OCR
Oxford Cambridge and RSA

Friday 23 June 2023 – Afternoon

Level 3 Free Standing Mathematics Qualification: Additional Mathematics

6993/01 Paper 1

Time allowed: 2 hours

8931169575

You must have:
- the Formulae Sheet for Level 3 Free Standing Mathematics Qualification: Additional Mathematics (inside this document)

You can use:
- a scientific or graphical calculator

6993 01

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **24** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
FSMQ Additional Mathematics (6993)

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n$, for positive integers, n,

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$, $r \leqslant n$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

Numerical methods

Trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Kinematics

Variable acceleration formulae

$v = \frac{\mathrm{d}s}{\mathrm{d}t}$

$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$

$s = \int v \, \mathrm{d}t$ and $v = \int a \, \mathrm{d}t$

Constant acceleration formulae

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

1 Find the term independent of x in the expansion of $\left(x+\frac{2}{x}\right)^6$. **[3]**

1	

2 Jack throws 4 ordinary six-sided dice numbered 1 to 6.

Find the probability that he throws at least one 3. **[3]**

2	

3 Use long division to find the quotient and the remainder when $x^3 + 3x^2 + 5x - 3$ is divided by $x + 1$. **[3]**

3	
	Quotient =
	Remainder =

4 Simplify the following.

(a) $\dfrac{1}{x-2}-\dfrac{2}{x+1}$ **[2]**

(b) In this question you must show detailed reasoning.

$\dfrac{2}{5-\sqrt{2}}+\dfrac{1}{5+\sqrt{2}}$ **[3]**

4(a)	
4(b)	

5 You are given that $\sin\theta = -0.6$ for $270° \leqslant \theta \leqslant 360°$.

(a) Find the value of θ. **[2]**

(b) Using Pythagoras' theorem, determine the **exact** value of $\tan\theta$. **[4]**

5(a)	
5(b)	

6 A car accelerates from rest in a straight line. At time t seconds its velocity, v m s^{-1}, is given by the equation $v = 20\left(1 - 2^{-\frac{t}{2}}\right)$.

(a) Calculate the velocity of the car when $t = 4$, 6 and 8 seconds. **[2]**

(b) Hence calculate an estimate of the acceleration of the car at $t = 6$ seconds. Give your answer correct to **2** significant figures. **[2]**

(c) Explain how this estimate could be improved. **[1]**

6(a)	
6(b)	
6(c)	

7 You are given that the equation $3^x - 4x^2 = 0$ has three roots, α, β and γ where $\alpha < 0$ and $\gamma > 3$.

(a) By considering the value of $3^x - 4x^2$ when $x = 0$ and $x = 1$, show that β lies between 0 and 1. **[2]**

(b) By considering appropriate values of x, determine the value of β correct to **1** decimal place. **[3]**

7(a)	
7(b)	

8 The triangle ABC is such that AB = 12 cm and angle A = 50°.

(a) Given that BC = 10 cm, determine the **two** possible values of angle C. **[4]**

(b) State **two** conditions for BC such that if **either** of them is satisfied there will be only **one** value for the angle C. **[2]**

8(a)	
8(b)	
	Condition 1:
	Condition 2:

9 The point A has the coordinate (3, 7) and the point B has the coordinate (7, 1).

Find the equation of the perpendicular bisector of AB. **[5]**

9	

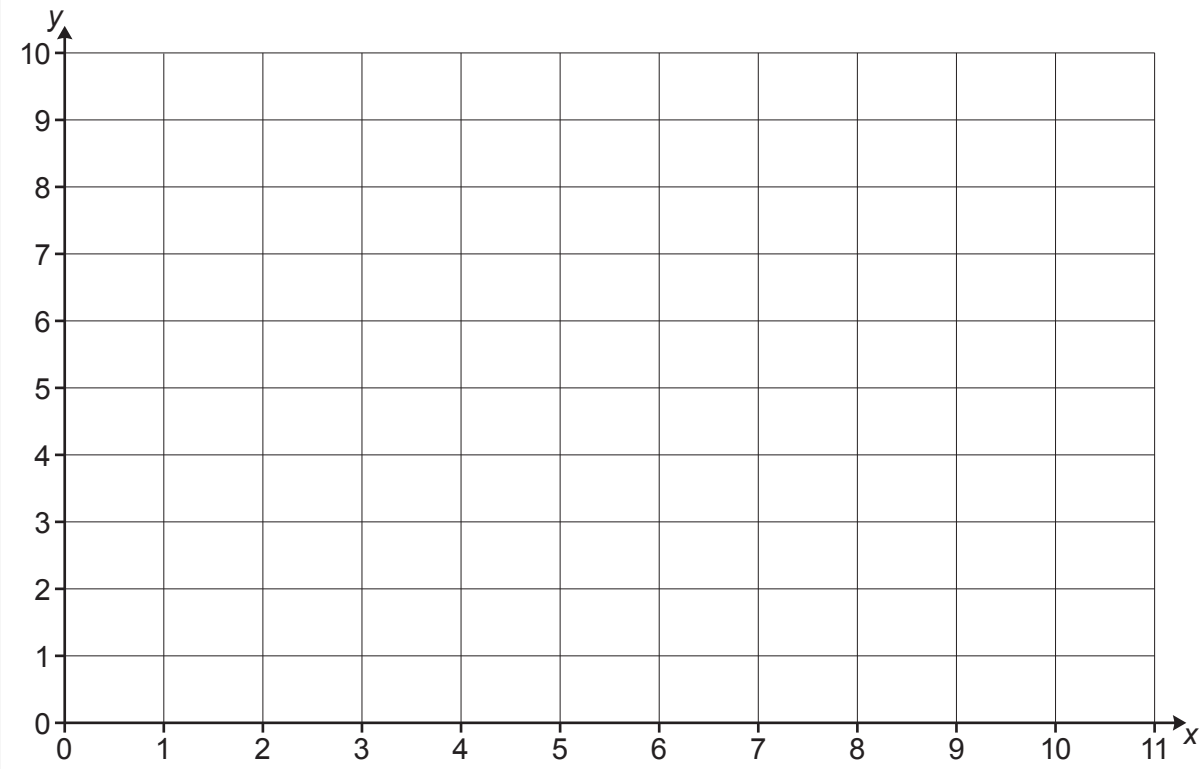
10 (a) On the grid below, indicate the region for which the following inequalities hold. You should shade the region that is **not** satisfied by the inequalities.

$y \geqslant x + 1$

$x \geqslant 1$

$x + 2y \leqslant 11$ **[4]**

10(a)



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(b) Find the maximum value of $x + y$ subject to these conditions. **[2]**

10(b)

11 Amir asked 80 people about their preferences for the drinks tea, coffee or hot chocolate.
The results of his investigation were as follows.

25 liked all three drinks.
3 liked tea and coffee but not hot chocolate.
4 liked hot chocolate and coffee but not tea.
5 liked tea but neither of the other two drinks.
35 liked tea and hot chocolate.
48 liked hot chocolate.
47 liked coffee.

(a) Draw a Venn diagram to represent these data. **[3]**

(b) Hence determine how many people said that they did **not** like any of the drinks. **[2]**

11(a)	
11(b)	

12 The point A (1, 3) lies on a circle with centre (4, 5).

(a) Determine the equation of the circle. **[2]**

B is a point on the circle such that AB is a diameter of the circle.

(b) Find the coordinates of B. **[2]**

D is the point (2, 8).

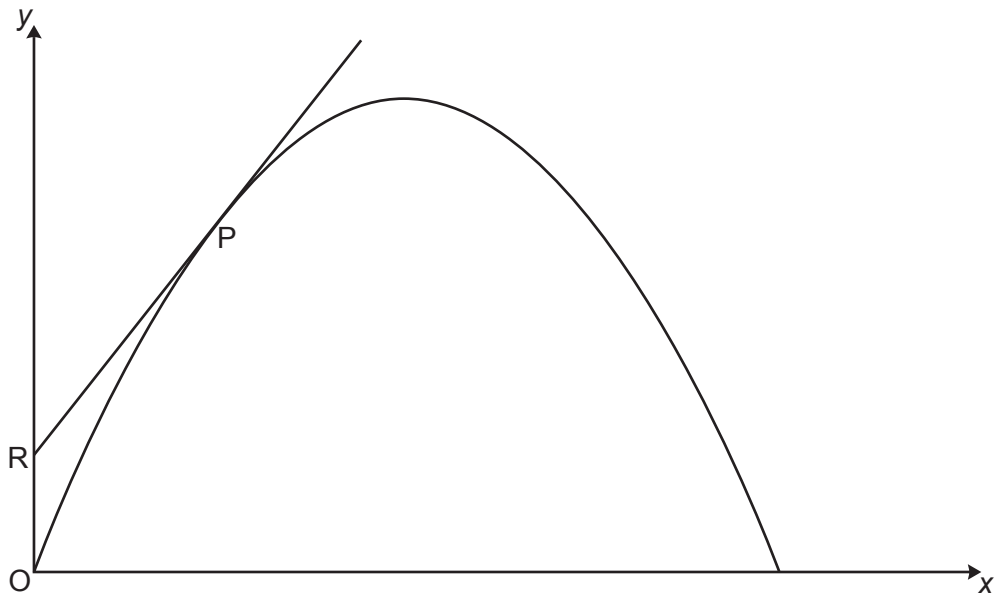
(c) Show that AD and DB are perpendicular. **[3]**

(d) Explain what this tells you about the point D. **[1]**

12(a)	
12(b)	
12(c)	
12(d)	

13 In this question you must show detailed reasoning.

The point P with coordinates (2, 12) lies on the curve $y = 8x - x^2$. The tangent to this curve at P meets the y-axis at the point R as shown in the diagram. The origin is O.



(a) Determine the coordinates of the point R. **[5]**

13(a)

(b) Determine the exact area of the region OPR that is bounded by the curve from O to P, the tangent PR and the *y*-axis. **[6]**

13(b)	

14 Sarah brings a saucepan of water to the boil. She leaves the water to cool, measuring its temperature every 10 minutes for 30 minutes.
The results are shown in the table below.

Time (t minutes)	0	10	20	30
Temperature (T °C)	100	60	40	30

Sarah believes that the temperature of the water as it cools can be modelled by the equation $T - 20 = A \times 2^{-\frac{t}{b}}$ where A and b are constants.

(a) (i) Explain the significance of the number 20 in this equation. **[1]**

(ii) Use the fact that the initial temperature of the water is 100 °C to determine the value of A. **[2]**

(b) By taking logs of both sides of Sarah's equation, show that plotting $\log_{10}(T - 20)$ against t will give a straight line. **[3]**

14(a)(i)

14(a)(ii)

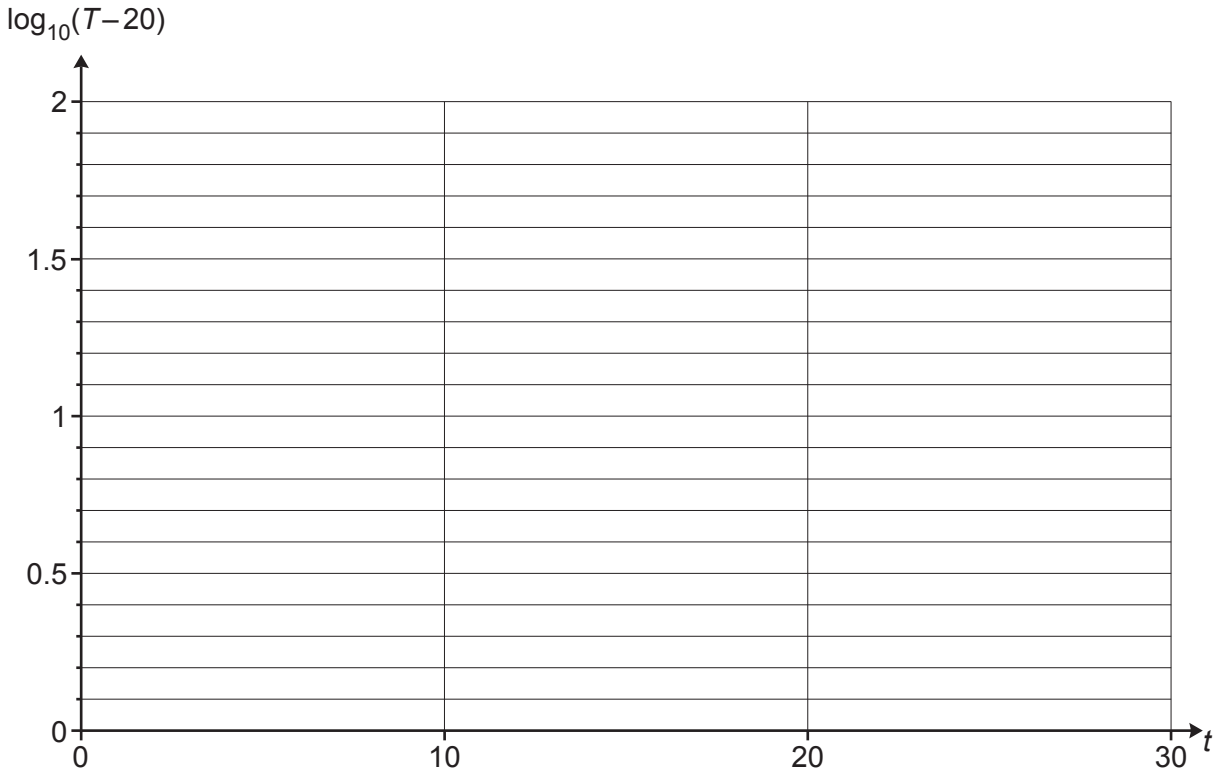
14(b)

(c) Complete the table below. **[2]**

Time (t minutes)	0	10	20	30
Temperature (T °C)	100	60	40	30
$T - 20$				
$\log_{10}(T - 20)$				

(d) Plot the values of $\log_{10}(T - 20)$ against t on the grid below. **[1]**

14(d)



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(e) Hence estimate the value of b. **[2]**

14(e)	

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15 In this question you must show detailed reasoning.

You are given that the curve $y = 2x^3 + 3x^2 - 12x + 8$ has two stationary points.

(a) (i) Show that one of the stationary points has coordinates (1, 1). **[4]**

(ii) Determine the nature of this stationary point. **[2]**

(b) Find the coordinates of the other stationary point. **[2]**

15(a)(i)	
15(a)(ii)	
15(b)	

16 I can drive my motor boat at a maximum speed of 4 kilometres per hour in still water.
One day I drive at maximum speed up a river from a point A to a point B, a distance of 9 km.
The constant speed of the current down the river is r kilometres per hour.

(a) Show that the time it takes me to drive up the river from A to B is $\frac{9}{4-r}$ hours. **[2]**

I also drive at maximum speed on my return journey down the river from B to A.

(b) Write down, in terms of r, the time it takes me to drive down the river from B to A. **[1]**

16(a)	
16(b)	

(c) Given that the difference between the time to drive up the river **(a)** and the time to drive down the river **(b)** is 1.2 hours, form an equation in r and show that it simplifies to $r^2 + 15r - 16 = 0$. **[4]**

(d) Hence find the speed of the current down the river. **[3]**

16(c)	
16(d)	

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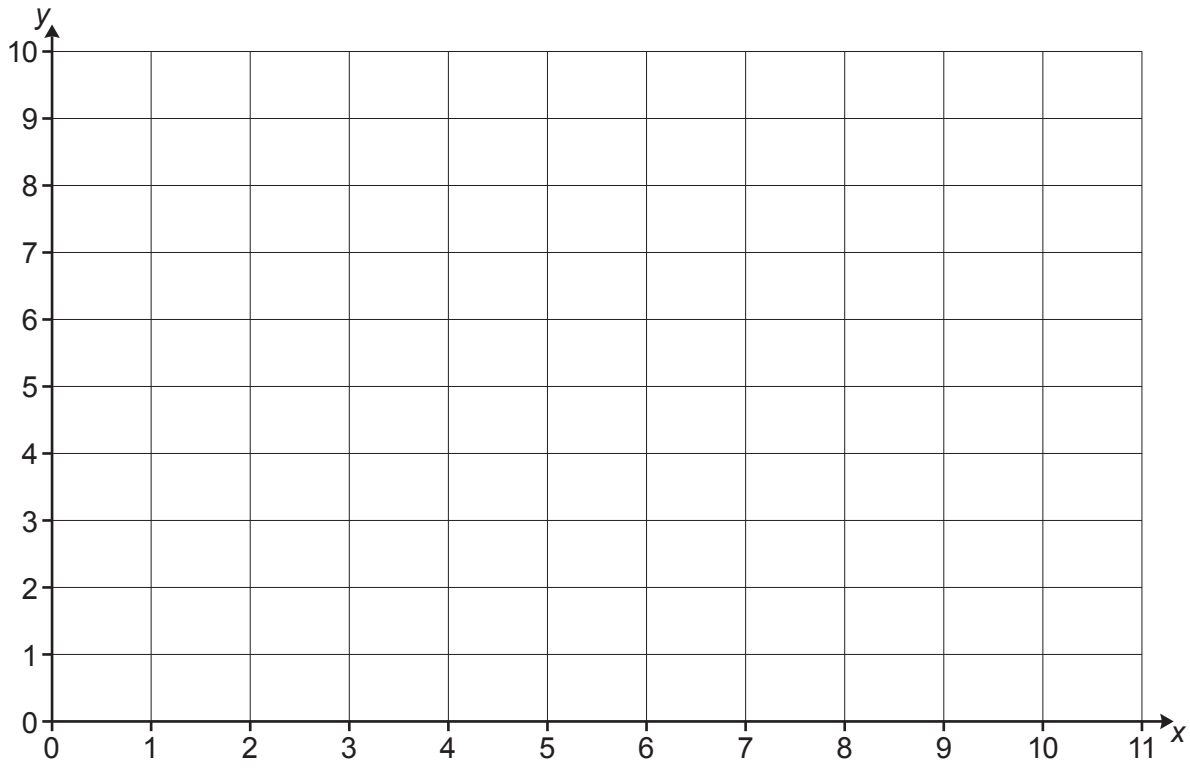
10(a)	

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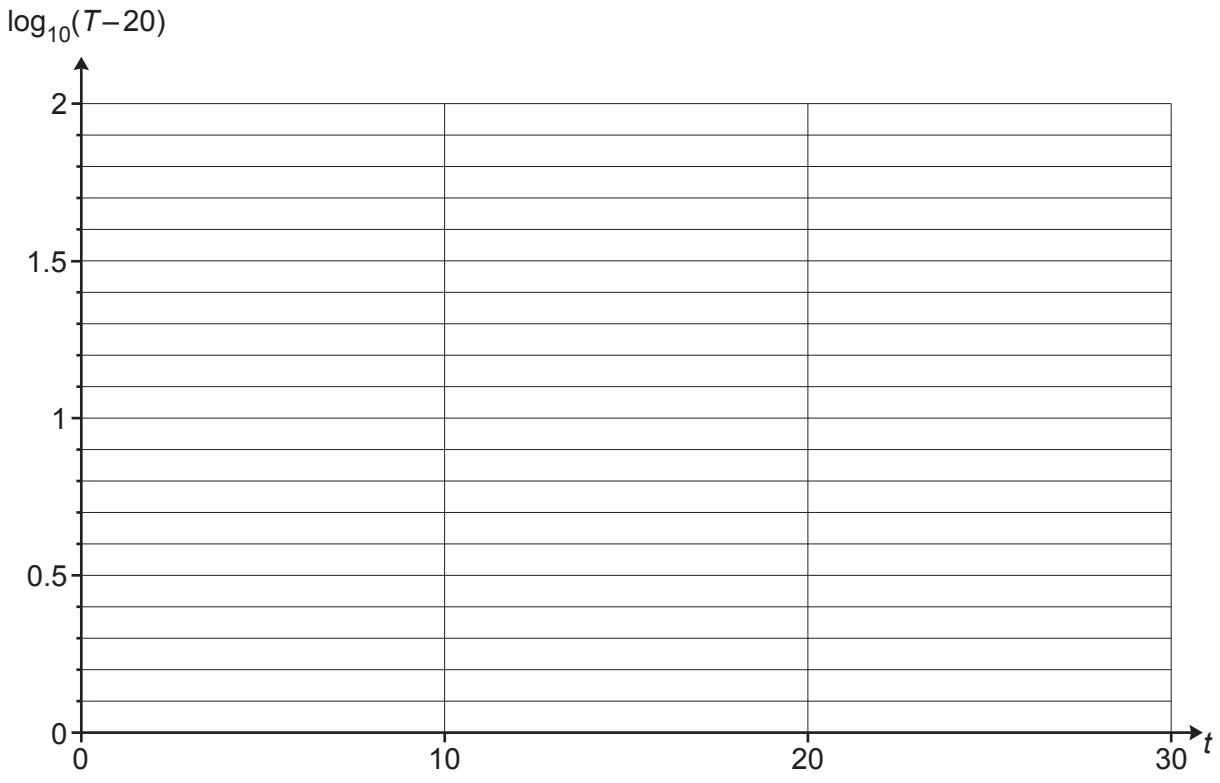
	This is a spare copy of the grid for question 14(d). Only write on this page if you want to offer a second attempt at the graph. If you do so, then you must cross through the first attempt on page 17.
14(d)	

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).