

GCE

Further Mathematics B (MEI)

Y413/01: MEI Modelling with algorithms

AS Level

Mark Scheme for June 2023

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ≭	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
Е	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in	Meaning
mark scheme	
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction In this question you must show detailed reasoning appears in the question.

2. Subject-specific Marking Instructions for AS/A Level Further Mathematics B (MEI)

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

Μ

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

В

Mark for a correct result or statement independent of Method marks.

Е

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it

Mark Scheme

easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
 - NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f"

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

- g Rules for replaced work and multiple attempts:
 - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
 - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
 - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" and "Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

	Quest	ion	Answer	Marks	AOs	Guidance
1	(a)		A supersource labelled S with arcs SA and SB with the following weight (or greater): $SA = 39$ and $SB = 7$ with arrows	B1	1.1	If B0 B0 then SC B1 for arrows or weights correct but must only have the correct 5 arcs
			A supersink labelled T with arcs IT, JT and KT with the following weights (or greater): $IT = 13$, $JT = 16$ and $KT = 19$ with arrows	B1	1.1	
				[2]		
1	(b)		Cut $(= 10 + 11 + 6 + 8 + 12 + 10 + 7) = 64$ (gallons per minute)	B1	2.2a	
				[1]		
1	(c)		40	B1	1.1	
				[1]		
1	(d)		The capacity of the cut which partitions the vertices into the sets {S, A, B, C, D, E}, {F, G, H, I, J, K, T} (is $10 + 2 + 2 + 6 + 8 + 12 = 40$)	B1*	3.1b	Condone without S and/or T - allow listing of cut arcs (CI, EI, EG, FE, CG, CK, AF)
			\therefore min. cut ≤ 40 by the maximum flow-minimum cut theorem the maximum flow is equal to the minimum cut and so therefore the maximum flow through the system is 40 (gallons per minute)	B1dep*	2.1	Requires correct cut, 40 stated and mention of max flow-min cut theorem
				[2]		

Mark Scheme

(Quest	ion	Answer	Marks	AOs	Guidance
2	(a)					
2	(b)	(i)	Key: Order of labelling Working values (do not cross out)	M1	1.1	All arcs present (allow one absent or one extra or one wrong arc)
			$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	A1	1.1	cao (including weights but no arrows)
				[2]		
			$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	M1	1.2	Correct working values at B
			38 14 7 5	A1	1.1a	Working values
			$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	A1	1.1 a	Labels
			43 (44) H	A1	1.1	Order of labelling
			Length of quickest route from A to H is 64 (mins)	A1ft	1.1	Follow through their Label at H only
				[5]		
2	(b)	(ii)	Quickest route from A to H is ACBGFDEH	B1	3.1b	cao
				[1]		
2	(c)		As the quickest route from A to H passes through all the nodes of the network (the length of the shortest path from A to H and the total length of the MST are the same)	B1	2.4	Explanation that the quickest route from A to H passes through all the nodes
			Total length of the arcs in MST is therefore 64	B1ft	1.1	Follow through their answer to (b)(i) provided their route from A to H in (b)(ii) passes through all the nodes
				[2]		_

	Quest	ion	Answer	Marks	AOs	Guidance
3	(a)		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	M1	1.1	17 used as the first pivot and in the correct
			23 20 25 18 17 15 9 14 12 11 (2 nd pass) 25 23 20 18 17 15 14 12 11 9 (3 rd pass) 25 23 20 18 17 15 14 12 11 9 (3 rd pass) 25 23 20 18 17 15 14 12 11 9 (4 th pass)	A1	1.1	position after the first pass First three passes correct – must be using quick (not slow) sort
			25 23 20 18 17 15 14 12 11 9 (5 th pass) (All sublists have only one element so sort is complete)	A1	1.1	Correct sort with (at least) a fifth pass in which no changes are made
				[3]		Mark ascending as a misread
3	(b)	(i)	23 has been placed in Bin 2 so therefore $m < 48$ (or $m \le 47$) but 20 has been placed in Bin 1 which implies that $m = 45,46$ or 47 (or $45 \le m < 48$)	B1	2.1	Considers bins 1 and 2 and the placement of (25), 23 and the 20
3	(b)	(ii)	The total of the items in Bin 3 is $17 + 15 + 14 = 46$ and so therefore <i>m</i> is either 46 or 47	[1] B1 [1]	2.2a	Correct deduction that <i>m</i> is either 46 or 47
3	(c)		Bin 1: 17 15 9 Bin 2: 18 23 Bin 3: 20 14 12 Bin 4: 25 11	M1 A1	1.1 1.1	First six numbers placed correctly (those in bold) cao (no additional/repeated values)
				[2]		

(Quest	ion	Answer	Marks	AOs	Guidance
3	3 (d) (i)		Worst case is any list for which at each pass the pivot is either the largest or the smallest value remaining 1^{st} pass: 1^{st} number is compared with all other numbers so (n-1) comparisons 2^{nd} pass: 2^{nd} number is compared with the remaining $(n-2)$ numbers so $(n-2)$ comparisons and so on	M1	2.1	Considers the worst case and the number of comparisons in each pass (at least two passes) e.g. list is already in the correct order e.g. list is already in reverse order etc.
			So total number of comparisons is = $(n-1)+(n-2)++3+2+1=\frac{1}{2}n(n-1)$	A1 [2]	2.2a	If M0 then SC B1 for the correct answer (either simplified or correct summation) without (or with an incorrect) explanation
	(d) (ii)		Complexity of quick sort is therefore $O(n^2)$	B1ft [1]	1.2	Allow quadratic complexity or n^2 Allow follow through from (d)(i) provided M mark earned

Mark Scheme

(Quest	ion		Answer		Marks	AOs	Guidance
4	(a)		Activity	Immediate Predecessor(s)				
			А	-				
			В	-				
			С	-				
			D	A		B1	1.1	Any 5 rows correct
			E	A				(not including rows A, B, C)
			F	A, B				
			G	A, B		B1	1.1	cao
			Н	C, G				
			Ι	C, E, F, G				
			J	C, E, F, G				
			K	D, I				
			L	H, J		[0]		
4						[2]		
4	(b)			4 7	20 20	M1	1.1	Forward pass – numbers increasing from source to sink (allow one slip) – all
				B(7) 7 7 F(7) C(5) G(3) H(10)	I(6) K(8) I4 I4 J(11) L(2)	M1	1.1	complete Backward pass – numbers decreasing from sink to source (allow one slip) – all complete (condone missing 0)
				10 14	5 26	A1	1.1	cao for backward and forward pass
				completion time is 28 (days) ivities are B, F, I and K		B1 B1 [5]	1.1 1.1	

Mark Scheme

	Quest	ion	Answer	Marks	AOs	Guidance
4	(c)		[•] 26 [•] - [•] 10 [•] - 10	M1	1.1	Correct method for finding the total float
			= 6 (days)	A1	1.1	for their activity H cao – correct answer of 6 with no working scores SC B1
				[2]		
4	4 (d)	(i)		M1 A1	3.3 1.1	Plausible histogram with no holes or overhangs. Must be attempted to at least day 14 and the first four days must have exactly 4 workers cao
			3	[2]		
4	(d)	(ii)	6	B1	3.4	Dependent on scoring the M mark in (d)(i)
				[1]		

(Question	Answer	Marks	AOs	Guidance
5	(a)	JA + JC + JR + JU = 1 ensures that Jamal can only be allocated one of the four swimming strokes	B1 [1]	3.3	Must mention Jamal (or J) and can <u>only</u> be allocated <u>one</u> of the four strokes (or <u>exactly</u> <u>one</u> of the four strokes)
5	(b)	Minimise 39JA + 32JC + 37JR +41 JU + 39KA + 34KC + 38KR + 42KU + 40LA + 36LC + 41LR + 40 LU + 42MA + 33MC + 42MR + 43MU	B1 B1 [2]	2.5 3.3	Either 'minimise' or 'min'
5	(c)	(37 + 39 + 40 + 33 =) 149 (seconds)	B1 [1]	3.5a	cao
5	(d)	The final four constraints (the constraints that guarantees all of the four stroke are completed) need Nina adding to them JA + KA + LA + MA + NA = 1 JC + KC + LC + MC + NC = 1 JR + KR + LR + MR + NR = 1 JU + KU + LU + MU + NU = 1	B1	3.5b	Correct explanation of how the final four constraints need modifying so that only one person can be allocated to each stroke – must explicitly see NA, NC, NR, NU being referred to
		Change = 1 to \leq 1 in the first four constraints (the constraints that restrict the swimmers to one stroke) and add a fifth constraint for N JA + JC + JR + JU \leq 1 KA + KC + KR + KU \leq 1 LA + LC + LR + LU \leq 1 MA + MC + MR + MU \leq 1 NA + NC + NR + NU \leq 1	B1	3.5c	Explanation regarding the other constraints required to allow 5 people to be allocated to only 4 activities – must explicitly see the constraint NA + NC + NR + NU \leq 1 stated and the changing in the first four constraints to \leq 1
		38NA + 43NC + 37NR + 42NU needs to be added to the objective function	B1	1.1	

(Questio	n Answer	Marks	AOs	Guidance
5	(d)	Alternative method The final four constraints need Nina adding to them JA + KA + LA + MA + NA = 1 JC + KC + LC + MC + NC = 1 JR + KR + LR + MR + NR = 1 JU + KU + LU + MU + NU = 1	B1		
		Add a dummy stroke X and add a fifth constraint for N. JA + JC + JR + JU + JX = 1 KA + KC + KR + KU + KX = 1 LA + LC + LR + LU + LX = 1 MA + MC + MR + MU + MX = 1 NA + NC + NR + NU + NX = 1 Add a constraint for the dummy stroke JX + KX + LX + MX + NX = 1 38NA + 43NC + 37NR + 42NU + k (JX + KX + LX + MX + NX) needs to be added to the objective function	B1 B1		Where k is a constant that is greater than 43
			[3]		(to ensure that it will not have an impact on the required allocation)
5	(e)	If J = C and, K = R then L should be chosen for U and N for A (leaving M not swimming) as these two times (40 and 38) are the quickest two times for these two strokes	B1	3.1b	Determining that $L = U$ and $N = A$ (or the correct calculation $32 + 38 + 40 + 38$ seen)
		The total time is $32 + 38 + 40 + 38 = 148$ therefore giving a difference of 1 (second)	B1 [2]	1.1	For 1 (second)

	Question		Answer									Guidance
6	(a)	$2y \le 5x$	$2y \le 5x \Longrightarrow -5x + 2y \le 0 \therefore -5x + 2y + s_1 = 0$								3.1 a	Convert given equations into equations
		3x+2y	$\leq 20 \Rightarrow$	3x+2y	$+s_2 = 2$	0						with slack variables: evidenced by one
		$3y \ge 2x$	$z \Rightarrow 2x -$	$3y \leq 0$.	$\therefore 2x - 3y$	$v + s_3 = 0$	C					correct equation with a slack variable from
		x+2y=	$\leq 12 \Rightarrow x$	x + 2y + x	$s_4 = 12$							a correct inequality stated or all four
												inequalities stated correctly
										A1	1.1	Any two slack equations explicitly stated from corresponding inequalities explicitly stated too
										A1	1.1	All four slack equations explicitly stated from corresponding inequalities explicitly stated too
		P = x +	$ky \Rightarrow P$	-x-ky	=0					B 1	3.1 a	Must be explicitly stated (and include = 0)
		P	x	у	<i>s</i> ₁	<i>s</i> ₂	<i>s</i> ₃	<i>s</i> ₄	RHS			
										M1	3.3	Two constraint rows correct (from correct
		1	-1	-k	0	0	0	0	0			inequalities/equations if seen)
		0	-5	2	1	0	0	0	0			and (from compating qualities / quations if
		0	3	2	0	1	0	0	20	A1	1.1	cao (from correct inequalities/equations if seen)
		0	2	-3	0	0	1	0	0			seen
		0	1	2	0	0	0	1	12			
										[6]		

(Quest	ion				A	nswer				Marks	AOs	Guidance
6	(b)		Р	x	У	<i>s</i> ₁	<i>s</i> ₂	<i>s</i> ₃	<i>s</i> ₄	RHS			
			1	0	0	0	$\frac{1}{2} - \frac{k}{4}$	0	$-\frac{1}{2}+\frac{3}{4}k$	4 + 4k	M1	3.4	Pivot (third) row and column (s_1) correct
			0	0	1	0	$-\frac{1}{4}$	0	$\frac{3}{4}$	4			(condone one slip on one value in the pivot row but the (s_1) column must be fully correct)
			0	0	0	1	3	0	-4	12			
			0	0	0	0	$-\frac{7}{4}$	1	$\frac{13}{4}$	4	A1	1.1	One of columns s_2, s_4 or RHS correct
			0	1	0	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	4	A1	1.1	cao
											[3]		

6	(c)	Optimal after the 3 rd iteration $\Rightarrow \frac{1}{2} - \frac{k}{4} \ge 0$ and $-\frac{1}{2} + \frac{3}{4}k \ge 0$	M1	3.4	Allow consideration of one of these two correct expressions with > 0 or ≥ 0 for this M mark
		$\frac{2}{3} \le k \le 2$	A1	1.1	
		Not optimal after the 2 nd iteration $\Rightarrow -\frac{1}{6} + \frac{k}{12} < 0$ therefore $k < 2$	B1	3.1 a	Ignore consideration of $\frac{1}{6} + \frac{5}{12}k$ - B0 if just stating that $k < 2$ without justification (i.e. must see correct inequality)
		So $\frac{2}{3} \le k < 2 \Longrightarrow \frac{20}{3} \le P < 12$	A1	2.2a	Dependent on all previous marks
			[4]		

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