Oxford Cambridge and RSA

Thursday 18 May 2023 – Afternoon

Level 3 Certificate Core Maths B (MEI)

H869/01 Introduction to Quantitative Reasoning

Insert

Time allowed: 2 hours

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INSTRUCTIONS
- Do **not** send this Insert for marking. Keep it in the centre or recycle it.

INFORMATION
- This Insert contains the pre-release material that you have already seen.
- This document has **8** pages.

A Containers

Containers for transporting goods have been used on a large scale for almost 70 years.
Transporting goods by container has several advantages. The goods are secure and as containers are made in standard sizes they are easily stacked on ships, so that loading and unloading is quick and efficient. **Fig. A.1** shows a typical modern container ship.

Fig. A.1

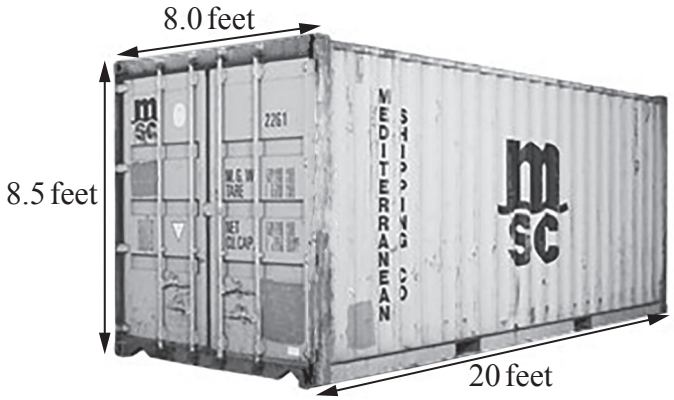
Containers are made from aluminium and steel and are usually airtight. Old ones can be adapted and put to many uses such as offices, housing units (**Fig. A.2**) or even small swimming pools **(Fig. A.3)**.

Fig. A.2

Fig. A.3

The dimensions of containers are convenient numbers when measured in imperial units.
The one in **Fig. A.4** measures 8.5 feet (2.591 m) by 8 feet (2.438 m) by 20 feet (6.096 m).
These containers are called TEUs (twenty-foot equivalents).

Fig. A.4



TEU containers weigh about 2 tonnes when empty.
They can hold a maximum of about 21 tonnes of goods.

The largest container ships can carry total loads of over 20 000 TEUs.

B Pedestrian accidents

Road accidents resulting in injury to pedestrians are a world-wide problem. A "serious injury" is one that involves the pedestrian being hospitalised. An injury that only requires, at most, first aid is described as a "slight injury". According to The World Health Organisation, each year well over 10 million pedestrians are seriously injured in the world.

Research suggests that reducing the number of road accidents involving pedestrians can be achieved by imposing driving speed limits. Countries have used various methods such as road bumps, road narrowing and marked pedestrian crossings to support this.

Road bumps, like that shown in **Fig. B.1**, are not popular with many motorists who suggest that they may damage cars.

Fig. B.1

Several cities in the world have declared whole city regions to be 20 mph (or 30 km/h) zones. These zones are clearly marked and publicised; no road bumps are used.

The table in **Fig. B.2** shows the accident figures for such a zone in a UK city. The data are for a period 81 months **before** the zone was put in place, and for 27 months **after** the zone was put in place.

Fig. B.2

Type of injury	81 months **before** the 20 mph speed limit imposed		27 months **after** the 20 mph speed limit imposed	
	Number of injuries	Mean annual number of injuries	Number of injuries	Mean annual number of injuries
Serious	128	19.0	36	16.0
Slight	811	120.1	207	92.0
Total	939	139.1	243	108.0

To help judge the effect of zoning, **Fig. B.2** includes the mean numbers of injuries per year over the two periods (before and after) as well as the total numbers.

For example, there was a total of 939 accidents in the 81 months before the 20 mph speed limit; 81 months is 6.75 years, so the mean annual number of accidents is 939 ÷ 6.75 which is 139.1 (1 dp).

The data in the table show that for this particular zone the mean annual number of serious injuries dropped from 19.0 to 16.0 after imposing the 20 mph limit.

C Passwords

Passwords are essential for online security. Unfortunately, many people use common passwords. These five appear top of popularity lists year after year: “123456”, “123456789”, “password”, “qwerty” and “abc123”. These are quite easily guessed, as are dates of birth or celebrity names.

Many techniques used by hackers involve “brute force”; trying all the possible passwords to find the correct one. Lists of several million of the most used passwords help them. Words appearing in dictionaries can also be tried. Although there are about 470 000 words in the larger English dictionaries, a computer can work through them in a relatively short time. Because of this, many cyber security organisations discourage using real words.

Imagine forming a 5-character password from 26 different characters.
Characters may be repeated within a password.

- There are 26 possibilities for the first character.
- For each one of these there are 26 possible second characters so, there are $26 \times 26 = 676$ possible different 2-character passwords.
- Similarly, there are $26 \times 26 \times 26 = 17\,576$ different 3-character passwords.
- The pattern continues giving $26^5 = 11\,881\,376$ different 5-character passwords from 26 different characters.

Using the 26 upper case and 26 lower case letters there are $26 + 26 = 52$ different characters to use giving $52^8 = 53\,459\,728\,531\,456$ different 8-letter passwords. These do not have to be proper words so yuSSoPRm counts as a password. Even a computer would take some time to try all these possible passwords.

A simple method to make it harder for hackers is called throttling. Your computer can be programmed to pause for 1 second between each incorrect attempt at a password. This severely slows down the rate at which a hacker’s computer can try passwords.

Another method is not to show how many characters long the password is. Using brute force, the hacker would need to try a number of different length passwords. For example, if the length of a password is 10, 11 or 12 out of 52 characters there are a total of 398 538 427 101 550 215 168 different passwords.

$52^{10} =$ 144 555 105 949 057 024 different 10-character passwords.
$52^{11} =$ 7 516 865 509 350 965 248 different 11-character passwords.
$52^{12} =$ 390 877 006 486 250 192 896 different 12-character passwords.

D Using indicators

International organisations, like the OECD and UNICEF, collect data on topics such as countries' poverty, education, medical provision, etc. The resulting information is presented as **indicators**; they can be used to highlight where improvements are needed, possibly involving greater support.

Topics can involve several features, for example, level of poverty might be expected to include at least these features: nutrition level, child mortality, life expectancy, number of people per doctor and average income.

To construct an indicator, you need to start with a scale of likely values. The smallest of these is called the "Minimum set global figure" and the largest is the "Maximum set global figure". These figures are set by experts, and need not necessarily be the actual maximum or minimum global values.

To make comparisons easier, the scale for the values of indicators is from 0 to 1.
The indicator for a particular feature, say A, is given by

$$\frac{\text{Value of Feature A} - \text{Minimum set global figure of Feature A}}{\text{Maximum set global figure of Feature A} - \text{Minimum set global figure of Feature A}}$$

Life expectancy has two features: life expectancy for males and life expectancy for females.
The minimum and maximum set global figures are shown in **Fig. D.1**.
This example shows the general rule that to find the indicator for a topic with several features, you take the mean of the values of the indicators for the features.

Fig. D.1

Feature	Minimum set global figure (years)	Maximum set global figure (years)
Female life expectancy	22.5	87.5
Male life expectancy	17.5	82.5

In the UK life expectancy for females is 83.3 years and for males is 80.2 years.
This gives:

Life expectancy indicator for UK males	Life expectancy indicator for UK females
$\frac{80.2-17.5}{82.5-17.5} = 0.9646$ (4 dp)	$\frac{83.3-22.5}{87.5-22.5} = 0.9354$ (4 dp)

The mean of these two indicators gives an overall indicator for UK life expectancy of $\frac{0.9646+0.9354}{2} = 0.9500$.

If a country exceeds the maximum value for an indicator, values are capped at the set global figure.
For example, in Hong Kong female life expectancy is 88.2 years, which is greater than the set global maximum of 87.5 years.

So, in this case, the female life expectancy is capped at 87.5 years giving an indicator of 1.
If the life expectancy was not capped at 87.5 the indicator

would be $\frac{88.2-22.5}{87.5-22.5} = \frac{65.7}{65} = 1.01$ (2 dp) which is outside the permitted range of 0 to 1.

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