

Thursday 25 May 2023 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Kinematics

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Section A

Pure Mathematics

- 1 The quadratic equation $kx^2 + 3x + k = 0$ has no real roots.

Determine the set of possible values of k .

[3]

- 2 **In this question you must show detailed reasoning.**

Solve the equation $x\sqrt{5} + 32 = x\sqrt{45} + 2x$. Give your answer in the form $a\sqrt{5} + b$, where a and b are integers to be determined. [4]

- 3 A Ferris wheel at a fairground rotates in a vertical plane. The height above the ground of a seat on the wheel is h metres at time t seconds after the seat is at its lowest point.

The height is given by the equation $h = 15 - 14 \cos(kt)^\circ$, where k is a positive constant.

(a) (i) Write down the greatest height of a seat above the ground. [1]

(ii) Write down the least height of a seat above the ground. [1]

(b) Given that a seat first returns to its lowest point after 150 seconds, calculate the value of k . [2]

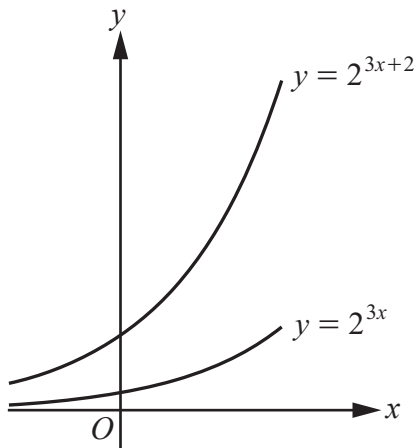
(c) Determine for how long a seat is 20 metres or more above the ground during one complete revolution. Give your answer correct to the nearest tenth of a second. [4]

- 4 (a) Find and simplify the first three terms in the expansion, in ascending powers of x , of $\left(2 + \frac{1}{3}kx\right)^6$, where k is a constant. [3]

(b) In the expansion of $(3 - 4x)\left(2 + \frac{1}{3}kx\right)^6$, the constant term is equal to the coefficient of x^2 .

Determine the exact value of k , given that k is positive. [3]

5



The diagram shows the graphs of $y = 2^{3x}$ and $y = 2^{3x+2}$. The graph of $y = 2^{3x}$ can be transformed to the graph of $y = 2^{3x+2}$ by means of a stretch.

- (a) Give details of the stretch. [2]

The point A lies on $y = 2^{3x}$ and the point B lies on $y = 2^{3x+2}$. The line segment AB is parallel to the y -axis and the difference between the y -coordinates of A and B is 36.

- (b) Determine the x -coordinate of A . Give your answer in the form $m \log_2 n$ where m and n are constants to be determined. [3]

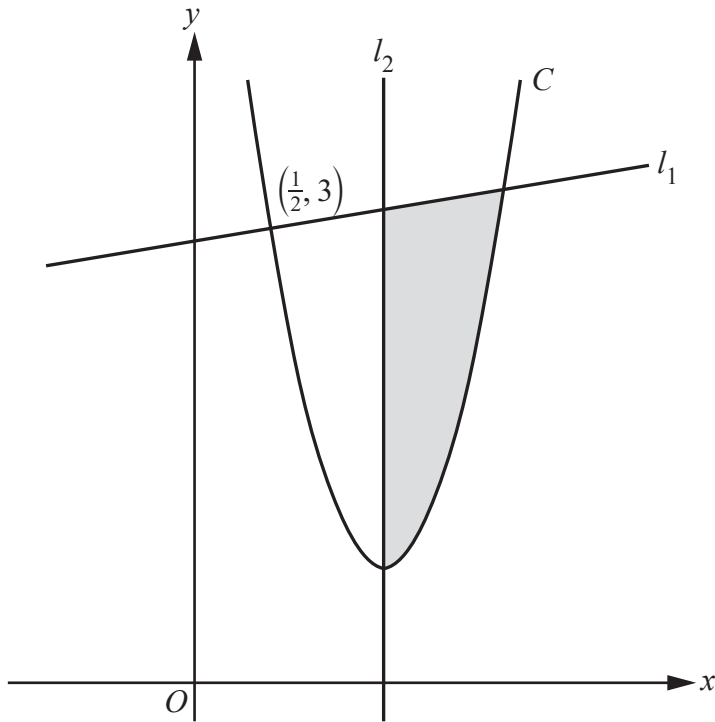
- 6 The vertices of triangle ABC are $A(-3, 1)$, $B(5, 0)$ and $C(9, 7)$.

- (a) Show that $AB = BC$. [2]
- (b) Show that angle ABC is **not** a right angle. [2]
- (c) Find the coordinates of the midpoint of AC . [1]
- (d) Determine the equation of the line of symmetry of the triangle, giving your answer in the form $px + qy = r$, where p , q and r are integers to be determined. [2]
- (e) Write down an equation of the circle with centre A which passes through B . [2]

This circle intersects the line of symmetry of the triangle at B and at a second point.

- (f) Find the coordinates of this second point. [1]

7



The diagram shows the curve C with equation $y = 4x^2 - 10x + 7$ and two straight lines, l_1 and l_2 . The line l_1 is the normal to C at the point $(\frac{1}{2}, 3)$. The line l_2 is the normal to C at the minimum point of C .

- (a) Determine the equation of l_1 , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers to be determined. [4]

The shaded region shown in the diagram is bounded by C , l_1 and l_2 .

- (b) Determine the inequalities that define the shaded region, including its boundaries. [3]

8 In this question you must show detailed reasoning.

Given that $\int_4^a \left(\frac{4}{\sqrt{x}} + 3 \right) dx = 7$, find the value of a . [7]

Section B

Mechanics

- 9 A cyclist travels along a straight horizontal road between house A and house B .

The cyclist starts from rest at A and moves with constant acceleration for 20 seconds, reaching a velocity of 15 m s^{-1} . The cyclist then moves at this constant velocity before decelerating at 0.3 m s^{-2} , coming to rest at B .

- (a) Find the time, in seconds, for which the cyclist is decelerating. [1]
- (b) Sketch a velocity-time graph for the motion of the cyclist between A and B . [Your sketch need not be drawn to scale; numerical values need not be shown.] [1]

The total distance between A and B is 1950 m.

- (c) Find the time, in seconds, for which the cyclist is moving at constant velocity. [2]

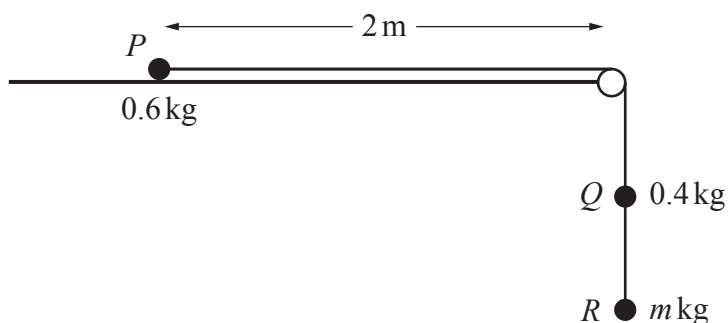
- 10 A particle P is moving in a straight line. At time t seconds, where $t \geq 0$, P has velocity $v \text{ m s}^{-1}$ and acceleration $a \text{ m s}^{-2}$ where $a = 4t - 9$. It is given that $v = 2$ when $t = 1$.

- (a) Find an expression for v in terms of t . [3]

The particle P is instantaneously at rest when $t = t_1$ and $t = t_2$, where $t_1 < t_2$.

- (b) Find the values of t_1 and t_2 . [2]
- (c) Determine the total distance travelled by P between times $t = 0$ and $t = t_2$. [3]

- 11 Two balls P and Q have masses 0.6 kg and 0.4 kg respectively. The balls are attached to the ends of a string. The string passes over a pulley which is fixed at the edge of a rough horizontal surface. Ball P is held at rest on the surface 2 m from the pulley. Ball Q hangs vertically below the pulley. Ball Q is attached to a third ball R of mass $m\text{ kg}$ by another string and R hangs vertically below Q (see diagram).



The system is released from rest with the strings taut. Ball P moves towards the pulley with acceleration 3.5 m s^{-2} and a constant frictional force of magnitude 4.5 N opposes the motion of P .

The balls are modelled as particles, the pulley is modelled as being small and smooth, and the strings are modelled as being light and inextensible.

- (a) By considering the motion of P , find the tension in the string connecting P and Q . [2]
- (b) Hence determine the value of m . Give your answer correct to 3 significant figures. [4]

When the balls have been in motion for 0.4 seconds the string connecting Q and R breaks.

- (c) Show that, according to the model, P does not reach the pulley. [6]

It is given that in fact ball P does reach the pulley.

- (d) Identify one factor in the modelling that could account for this difference. [1]

END OF QUESTION PAPER

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