Thursday 18 January 2024 – Afternoon

Level 3 Cambridge Technical in Engineering

05823/05824/05825/05873 Unit 23: Applied mathematics for engineering

Time allowed: 2 hours
C305/2401

You must have:
- the Formula Booklet for Level 3 Cambridge Technical in Engineering (inside this document)
- a ruler (cm/mm)
- a scientific calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ____________________

Last name ____________________

Date of birth D D M M Y Y Y Y

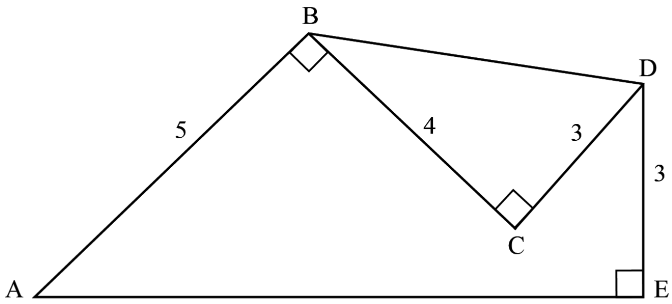
INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

INFORMATION
- The total mark for this paper is **80**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

1 **Fig. 1** shows a diagram of a roof supporting truss made from six straight, rigid members.

Fig. 1



The lengths of members AB, BC, CD and DE are 5 m, 4 m, 3 m and 3 m respectively.
Member AE is horizontal and spans the entire width of the truss. Angles ABC, BCD and AED are all right angles.

(i) Calculate the length BD.

....

.... **[1]**

(ii) Show that the angle ABD is approximately 126.9°.

....

....

....

....

.... **[3]**

(iii) By first finding the straight line distance AD, calculate the length AE.

....

....

....

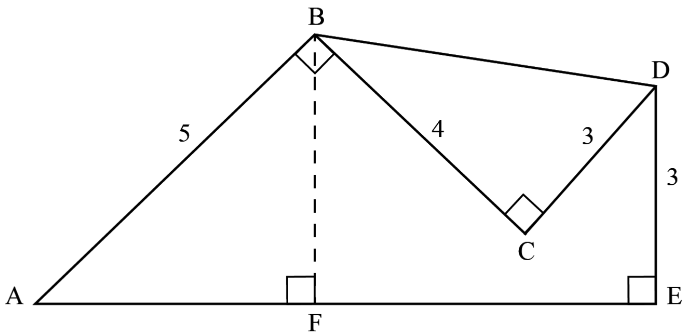
....

....

.... **[4]**

(iv) A new vertical supporting member, BF, is to be inserted into the truss as shown in **Fig. 2.**

Fig. 2



Not to scale

Calculate the length BF.

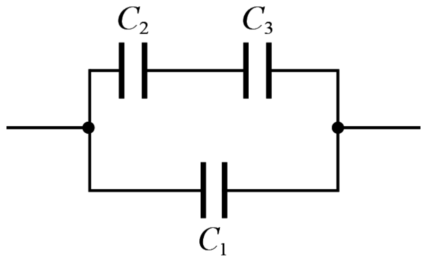
....

....

.... **[4]**

2 **Fig. 3** shows a circuit diagram containing three capacitors with capacitance values, measured in μF, of C_1, C_2 and C_3.

Fig. 3



The value C_1 is 1; the values C_2 and C_3 are to be calculated.

The total capacitance across this circuit is given by the following equation.

$$1+\frac{C_2C_3}{C_2+C_3}=3$$

(i) Show that

$$C_2=\frac{2C_3}{C_3-2}\ .$$

....

....

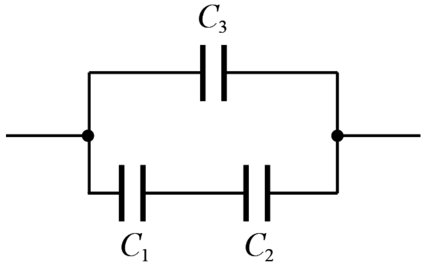
....

....

.... **[2]**

Fig. 4 shows another circuit diagram containing the same capacitors but in a different configuration.

Fig. 4



The total capacitance across this circuit is given by the following equation.

$$\frac{C_2}{C_2+1}+C_3=6.75$$

(ii) Using this equation and the result given in part **(i)** show that

$3(C_3)^2 - 20.25C_3 + 13.5 = 0.$

....

....

....

....

....

....

....

....

....

....

....

.... **[4]**

(iii) Calculate the values C_2 and C_3, given that capacitance values cannot be negative.

....

....

....

....

....

....

....

....

....

....

....

.... **[4]**

3 **Fig. 5** shows a triangle on which dotted lines have been drawn between the midpoint of each side and the opposite vertex. The point at which the dotted lines cross is called the centroid of the triangle. The coordinates of the centroid of the triangle can be found by finding the intersection of any two of these dotted lines.

Fig. 5

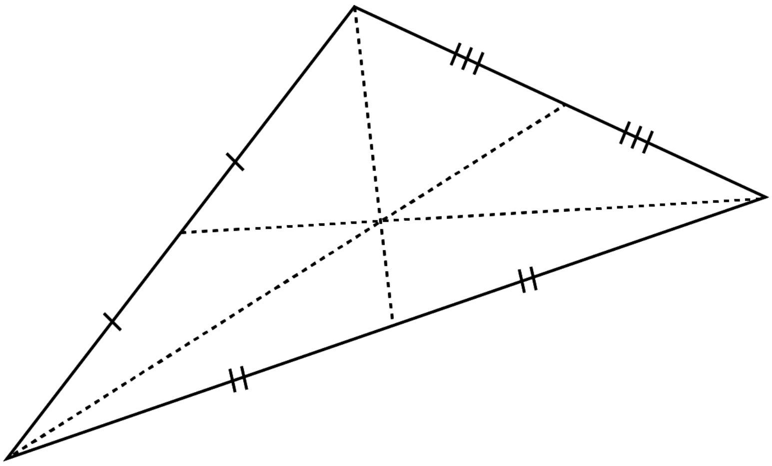
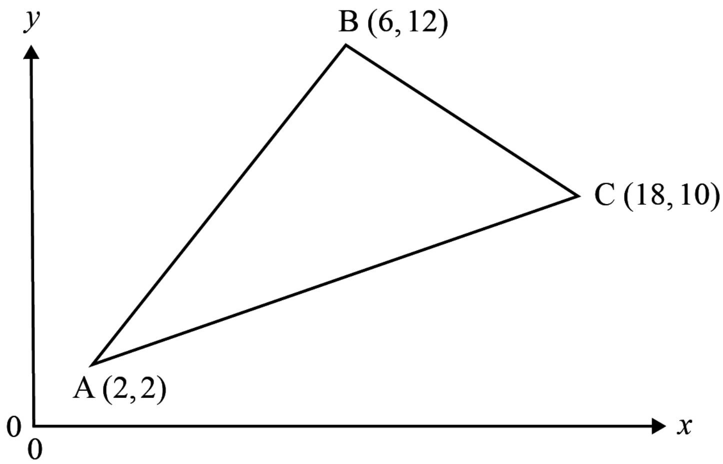


Fig. 6 shows a triangle (A, B, C) aligned within a Cartesian coordinate system (x, y). The coordinates of the vertices A, B and C are (2, 2), (6, 12) and (18, 10) respectively.

Fig. 6



Not to scale

(i) Find the coordinates of the midpoint of AB.

....

.... **[1]**

(ii) Find the coordinates of the midpoint of BC.

....

.... **[1]**

(iii) Using the coordinates of the midpoint of AB and the vertex C, show that the coordinates (x, y) of the centroid of the triangle satisfy the following equation.

$3x - 14y = -86$

....

....

....

.... **[2]**

(iv) Derive another similar equation satisfied by the coordinates of the centroid of the triangle.

....

....

....

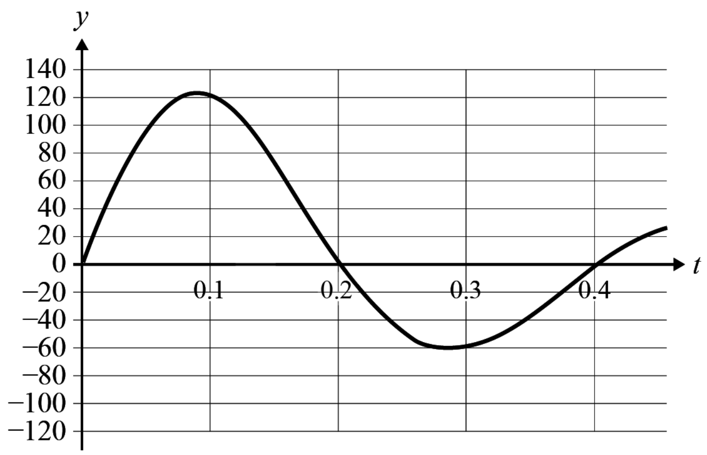
.... **[2]**

(v) Use a matrix method to find the coordinates (x, y) of the centroid.

.... **[5]**

4 The displacement, y mm, from a fixed point, at time t s, of an oscillating mass connected to a spring and damper mechanism is shown in **Fig. 7**.

Fig. 7



The value of y at time t satisfies the following formula.

$y = Ae^{-at} \sin bt$,

where A, a and b are constants.

(i) Use **Fig. 7** to calculate the frequency of oscillation of the mass. Give the units of your answer.

....

....

....

....

.... **[2]**

(ii) Calculate the value of b.

....

....

....

....

....

....

.... **[2]**

(iii) Using the formula above with $y = 120$ when $t = 0.1$ and $y = -60$ when $t = 0.3$ calculate the value of a.

.... **[4]**

(iv) Calculate the value of A.

.... **[3]**

(v) Calculate the time at which y attains its maximum value, giving your answer correct to **4** decimal places.

.... **[4]**

5 Flight controllers wish to establish whether two aircraft flying at the same altitude will remain at a safe distance apart of at least 5 km if they continue to travel at their existing speeds and in their existing directions. The displacements from a common origin, in units of 50 km, of the two aircraft at time *t* hours are given by the following position vectors.

Aircraft 1 $(1+3t)\mathbf{i}+(14t)\mathbf{j}$

Aircraft 2 $(5t)\mathbf{i}+(10-11t)\mathbf{j}$

where **i** and **j** are unit vectors associated with directions East and North respectively.

(i) Derive a formula for the distance between the two aircraft as a function of *t*.

....

....

....

....

....

....

....

....

.... **[3]**

(ii) Calculate the time at which the distance between the aircraft is a minimum. Give your answer in hours, correct to **2** decimal places.

....

....

....

....

....

....

....

....

....

.... **[3]**

(iii) By calculating the minimum distance in km that the aircraft are apart, determine whether or not the two aircraft will remain at a safe distance from each other.

....

....

....

....

....

....

....

....

.... **[3]**

6 For this question you may use the following trigonometric identity.

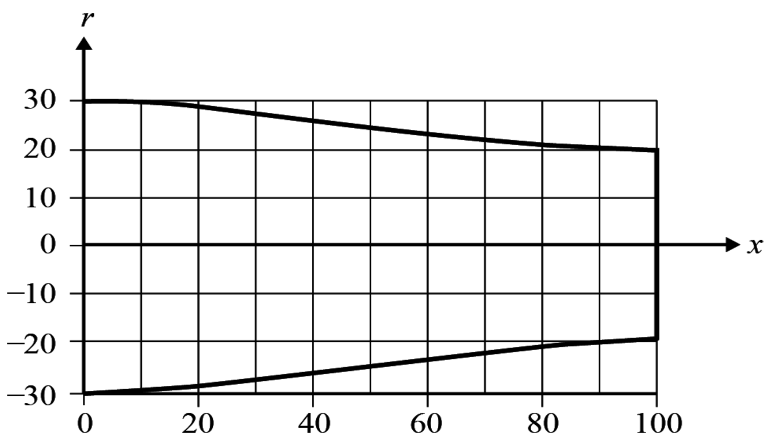
$$\cos^2 A = \frac{1}{2}(\cos 2A + 1)$$

Fig. 8 shows a section of pipe which is 100 mm long and has an internal diameter of 60 mm at one end and 40 mm at the other end.

Fig. 8

The internal shape of the section is shown in **Fig. 9** aligned within a Cartesian coordinate system (x, r) with origin (0, 0).

Fig. 9



The internal radius of the pipe, r mm, at a distance x mm from the origin is given by the following.

$$r = 5\cos\left(\frac{x\pi}{100}\right) + 25$$

The internal volume of the section of pipe shown above, V mm^3, is given by the following.

$$V = \int_0^{100} \pi r^2 \, dx$$

(i) Calculate this volume. Give your answer in cubic metres.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

.... **[8]**

(ii) The section of pipe is completely full of water which is flowing at a speed of $0.5\,m\,s^{-1}$ at the point where its diameter is 60 mm. Assuming that the volume flow rate of water remains constant throughout the section, calculate the speed of the water where the diameter is 40 mm.

....

....

....

....

.... **[3]**

7 For this question you are given the following information.

Density of water = $1000\,\text{kg}\,\text{m}^{-3}$

1 J (Joule) = 1 Ws (Watt second)

Specific heat capacity of water, $c = 4183\,\text{J}\,\text{C}^{-1}\,\text{kg}^{-1}$

(i) The heat energy, Q J, required to raise the temperature of liquid water with mass m kg and specific heat capacity $c\,\text{J}\,\text{C}^{-1}\,\text{kg}^{-1}$ by a temperature of ΔT °C is given by

$Q = mc\,\Delta T$.

A domestic cylindrical hot water tank with an internal diameter of 400 mm and an internal height of 800 mm is completely full of water.

Assuming that no heat energy is lost through the surface of the tank, calculate the time it would take a heating element in the tank producing 2.5 kW of power to raise the temperature of the water from 20 °C to 65 °C.

....

....

....

....

....

....

....

.... **[3]**

For the next part of the question you are given the following integral.

$$\int \frac{1}{a+bx}\,\mathrm{d}x = \frac{\ln|a+bx|}{b} + C$$

(ii) In practice heat is lost through the surface of the tank. While water in the tank is being heated its temperature T °C at a time t s after the heating started satisfies the following differential equation.

$$mc\frac{\mathrm{d}T}{\mathrm{d}t} = q - K(T - A),$$

where q W is the heat energy flow into the water produced by the heating element, A °C is the ambient temperature of the air surrounding the tank and $K\,\text{W}\,°\text{C}^{-1}$ is a constant.

Assuming that q and A remain constant and that $T = A$ when $t = 0$, solve the differential equation to produce a formula for T in terms of t, m, c, q, K and A.

[9]

END OF QUESTION PAPER

EXTRA ANSWER SPACE

If you need extra space use these lined pages. You must write the question numbers clearly in the margin.