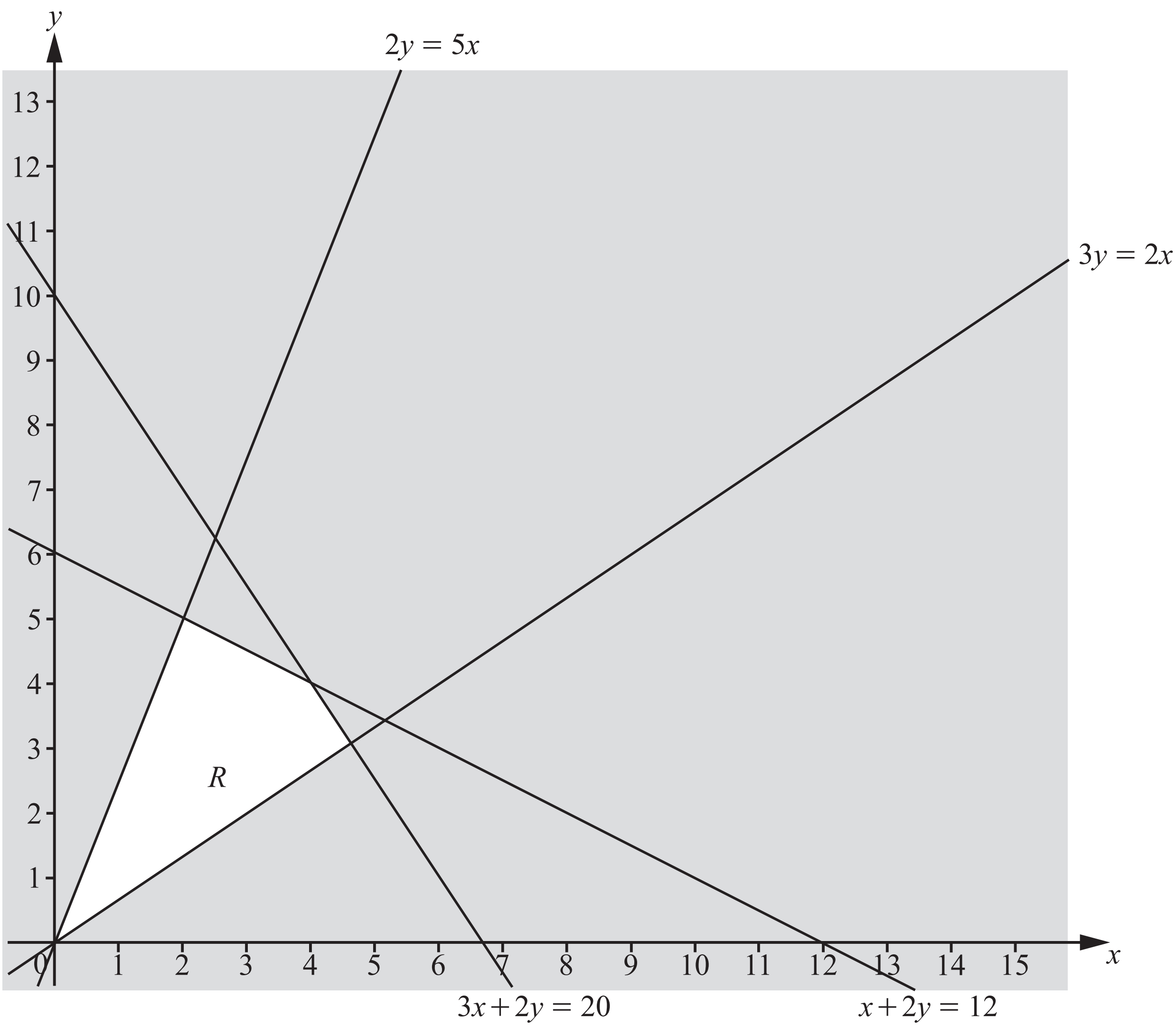


Fig. 6.1 shows the constraints of a linear programming problem, in which the objective is to maximise $P = x + ky$, where k is a positive constant.

The feasible region, R, is the unshaded region together with its boundaries.

(a) The simplex method could be used to solve this problem.

- Show how the constraints for the problem can be made into equations using slack variables.
- Show how the row for the objective function can be formed.
- Complete the initial tableau in the Printed Answer Booklet so that the simplex method may be used to solve this problem. **[6]**

After two iterations of the simplex method a computer produces the tableau in **Fig. 6.2**. It is given that the tableau in **Fig. 6.2** does not give an optimal solution to the LP problem.

Fig. 6.2

P	x	y	s_1	s_2	s_3	s_4	RHS
1	0	0	$-\frac{1}{6}+\frac{1}{12}k$	0	0	$\frac{1}{6}+\frac{5}{12}k$	$2+5k$
0	0	1	$\frac{1}{12}$	0	0	$\frac{5}{12}$	5
0	0	0	$\frac{1}{3}$	1	0	$-\frac{4}{3}$	4
0	0	0	$\frac{7}{12}$	0	1	$\frac{11}{12}$	11
0	1	0	$-\frac{1}{6}$	0	0	$\frac{1}{6}$	2

(b) Perform a third iteration of the simplex method. **[3]**

(c) Given that an optimal solution to the LP problem is found after a third iteration of the simplex method, determine the range of possible values of P. **[4]**

END OF QUESTION PAPER